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The effect of external influences in innovation diffusion models

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Abstract

The aim of this paper is to describe mathematical model which represents the innovation of technology and gives decision about the launching of a new product in the market, making right kind of investment in apropriate right time, pricing the product as it influences the number of potential adopters. This study is very important in changing technological scenario.

Keywords-Innovation diffusion, Fisher-Pry model, Bass model, External influence, internal influence.

1. Introduction

An industry manufactures a new technological product or develops a new procuct like cell phone, personal computers, colour TV, a new steel making process, a dish washer, a micro-oven, a vacuum cleaner, a new type of car and wants to know at what rate the new process or product can penetrate the market so that it can regulate the production strategy accordingly or so that the government can license to produce it in order to satisfy the future demands. The theory of innovation diffusion is concerned with the demand for an innovation by relevant in the following words: In the context of the developing countries where most of the technological innovation have been transferred from the developed countries, they can be perceived as new and can be treated as an innovation for the purpose of diffusion in the recipient society. The extent to a new idea or product spread is a measure of diffusion. The diffusion itself is referred, more conventionally to the process by which an innovation is communicated to possible adopters. The diffusion of the technology can also be viewed as an evolutionary process of replacement of an old technology by newer one for accomplishing similar objectives or doing similar jobs. In this paper attempt has been made to explain the process of technological change and diffusion with the help of mathematical model and the procedure for estimating their parameters.

2.Innovation diffusion models

We have to identify the variables and parameters of innovation diffusion model. The number of adopters of new innovation (t) till time t is a variable and the total number of potential adopter N is parameter. We assume that the innovation spreads by word of mouth that is communication of information about the product from those who have adopted it to those who have not yet adopted.

At any time t, n(t) is number of adopters and N -n(t) is non-adopters. If the number of successful adopters who can communicate the new innovation in efficient manner, is large, the greater will be the manner who can possibly adopt it and larger will be the rate of change n (t) so that we assume (Kapur, 1992)

$$\frac{\mathbf{dn}}{\mathbf{dt}} = \mathbf{cn(t)[N - n(t)]} \qquad (1)$$

c is constant. This gives the first mathematical model.

Assuming $\frac{\boldsymbol{n(t)}}{\boldsymbol{N}} = \boldsymbol{f(t)}$

where f (t) is the fraction of potential adopters who have adopted the technology by time t, we have

$$\frac{\boldsymbol{df}}{\boldsymbol{dt}} = \boldsymbol{c'f(1-f)} \qquad (2)$$

We investigate this model.

Since $\boldsymbol{n(t) \leq N(t),\ f(t) \leq, \frac{df}{dt} \leq 0}$

$$\frac{\boldsymbol{d^2f}}{\boldsymbol{dt^2}} > \boldsymbol{c'(1-2f)\frac{df}{dt}} \qquad (3)$$

$$\frac{d^2f}{dt^2} > 0 \ \ for \ \ f < \frac{1}{2}$$

$$\frac{d^2f}{dt^2} = 0 \quad for \quad f = \frac{1}{2}$$

$$\frac{d^2f}{dt^2} < 0 \quad for \quad f > \frac{1}{2}$$

$\frac{df}{dt}$ increases when $f < \frac{1}{2}$

$\frac{df}{dt}$ decreases when $f > \frac{1}{2}$

in other words f (t) or n (t) increases at increasing rate when n (t)< N/2 and it increases but at decreasing rate when n(t)> N/2 and there is a point of inflection when f(t)=1/2 or n(t) =N/2. At point of inflexion at which almost half of the potential adopter have adopted the innovation but the number n(t) continuous increase with decresing rate this continuous till almost all the potential adopters adopt the innovation.

From (2), we get $\frac{\boldsymbol{f}}{\boldsymbol{1-f}} = \frac{\boldsymbol{f_0}}{\boldsymbol{1-f_0}}\boldsymbol{e^{c't}}$.

Thus f (t)$\leq$1, and f (t) $\to$ 1, when t$\to \infty$ or n(t)$\to$ N, when t $\to \infty$.

That is innovation has to wait very very long time till all the potential adopter adopt it. In the fast growing technological age adopter has always possibilities to adopt new innovation hence old one will not adopted by all potential adopters.

The model (2) is called Fisher-Pry model and very successful model in the study of innovation diffusion.

3.Influence of diffusion models

The innovation spread by words of mouth but they can also spread by external publicity by T.V, radio, newspaper etc. this influence does not depend on the number of adopters but is directly proportional to the number of non-adopters.The growth of a new durable product (innovation) due to the diffusion effect, Bass (1969) used the following differential equation with constant coefficients as follows

$$\frac{dn}{dt} = p\big(N - n(t)\big) + \frac{qn(t)}{N}\big(N - n(t)\big) \tag{4}$$

$$m(t) = \frac{dn}{dt}$$

$$m(t) = p\big(N - n(t)\big) + \frac{qn(t)}{N}\big(N - n(t)\big)$$

where n (t) and m(t) are cumulative and non-cumulative number of adopters of a new product at time t

- p[N-n(t)] represents the adoption due to external-influence
- $q\frac{n(t)}{N}[N - n(t)]$ represents the adoption due to external-influence

The constant p is defined as the coefficient of innovation or external influence, emanating from the outside of a social system (Coleman *et al.,* 1966), Robertson (1971). It depends directly on mass media communication regarding innovation, formulated by market agents, government agencies etc, and aimed at potential users of imitation. The constant q is defined as the coefficient of imitation, reflects the interactions of prior adopters with potential adopters. Therefore, the decision by potential users to adopt an innovation depends directly on the information formulated by existing users.

Assuming, where f (t) = $\frac{n(t)}{N}$ is the fraction of potential adopters who love adopted the technology by time t. The model (4) can be restated as:

$$\frac{df}{dt} = p[1 - f(t) + \ qf(t)[1 - f(t)] \tag{5}$$

is the solution of (4)

$$n(t) = \frac{\frac{N - p(N - n_0)}{p + \frac{q}{N}n_0} e^{-(p+q)t}}{1 + \frac{\frac{q}{N}(N - n_0)}{p + \frac{q}{N}n_0} e^{-(p+q)t}} \tag{6}$$

n (t) depends on the parameters N, p & q, we shall write equation (6) as follows

$$n(t,N,p,q) = \frac{\frac{N-p(N-n_0)}{p+\frac{q}{N}n_0}e^{-(p+q)t}}{1+\frac{\frac{q}{N}(N-n_0)}{p+\frac{q}{N}n_0}e^{-(p+q)t}} \quad , \quad \text{where } n_0=n(0)$$

For the diffusion of innovation (6), the point of inflection occurs when,

$$N(t^*) = N\left(\frac{1}{2}-\frac{p}{2q}\right)$$

$$t^* = -\frac{1}{p+q}\log(p/q)$$

$$m(t^*) = N\left(\frac{q}{4}+\frac{p}{2}+\frac{1}{4}\frac{p^2}{q}\right)$$

In this case the point of inflection occurs before half the final population size is reached. This model is not sufficiently flexible since it cannot represent those situations in which the point of inflection occurs after half the population size is reached.

4. Estimating the parameters of diffusion models

The ordinary least squares procedure suggested by bass (1969) is one of the earliest procedures for estimating the parameters. This procedure involves estimation of the parameters by describing the following differential equation as follows

$$\frac{df}{dt} = p(1-f(t)) + qf(t)(1-f(t))$$

or

$$\frac{df}{dt} = (p + qf)\,(1\text{-}f) = p+ (q-p)f - qf^2$$

If F (t) is the probability density function of random variable t, the adoption time of new product is given by

$$F(t) = \frac{df}{dt}$$

$$F(t) = p+(q-p)f - qf^2$$

Let x (t) and y(t) denote the rate of adoption of aninnovation cumulative rate of adoption of the new product respectively at time t. Assuming that the product rate adoption are

$$x(t) = F(t)$$

$$x(t) = pN + (q-p)Nf - qNf^2$$

$$= pN + (q-p)Nf - \frac{q}{N}(Nf^2)$$

Estimation of this model involves a quadrate regression

$$x(t) = a + bY(t) + cY^2(t)$$

The parameters p, q and N are estimated from the regression coefficient using the relationships.

$$a = pN,\ b = q-p,\ c = -q/N$$

Given regression coefficients a, b, c, the estimators of the parameters p, q, N can be easily obtained as follows:

$$\bar{p} = -\frac{-\bar{b} \pm \sqrt{\overline{(b)}^2 - 4\bar{a}\bar{c}}}{2}$$

$$\bar{q} = -\frac{-\bar{b} \pm \sqrt{\overline{(b)}^2 - 4\bar{a}\bar{c}}}{2}$$

$$\bar{N} = -\frac{-\bar{b} \pm \sqrt{\overline{(b)}^2 - 4\bar{a}\bar{c}}}{2c}$$

The ordinary least squares procedure is easy to implement, but it has three short comings.

First, when only a few data points are available, due to the likely multi colinearity between regression may obtain parameter estimates that the unstable or possess wrong signs. Second factors for the standard errors of the estimates are not available. Third a time intervals bias is present, since discrete time-series data are used to estimate a continuous time model.

5. Conclusion

We have studied the models of innovatin diffusion starting from simple case and taken into account internal and external influences which are coming from real life situation. In Mathematical form these models are differential equation. The parameters of the models were estimated by ordinary least squares method. If we take many parameters at a time model becomes complex and the object of modeling is not fulfilled. The ultimate object of mathematical modeling is not the display of mathematical power but obtaining the insight and understanding. These models and such type of studies help in preocess of taking decision about the launching of a new product in the market, making right kind of investment in the appropriate technology at right time, pricing the product as it influences the number of potential adopters.

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