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Transformations of q - hypergeometric series using mock-theta functions of order seven and ten

Ravindra Kumar Yadav and Pradhyot Kumar Mishra
Department of Mathematics
Sobhasaria Group of Institutions, NH-11, Bajajgram, Sikar
Gokulpur (Rajasthan), India
E-mail: rkyadav81@yahoo.com, mishrapk69@gmail.com

Abstract

In this paper, making use of certain known identities and mock-theta functions and partial mock-theta functions of order three and five, an attempt has been made to established transformations for basic hypergeometric series.

Keywords- Transformation, q-hypergeometric series, mock-theta funtion.

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1. Introduction, Notations and Definitions

Throughout this paper we shall adopt the following notations and definitions.
For any numbers a and q real or complex and $|q| < 1$,

$$[\alpha; q]_n = [\alpha]_n = \begin{cases} (1-\alpha)(1-\alpha q)(1-\alpha q^2)\ldots(1-\alpha q^{n-1}); & n > 0 \\ 1 & ; n = 0 \end{cases} \tag{1.1}$$

Accordingly, we have $[\alpha; q]_\infty = \prod_{r=0}^{\infty}[1-\alpha q^r]$

Also, $[a_1, a_2, a_3 \ldots a_r; q]_n = [a_1; q]_n\,[a_2; q]_n [a_3; q]_n \ldots [a_r; q]_n$

Following Gasper and Rahman [2], we define a basic hypergeometric series,

$$ {}_r\Phi_s\left[\begin{matrix} a_1, a_2, a_3 \ldots a_r; q; z \\ b_1, b_2, b_3 \ldots b_s \end{matrix}\right] = \sum_{n=0}^{\infty} \frac{[a_1, a_2, a_3 \ldots a_r; q]_n\, z^n}{[q, b_1, b_2, b_3 \ldots b_s; q]_n} \left\{(-1)^n q^{n(n-1)/2}\right\}^{1+s-r} \tag{1.2}$$

Where $0 < |q| < 1$ and $r < s+1$

In this paper we have established certain transformation formulae for basic hypergeometric functions by make use of summations of truncated series and following identity,

$$A(q)\sum_{m=0}^{\infty} B_m(q)\sum_{r=0}^{m}\alpha_r + C_\infty(q)\sum_{m=0}^{\infty}\alpha_m = \sum_{m=0}^{\infty} C_m(q)\alpha_m . \tag{1.3}$$

where,

$$A(q) = \frac{(aq-e)(e-bq)}{(q-e)(e-abq)},$$

$$B_m(q)=\frac{(a,b;q)_m\, q^m}{\left(e,\frac{abq^2}{e};q\right)_m}$$

$$C_m(q)=\frac{(a,b;q)_m}{\left(\frac{e}{q},\frac{abq}{e};q\right)_m}\qquad C_\infty(q)=\frac{(a,b;q)_\infty}{\left(\frac{e}{q},\frac{abq}{e};q\right)_\infty}$$

In this paper, we shall use the identity (1.3) in order to establish new representations of mock theta function of order seven, ten are discussed.

Mock theta functions of order seven. Partial mock theta functions of order seven.

(i)
$$\Im_o(q)=\sum_{k=o}^{\infty}\frac{q^{k^2}}{\left[q^{k+1};q\right]_k};\qquad \Im_{o,n}(q)=\sum_{k=o}^{n}\frac{q^{k^2}}{\left[q^{k+1};q\right]_k}\tag{1.4}$$

(ii)
$$\Im_1(q)=\sum_{k=o}^{\infty}\frac{q^{(k+1)^2}}{\left[q^{k+1};q\right]_{k+1}};\qquad \Im_{1,n}(q)=\sum_{k=o}^{n}\frac{q^{(k+1)^2}}{\left[q^{k+1};q\right]_{k+1}}\tag{1.5}$$

(iii)
$$\Im_2(q)=\sum_{k=o}^{\infty}\frac{q^{k^2+k}}{\left[q^{k+1};q\right]_{k+1}};\qquad \Im_{2,n}(q)=\sum_{k=o}^{n}\frac{q^{k^2+k}}{\left[q^{k+1};q\right]_{k+1}}\tag{1.6}$$

Mock theta functions of order ten. Partial mock theta functions of order ten.

(i)
$$\chi_{Lc}(q)=\sum_{k=o}^{\infty}\frac{(-1)^k q^{k^2}}{[-q;q]_{2k}};\qquad \chi_{Lc,n}(q)=\sum_{k=o}^{n}\frac{(-1)^k q^{k^2}}{[-q;q]_{2k}}\tag{1.7}$$

(ii)
$$\chi'_{Lc}(q)=\sum_{k=o}^{\infty}\frac{(-1)^k q^{(k+1)^2}}{[-q;q]_{2k+1}};\qquad \chi'_{Lc,n}(q)=\sum_{k=o}^{n}\frac{(-1)^k q^{(k+1)^2}}{[-q;q]_{2k+1}}\tag{1.8}$$

(iii)
$$\Phi_{Lc}(q)=\sum_{k=o}^{\infty}\frac{q^{k(k+1)/2}}{\left[q;q^2\right]_{k+1}};\qquad \Phi_{Lc,n}(q)=\sum_{k=o}^{n}\frac{q^{k(k+1)/2}}{\left[q;q^2\right]_{k+1}}\tag{1.9}$$

(iv)
$$\Psi_{Lc}(q)=\sum_{k=o}^{\infty}\frac{q^{(k+1)(k+2)/2}}{\left[q;q^2\right]_{k+1}};\qquad \Psi_{Lc,n}(q)=\sum_{k=o}^{n}\frac{q^{(k+1)(k+2)/2}}{\left[q;q^2\right]_{k+1}}\tag{1.10}$$

2. Main Results

In this section we shall establish new representations of mock theta functions of order seven and ten.

(i) Taking $\alpha_m=\dfrac{q^{m^2}}{\left[q^{m+1};q\right]_m}$ in (1.3) and using (1.4), we get.

$$\Im_o(q) = \frac{\left(\frac{e}{q}, \frac{abq}{e}; q\right)_\infty}{(a,b;q)_\infty}\left[\sum_{m=o}^{\infty} \frac{(a,b;q)_m}{\left(\frac{e}{q}, \frac{abq}{e}; q\right)_m} \frac{q^{m^2}}{\left[m^{m+1};q\right]_m} - \frac{(aq-e)(e-bq)}{(q-e)(e-abq)} \sum_{m=o}^{\infty} \frac{(a,b;q)_m\, q^m}{\left(e, \frac{abq^2}{e}; q\right)_m} \Im_{o,m}(q)\right] \tag{2.1}$$

(ii) Taking $\alpha_m = \dfrac{q^{(m+1)^2}}{\left[q^{m+1},q\right]_{m+1}}$ in (1.3) and using (1.5), we get.

$$\Im_1(q) = \frac{\left(\frac{e}{q}, \frac{abq}{e}; q\right)_\infty}{(a,b;q)_\infty}\left[\sum_{m=o}^{\infty} \frac{(a,b;q)_m}{\left(\frac{e}{q}, \frac{abq}{e}; q\right)_m} \frac{q^{(m+1)^2}}{\left[q^{m+1};q\right]_{m+1}} - \frac{(aq-e)(e-bq)}{(q-e)(e-abq)} \sum_{m=o}^{\infty} \frac{(a,b;q)_m\, q^m}{\left(e, \frac{abq^2}{e}; q\right)_m} \Im_{1,m}(q)\right] \tag{2.2}$$

(iii) Taking $\alpha_m = \dfrac{q^{m^2+m}}{\left[q^{m+1},q\right]_{m+1}}$ in (1.3) and using (1.6), we get.

$$\Im_2(q) = \frac{\left(\frac{e}{q}, \frac{abq}{e}; q\right)_\infty}{(a,b;q)_\infty}\left[\sum_{m=o}^{\infty} \frac{(a,b;q)_m}{\left(\frac{e}{q}, \frac{abq}{e}; q\right)_m} \frac{q^{m^2+m}}{\left[q^{m+1};q\right]_{m+1}} \right. \tag{2.3}$$

$$\left. - \frac{(aq-e)(e-bq)}{(q-e)(e-abq)} \sum_{m=o}^{\infty} \frac{(a,b;q)_m\, q^m}{\left(e, \frac{abq^2}{e}; q\right)_m} \Im_{2,m}(q)\right]$$

(iv) Taking $\alpha_m = \dfrac{(-1)^m q^{m^2}}{[-q,q]_{2m}}$ in (1.3) and using (1.7), we get.

$$\chi_{Lc}(q) = \frac{\left(\frac{e}{q}, \frac{abq}{e}; q\right)_\infty}{(a,b;q)_\infty}\left[\sum_{m=o}^{\infty} \frac{(a,b;q)_m}{\left(\frac{e}{q}, \frac{abq}{e}; q\right)_m} \frac{(-1)^m q^{m^2}}{[-q;q]_{2m}} - \frac{(aq-e)(e-bq)}{(q-e)(e-abq)} \sum_{m=o}^{\infty} \frac{(a,b;q)_m\, q^m}{\left(e, \frac{abq^2}{e}; q\right)_m} \chi_{Lc,m}(q)\right] \tag{2.4}$$

(v) Taking $\alpha_m = \dfrac{(-1)^m q^{(m+1)^2}}{[-q,q]_{2m+1}}$ in (1.3) and using (1.8), we get.

$$\chi'_{Lc}(q)=\frac{\left(\frac{e}{q},\frac{abq}{e};q\right)_\infty}{(a,b;q)_\infty}\left[\sum_{m=o}^{\infty}\frac{(a,b;q)_m}{\left(\frac{e}{q},\frac{abq}{e};q\right)_m}\frac{(-1)^m q^{(m+1)^2}}{[-q;q]_{2m+1}}\right.\tag{2.5}$$

$$\left.-\frac{(aq-e)(e-bq)}{(q-e)(e-abq)}\sum_{m=o}^{\infty}\frac{(a,b;q)_m q^m}{\left(e,\frac{abq^2}{e};q\right)_m}\chi'_{Lc,m}(q)\right]$$

(vi) Taking $\alpha_m=\frac{q^{m(m+1)/2}}{\left[q,q^2\right]_{m+1}}$ in (1.3) and using (1.9), we get.

$$\Phi_{Lc}(q)=\frac{\left(\frac{e}{q},\frac{abq}{e};q\right)_\infty}{(a,b;q)_\infty}\left[\sum_{m=o}^{\infty}\frac{(a,b;q)_m}{\left(\frac{e}{q},\frac{abq}{e};q\right)_m}\frac{q^{m(m+1)/2}}{\left[q;q^2\right]_{m+1}}\right.\tag{2.6}$$

$$\left.-\frac{(aq-e)(e-bq)}{(q-e)(e-abq)}\sum_{m=o}^{\infty}\frac{(a,b;q)_m q^m}{\left(e,\frac{abq^2}{e};q\right)_m}\Phi_{Lc,m}(q)\right]$$

(vii) Taking $\alpha_m=\frac{q^{(m+1)(m+2)/2}}{\left[q;q^2\right]_{m+1}}$ in (1.3) and using (1.10), we get.

$$\Psi_{Lc}(q)=\frac{\left(\frac{e}{q},\frac{abq}{e};q\right)_\infty}{(a,b;q)_\infty}\left[\sum_{m=o}^{\infty}\frac{(a,b;q)_m}{\left(\frac{e}{q},\frac{abq}{e};q\right)_m}\frac{q^{(m+1)(m+2)/2}}{\left[q;q^2\right]_{m+1}}\right.\tag{2.7}$$

$$\left.-\frac{(aq-e)(e-bq)}{(q-e)(e-abq)}\sum_{m=o}^{\infty}\frac{(a,b;q)_m q^m}{\left(e,\frac{abq^2}{e};q\right)_m}\Psi_{Lc,m}(q)\right]$$

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