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Analytical solution of the Darcy equation in parabolic, bi-polar cylindrical and parabolic cylindrical coordinates

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Abstract

The analytical solution (stream function) of the Darcy equation in specific orthogonal curvilinear coordinates, including parabolic coordinates, parabolic cylindrical coordinates, and bi-polar cylindrical coordinates, is the subject of the current research. Stream function solution of $(\nabla^2 - \alpha^2)\psi(u, v) = 0$, are also obtained analytically in parabolic co-ordinates and parabolic cylindrical co-ordinates. The resulting analytical expressions of the stream function solution are a suitable combination of trigonometric, exponential, Modified Bessel, Parabolic Cylinder, Whittaker, and Laguerre functions.

Key words- Porous media, Orthogonal curvilinear coordinates, Modified Bessel functions, Whittaker function, Laguerre function

MSC (2020): 76A05, 76S05, 76D05, 35G05, 35C05

Nomenclature

Symbols	Description
p	Fluid pressure
k	Permeability
μ	Dynamic viscosity
ψ	Stream function
$\boldsymbol{q}$	Fluid velocity vector
∇	Gradient operator
∇^2	Laplacian operator
(h_1, h_2, h_3)	Scale factors
$(\hat{e}_1, \hat{e}_2, \hat{e}_3)$	Unit vectors
(x, y, z)	Cartesian coordinates
(u, v, ϕ)	Parabolic coordinates
(θ, ϕ, z)	Bi-polar cylindrical coordinates
(ξ, η, z)	Parabolic cylindrical coordinates
(u_1, u_2, u_3)	Orthogonal curvilinear coordinates
$D_v(\xi)$	Parabolic cylinder function
$W_{r,s}(\xi)$	Whittaker function
$L_n^{\alpha}(x)$	Laguerre polynomial

$I_1(.)$	Modified Bessel function of first order of first kind
$K_1(.)$	Modified Bessel function of first order of second kind
$U(a, b, x)$	Confluent hypergeometric function of first kind

1. Introduction

Let $\boldsymbol{q}$ be the average fluid velocity over volume element consisting of the fluid and porous material, named as seepage or filtration velocity, given in the book of Nield and Bejan (2006). Assuming that$p$is fluid velocity,$\mu$is dynamic coefficient of viscosity and $k$is permeability of porous medium. The seepage velocity of fluid flow through porous medium is proportional to the driving pressure gradient commonly known as Darcy law and the mathematical form of Darcy's law is expressed by Darcy equation

$$\nabla p = -\frac{\mu}{k}\boldsymbol{q}. \tag{1.1}$$

Joseph et al. (1982) proposed that the generalization of Darcy's equation, whenever the inertial effects are included. Deo and Tiwari (2008) obtained the analytical solution of the partial differential equation $E^2\psi = 0$ in bispherical polar coordinates which is arising in the irrotational fluid flow. Khuri and Wazwaz (1996) studied the irrotational fluid motion partial differential equation $E^2\psi = 0$ is investigated in the toroidal coordinates. Dassios et al. (1994) investigated the Stokes equation in spheroidal coordinates by using the semi-separable nature of spheroidal coordinates.

Brinkman (1947) formulated the generalized version of Darcy equation, named as Brinkman equation which is the governing equation of fluid flow through porous medium. In the cylindrical polar coordinates, Deo and Maurya (2019) presented the stream function solution the Brinkman equation in the generalized form which is containing the hypergeometric, trigonometric, and modified Bessel functions. Expressions for velocity and acceleration in the parabolic cylindrical coordinates are reported earlier by Omonile et al. (2015). Zaytoon *et al.* (2016) reported the solution of Weber's differential equation for both initial value problems and boundary value problems. Stream function solution of the Brinkman equation and the Stokes equation in the parabolic cylindrical coordinates, obtained by Maurya and Deo (2020). Maurya *et al.* (2021) investigated the Stokes flow of micropolar fluid (*i.e.,* non-Newtonian) through porous cylinder for two types of boundary value problems and presented a comparison for both BVPs graphically. Deo *et al.* (2021) reported the influence of external and uniform magnetic field on hydrodynamic permeability of biporous membrane relative to the flow of micropolar fluid using cell models. Deo and Maurya (2022) investigated the MHD impacts on micropolar–Newtonian fluid flow through composite porous channel and reported the numerical value of flow rate, wall shear stresses and couple stresses at respective porous interfaces. Deo *et al.* (2020) reported the result about Stokesian flow of a non-Newtonian liquid in a cylindrical pipe enclosing a solid/impermeable core coated with porous layer in the presence of magnetic field. Maurya and Deo (2022) investigated the effectiveness of the magnetic field on Newtonian fluid sandwiched between two porous cylindrical pipes which are filled with micropolar liquids.

2. Formulation of Problem

Our goal is to solve the Darcy equation (1.1) using the general stream function in a specific orthogonal coordinate system. The estimated coordinates are:

- Parabolic coordinates(u, v, ϕ)
- Parabolic cylindrical coordinates(θ, ϕ, z)
- Bi-polar cylindrical coordinates(ξ, η, z)

3. Solution in Parabolic Coordinates$(\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{\phi})$

Transformation equations between 3-dimensional Cartesian coordinates(x, y, z) and 3-dimensional Parabolic coordinates(u, v, ϕ) are:

$$x = uv\cos\phi\,, y = uv\sin\phi\,, z = \frac{1}{2}(v^2 - u^2), \tag{3.1}$$

where$0 \le u < \infty, 0 \le v < \infty$ and $0 \le \phi \le 2\pi$.

In the parabolic coordinates, scale factors are:

$$h_1 = \sqrt{u^2 + v^2}, h_2 = \sqrt{u^2 + v^2}, h_3 = uv. \tag{3.2}$$

Defining a differential operator E^2 in the orthogonal curvilinear coordinates (u_1, u_2, u_3):

$$E^2 = \frac{h_3}{h_1 h_2}\left[\frac{\partial}{\partial u_1}\left(\frac{h_2}{h_3 h_1}\frac{\partial}{\partial u_1}\right) + \frac{\partial}{\partial u_2}\left(\frac{h_1}{h_2 h_3}\frac{\partial}{\partial u_2}\right)\right]. \tag{3.3}$$

Fluid motion is only conceivable when the conservation of mass principle is true, or when the equation of continuity is satisfied. For incompressible fluid,

$$\nabla \cdot \boldsymbol{q} = 0. \tag{3.4}$$

In two-dimensional fluid motion, fluid velocity vector $\boldsymbol{q} = (q_1(u, v), q_2(u, v), 0)$ can be assumed. Introducing the scalar valued function $\psi(u, v)$ is such a way that equation of continuity is automatically satisfied. For this, we may choose

$$q_1 = -\frac{1}{uv\sqrt{u^2+v^2}}\left(\frac{\partial\psi}{\partial v}\right) \text{and } q_2 = \frac{1}{uv\sqrt{u^2+v^2}}\left(\frac{\partial\psi}{\partial u}\right). \tag{3.5}$$

By the 2nd order vector identity of the form

$$\nabla \times (\nabla p) = 0. \tag{3.6}$$

Applying the curl operator on equation (1.1), we get

$$\nabla \times \boldsymbol{q} = \boldsymbol{0}, \Rightarrow E^2\psi = 0,$$

where, differential operator $E^2$is given by

$$E^2 = \frac{uv}{u^2+v^2}\left[\frac{\partial}{\partial u}\left(\frac{1}{uv}\frac{\partial\psi}{\partial u}\right) + \frac{\partial}{\partial v}\left(\frac{1}{uv}\frac{\partial\psi}{\partial v}\right)\right].$$

Therefore, we have

$$\frac{\partial^2\psi}{\partial u^2} + \frac{\partial^2\psi}{\partial v^2} - \frac{1}{u}\frac{\partial\psi}{\partial u} - \frac{1}{v}\frac{\partial\psi}{\partial v} = 0. \tag{3.7}$$

4. Solution of Problem (3.7)

In this section, we wish to obtain the solution of partial differential equation (3.7) with the help of method of separation. For this, we can define the stream function ψ as follows:

$$\psi(u, v) = uv\xi(u)\varsigma(v). \tag{4.1}$$

Substituting the value of equation (4.1) in the equation (3.7), we get the following differential equations:

$$u^2\xi''(u) + u\xi'(u) - (1 + (nu)^2)\xi(u) = 0, \tag{4.2}$$

and $v^2\varsigma''(u) + v\varsigma'(v) - (1-(nv)^2)\varsigma(v) = 0.$ (4.3)

Equation (4.2) is a special type of modified Bessel differential equation whose two linearly independent solutions are $I_1(nu)$ and $K_1(nu)$. On the other hand, equation (4.3) is a Bessel differential equation whose two solutions (linearly independent) are $J_1(nv)$ and $Y_1(nv)$. Hence, the analytical solution of the Darcy equation (1.1) will be

$$\psi(u,v) = \sum_{n=0}^{\infty}[A_n I_1(nu) + B_n K_1(nu)]_{Y_1(nv)}^{J_1(nv)}, \quad (4.4)$$

where, A_n's and B_n's are arbitrary parameters.

5. Solution of $(\nabla^2 - \alpha^2)\psi(u,v) = 0$ in Parabolic Coordinates

The Laplacian operator ∇^2 in parabolic coordinates (u, v, ϕ) is

$$\nabla^2 = \frac{1}{uv(u^2+v^2)}\left[\frac{\partial}{\partial u}\left(uv\frac{\partial}{\partial u}\right) + \frac{\partial}{\partial v}\left(uv\frac{\partial}{\partial v}\right) + \frac{\partial}{\partial \phi}\left(\frac{u^2+v^2}{uv}\frac{\partial}{\partial \phi}\right)\right]. \quad (5.1)$$

For axi-symmetric flow,

$$\frac{\partial \boldsymbol{q}}{\partial \phi} = 0.$$

Therefore, equation $(\nabla^2 - \alpha^2)\psi(u,v) = 0$, will imply that

$$\frac{\partial^2\psi}{\partial u^2} + \frac{1}{u}\frac{\partial\psi}{\partial u} + \frac{\partial^2\psi}{\partial u^2} + \frac{1}{v}\frac{\partial\psi}{\partial v} - \alpha^2(u^2+v^2)\psi = 0. \quad (5.2)$$

It is a partial differential equation of order 2 with variable coefficients. With the help of method of separation of variables $\psi(u,v) = U(u)V(v)$, it can be reduced into two ordinary differential equations with variable coefficients of the form:

$$u^2U''(u) + uU'(u) - u^2(\alpha^2u^2 + n^2)U(u) = 0 \quad (5.3)$$

and $$v^2V''(v) + vV'(v) - v^2(\alpha^2v^2 - n^2)V(v) = 0. \quad (5.4)$$

The equations (5.3) and (5.4) are special cases of Laguerre differential equation. Two linearly independent solutions of these equations are Laguerre polynomial $L_n^\alpha(x)$ and confluent hypergeometric function of first kind $U(a,b,x)$. Therefore, the general solution of differential equations (5.3) and (5.4) will be

$$U(u) = \left[C_1U\left(\frac{n^2}{4l}+\frac{1}{2}, 1, lu^2\right) + C_2L^0_{-\left(\frac{n^2}{4l}+\frac{1}{2}\right)}(lu^2)\right]exp\left(-\frac{lu^2}{2}\right), \quad (5.5)$$

$$V(v) = \left[C_3U\left(-\frac{n^2}{4l}+\frac{1}{2}, 1, lv^2\right) + C_4L^0_{-\left(-\frac{n^2}{4l}+\frac{1}{2}\right)}(lu^2)\right]exp\left(-\frac{lv^2}{2}\right), \quad (5.6)$$

where $C_i, i = 1,2,3,4$ are arbitrary parameters.

Hence, stream function solution of $(\nabla^2 - \alpha^2)\psi(u,v) = 0$ in parabolic coordinates (u, v, ϕ) will be

$$\psi(u,v) = \sum_n\left[P_nU\left(\frac{n^2}{4l}+\frac{1}{2}, 1, lu^2\right) + Q_nL^0_{-\left(\frac{n^2}{4l}+\frac{1}{2}\right)}(lu^2)\right]$$
$$S_nU\left(-\frac{n^2}{4l}+\frac{1}{2}, 1, lu^2\right) + S_nL^0_{-\left(\frac{-n^2}{4l}+\frac{1}{2}\right)}(lu^2)]\, exp\left(-\frac{l(u^2+v^2)}{2}\right), \quad (5.7)$$

whereP_n, Q_n, R_n and S_n are arbitrary parameters.

6. Analytical Solution in Parabolic Cylindrical Coordinates (u, v, z)

The equations connecting the 3-dimensional Cartesian coordinates (x, y, z) and parabolic cylindrical coordinates(u, v, z) are given by

$$x = c\,(u^2 - v^2), y = 2cuv, z = z, \tag{6.1}$$

where $-\infty < u < \infty, 0 \leq v < \infty, -\infty < z < \infty$ and $c > 0$.

In this case, scale factors are

$$h_1 = 2c\sqrt{u^2 + v^2}, h_2 = 2c\sqrt{u^2 + v^2}, h_3 = 1.$$

Assuming that fluid velocity vector is $\boldsymbol{q} = (q_1(u, v), q_2(u, v), 0)$. Introducing the scalar valued function $\psi(u, v)$ in such a way that equation of continuity is automatically satisfied. For this, we may choose

$$q_1 = -\frac{1}{2c\sqrt{u^2+v^2}}\left(\frac{\partial \psi}{\partial v}\right) \text{and } q_2 = \frac{1}{2c\sqrt{u^2+v^2}}\left(\frac{\partial \psi}{\partial u}\right). \tag{6.2}$$

Taking the curl on both sides on Darcy equation (1.1), then using 2nd order vector identity (3.6), we get

$$E^2\psi(u, v) = 0, \tag{6.3}$$

where, $E^2 = \frac{uv}{u^2+v^2}\left[\frac{\partial}{\partial u}\left(\frac{1}{uv}\frac{\partial \psi}{\partial u}\right) + \frac{\partial}{\partial v}\left(\frac{1}{uv}\frac{\partial \psi}{\partial v}\right)\right]$.

Therefore, on simplification, we will obtain that stream function is satisfying the two-dimensional Laplace equation in variables *u* and *v* of the form:

$$\frac{\partial^2 \psi}{\partial u^2} + \frac{\partial^2 \psi}{\partial v^2} = 0. \tag{6.4}$$

7. Solution of Problem (6.4)

To obtain the solution of partial differential equation (6.4) with the help of method of separation, we may assume

$$\psi(u, v) = U(u)V(v). (7.1)$$

Substituting the value of ψ from the equation (7.1) in the Laplace equation (6.4), we will get

$$\frac{1}{U}\frac{d^2U}{du^2} = -\frac{1}{U}\frac{d^2V}{dv^2} = -n^2. (7.2)$$

Therefore, we will get a pair of differential equations as follows:

$$\frac{d^2U}{du^2} - n^2U = 0, \tag{7.3}$$

and $\frac{d^2V}{du^2} + n^2V = 0. (7.4)$

So, analytical solution of differential equations (7.3)-(7.4) are:

$$U(u) = A_1\, cosh(nu) + B_1\, sinh(nu), \tag{7.5}$$

and $V(v) = A_2 \cos(nv) + B_2 \sin(nv).$ (7.6)

Therefore, general stream function solution of Darcy equation (1.1) can be expressed as:

$$\psi(u,v) = \sum_{n=0}^{\infty}[A_n \cosh(nu) + B_n \sinh(nu)]_{\cos(nv)}^{\sin(nv)}, \tag{7.7}$$

where, A_n's and B_n's are arbitrary parameters.

8. Solution of $(\nabla^2 - \alpha^2)\psi(u,v) = 0$ in Parabolic Cylindrical Coordinates

The Laplacian operator (∇^2) in parabolic cylindrical coordinates (u, v, z) is

$$\nabla^2 = \frac{1}{4c^2(u^2+v^2)}\left[\frac{\partial^2}{\partial u^2} + \frac{\partial^2}{\partial v^2} + 4c^2(u^2+v^2)\frac{\partial^2}{\partial z^2}\right].$$

Therefore, mathematical equation $(\nabla^2 - \alpha^2)\psi(u,v) = 0$ can be expressed as:

$$\frac{\partial^2\psi}{\partial u^2} + \frac{\partial^2\psi}{\partial v^2} - \beta^2(u^2+v^2)\psi = 0, \tag{8.1}$$

where $\beta^2 = 4c^2\alpha^2$.

For analytical solution of the equation (8.1), we will apply the technique of separation of variable by choosing $\psi(u,v)$ of the form:

$$\psi(u,v) = U(u)V(v). \tag{8.2}$$

Substituting the value of $\psi(u,v)$ from equation (8.2) in the differential equation (8.1), we will get a pair of differential equations

$$\frac{d^2U}{du^2} - (\beta^2u^2 + n)U = 0, \tag{8.3}$$

and $\frac{d^2V}{dv^2} - (\beta^2v^2 - n)V = 0.$ (8.4)

Above differential equations are particular cases of Weber differential equations, written from a classical book by Murphy (1969). In which, two linearly independent solutions of differential equation (8.3) are parabolic cylinder functions $D_{\frac{-n-\beta}{2\beta}}(u\sqrt{2\beta})$ and $D_{\frac{n-\beta}{2\beta}}(iu\sqrt{2\beta})$. To get real solutions, we shall transform the parabolic cylinder function $D_\eta(\xi)$ in the Whittaker function $W_{r,s}(\xi)$ by using the following relation

$$D_\eta(\xi) = 2^{\frac{\eta}{2}+\frac{1}{4}}\xi^{-\frac{1}{2}}W_{\frac{\eta}{2}+\frac{1}{4},-\frac{1}{4}}\left(\frac{\xi^2}{2}\right). \tag{8.5}$$

Hence, analytical solution of the equation $(\nabla^2 - \alpha^2)\psi(u,v) = 0$ comes out as:

$$\psi(u,v) = \sum_{n=0}^{\infty}\left(P_n u^{-\frac{1}{2}}W_{-\frac{n}{4\beta},-\frac{1}{4}}(u^2\beta) + Q_n u^{-\frac{1}{2}}W_{\frac{n}{4\beta},-\frac{1}{4}}(-u^2\beta)\right)$$
$$\left(R_n v^{-\frac{1}{2}}W_{\frac{n}{4\beta},-\frac{1}{4}}(v^2\beta) + S_n v^{-\frac{1}{2}}W_{-\frac{n}{4\beta},-\frac{1}{4}}(-v^2\beta)\right), \tag{8.6}$$

where P_n, Q_n, R_n and S_n are arbitrary parameters.

9. Analytical Solution in Bi-polar Cylindrical Coordinates (θ, ϕ, z)

The bi-polar cylindrical coordinates (θ, ϕ, z) and 3-dimensional Cartesian coordinates (x, y, z) are given in the book of Happel and Brenner (1983). Connections transformed by following equations:

$$x = \frac{c \sinh \phi}{\cosh \phi - \cos \theta}, y = \frac{c \sin \theta}{\cosh \phi - \cos \theta}, z = z, \tag{9.1}$$

The scale factors in the bi-polar cylindrical coordinates h_1, h_2 and h_3 are

$$h_1 = \frac{c}{\cosh \phi - \cos \theta}, h_2 = \frac{c}{\cosh \phi - \cos \theta}, h_3 = 1.$$

Assuming that velocity vector of Newtonian fluid is $\boldsymbol{q} = (q_1(\theta, \phi), q_2(\theta, \phi), 0)$Introducing the stream function $\psi(\theta, \phi)$ as:

$$q_1 = -\left(\frac{\cosh \phi - \cos}{c}\right)\frac{\partial \psi}{\partial \phi} and q_2 = \left(\frac{\cosh \phi - \cos \theta}{c}\right)\frac{\partial \psi}{\partial \theta}. \tag{9.2}$$

Applying the curl operator both sides on equation (1.1) and with the help of equation (3.6), we get

$$\frac{\partial^2 \psi}{\partial \phi^2} + \frac{\partial^2 \psi}{\partial \theta^2} = 0. \tag{9.3}$$

Therefore, stream function solution of the Darcy equation in the bi-polar cylindrical coordinates will be

$$\psi(\theta, \phi) = \sum_{n=0}^{\infty} [A_n \exp(n\phi) + B_n \exp(n\phi)]_{\cos(n\theta)}^{\sin(n\theta)}, \tag{9.4}$$

where,A_n's and B_n's are arbitrary parameters.

Conclusion

Analytical solutions to the Darcy equation's stream function are found for some orthogonal curvilinear coordinate systems, including parabolic coordinates, parabolic cylindrical coordinates, and bi-polar cylindrical coordinates. The two-dimensional fluid flow passing through a porous media can be handled using such solutions. Stream function solution of the mathematical equation $(\nabla^2 - \alpha^2)\psi(u, v) = 0$, are evaluated analytically in parabolic and parabolic cylindrical co-ordinates. Combinations of trigonometric, hyperbolic, exponential, Laguerre polynomial, parabolic cylinder, and Whittaker functions are employed in the analytical equations of the resultant stream function.

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