ISSN:0976-4933
Journal of Progressive Science
A Peer-reviewed Research Journal
Vol.13, No.01 & 02, pp 38-44 (2022)

On quasi – conformally flat LP-Sasakian manifold with a coefficient α

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Abstract

The notion of Lorentzian almost paracontact manifolds with a coefficient α has been introduced and studied by De et al (2002). In the present paper we investigate some properties of quasi-conformally flat LP-Sasakian manifold with a coefficient α.

Key words- LP-Sasakian manifold with a coefficient α, quasi-conformal curvature tensor and manifold of constant curvature.

1. Introduction

In 1989, Matsumoto introduced the notion of LP-Sasakian manifolds. Then Mihai and Rosca (1992) introduced the same notation independently and they obtained several results on this manifold. In a recent paper De, Shaikh and Sengupta (2002) introduced the notion of LP-Sasakian manifold with a coefficient α which generalized the notion of LP-Sasakian manifolds. Recently T.Ekawa and his co-authors (1996 and 1997) studied Sasakian manifold with a Lorentzian metric and obtained several results on this manifold. De, Jun and Shaikh (2002) and Das and Sengupta (2006) studied conformally flat LP-Sasakian manifold with a coefficient α. In 2007, Bagewadi, Prakasha and Venkatesha studied the pseudo projectively flat LP-Sasakian manifold with a coefficient α. The existence of a product submanifold of LP-Sasakian manifold with a coefficient α have been proved by Sengupta, De and Jun in 2003. Moreover, certain curvature conditions on an LP-Sasakian manifold with a coefficient α obtained by De and Arslan (2009). Recently in 2017, Das proved that a second order symmetric parallel tensor an LP-Sasakian manifold with a coefficient α is a constant multiple of the associated metric tensor. In the present paper, we study quasi conformally flat LP-Sasakian manifold with a coefficient α. Here we prove that in a quasi-conformally flat LP-Sasakian manifold with a coefficient α the characteristic vector field is a concircular vector field if and only if the manifold is η-Einstein and a quasi-conformally LP-Sasakian manifold with a coefficient α is a manifold of constant curvature if scalar curvature r is a constant, provided $a + 2(n-1)b \neq 0$.

2. Preliminaries

Let M be an n-dimensional differentiable manifold endowed with a (1,1) tensor field ϕ, a contravariant vector field ξ, a covariant vector field η and a Lorentzian metric of type (0,2) such that for each point $p\epsilon M$, the tensor $g_p: T_pM \times T_pM \rightarrow R$ is non-degenerate inner product of signature (-,+, · · ·,+), where T_pM denotes the tangent vector space of M at p and R is the real number space which satisfies

$$\eta(\xi) = -1,\ \phi^2 X = X + \eta(X)\xi,\ g(X,\xi) = \eta(X),\ D_X\xi = \phi X, \tag{2.1}$$

$$g(\phi X, \phi Y) = g(X,Y) + \eta(X)\,\eta(Y), \tag{2.2}$$

for all vector fields X and Y. Then such a structure (ϕ, ξ, η, g) is termed as Lorentzian almost paracontact structure and the manifold M with the structure (ϕ, ξ, η, g) is called Lorentzian almost paracontact manifold M. In the Lorentzian almost paracontact manifold, the following relation hold De, Shaikh and Sengupta (2002)

$$\phi\xi = 0, \eta(\phi X) = 0, \text{rank } \phi = n - 1. \tag{2.3}$$

If we put

$$\omega(X, Y) = \omega(Y, X), \tag{2.4}$$

where $\omega(X, Y)$ is a symmetric (0, 2) type tensor field and $\omega(X, Y) = g(X, \phi Y)$.

In the Lorentzian almost paracontact manifold M, if it satisfies the following relations:

$$(D_Z\, \omega)(X, Y) = \alpha[\{g\,(X, Z) + \eta(X)\,\eta(Z)\}\eta(Y) + \{g\,(Y, Z) + \eta(Y)\,\eta(Z)\}\eta(X)], \tag{2.5}$$

and

$$\omega(X, Y) = \frac{1}{\alpha}(D_X\, \eta)(Y), \tag{2.6}$$

hold, where D denotes the operator of covariant differentiation with respect to the Lorentzian metric g, then M is called an LP-Sasakian manifold with coefficient α. An LP-Sasakian manifold with coefficient 1 is an LP-Sasakian manifold Matsumoto (1989). If a vector field V satisfies the equation of the form $(D_X\, V) = \beta X + T(X)V$, where β is a non-zero scalar function and T is a covariant vector field, then V is called a torse forming vector field Yano (1944). In a Lorentzian manifold M, if we assume that ξ is a unit torse forming vector field, then

$$(D_X\, \eta)Y = \alpha[g\,(X, Y) + \eta(X)\,\eta(Y)], \tag{2.7}$$

where α is a non-zero scalar function. Hence the manifold admitting a unit torse forming vector field satisfying (2.7) is an LP-Sasakian manifold with coefficient α. And if η satisfies

$$(D_X\, \eta)Y = \varepsilon[g\,(X, Y) + \eta(X)\,\eta(Y)], \varepsilon^2 = 1. \tag{2.8}$$

Then M is called an LSP- Sasakian manifold Matsumoto (1989). In particular, if α satisfies equation (2.7) and the equation of the following form

$$\alpha(X) = P\,\eta(X), \alpha(X) = D_X\alpha, \tag{2.9}$$

where P is a scalar function, then ξ is called a concircular vector field. Let us consider an LP-Sasakian manifold M with a structure (ϕ, ξ, η, g) and with a coefficient α. Then we have:

$$\eta(R(X, Y)Z) = -\,\alpha(X)\,\omega(Y, Z) + \alpha(Y)\,\omega(X, Z) + \alpha^2[\,\eta(X)\,g(Y, Z) - \eta(Y)\,g(X, Z)], \tag{2.10}$$

and

$$Ric\,(X, \xi) = -\,\psi\,\alpha(X) + (n-1)\,\alpha^2\,\eta(X) + \alpha\,\phi X, Trace\,(\phi) = \psi. \tag{2.11}$$

where R and Ric denote respectively the curvature tensor and Ricci tensor of the manifold.

Furthermore, quasi-conformal curvature tensor on a Riemannian manifold introduced by Chaki and Ghosh (1997) as follows

$$\tilde{C}(X, Y)Z = a\,R(X, Y)Z + b[\,Ric(Y, Z)X - Ric(X, Z)Y + g(Y, Z)QX - g(X, Z)QY] - \frac{r}{n}\left(\frac{a}{n-1} + 2b\right)(\,g(Y, Z)X - g(X, Z)Y), \tag{2.12}$$

where a and b are constant Q and r are the Ricci operator defined by $Ric(X,Y) = g(QX,Y)$ and scalar curvature of the manifold. If $a = 1$ and $b = -\frac{1}{n-2}$ then (2.12) takes the form

$$\tilde{C}(X,Y)Z = R(X,Y)Z - \frac{1}{n-2}[Ric(Y,Z)X - Ric(X,Z)Y + g(Y,Z)QX - g(X,Z)QY] + \frac{r}{(n-1)(n-2)}[g(Y,Z)X - g(X,Z)Y] = C = \text{Conformal curvature tensor.}$$

Now, we state the following results which are used in the later section.

Lemma (2.1) (De, Shaikh and Sengupta, 2002). In an LP-Sasakian manifold with a non constant coefficient α, one of the following cases occurs:

$(i)\ \psi^2 = (n-1)^2 \quad (ii)\ \alpha(Y) = -P\,\eta(Y)$, where $P = \alpha(\xi)$.

Lemma (2.2) (De, Shaikh and Sengupta, 2002). In Lorentzian almost paracontact manifold with structure (ϕ,ξ,η,g) satisfying $\omega(X,Y) = \frac{1}{\alpha}(D_X\,\eta)(Y)$, where α is a non-zero scalar function the vector field ξ is torse forming if and only if $\psi^2 = (n-1)^2$ holds.

3. Quasi-conformally flat LP-Sasakian manifold with a coefficient α

Let us consider a quasi-conformally flat LP-Sasakian manifold M with a coefficient α. First suppose that α is not constant. Then since quasi- conformal curvature vanishes, then from (2.12), we get

$$'R(X,Y,Z,W) = -\frac{b}{a}[Ric(Y,Z)g(X,W) - Ric(X,Z)g(Y,W) + g(Y,Z)Ric(X,W) - g(X,Z)Ric(Y,W)] - \frac{r}{n}\left(\frac{1}{n-1} + \frac{2b}{a}\right)(g(Y,Z)\,g(X,W) - g(X,Z)\,g(Y,W), \quad (3.1)$$

where $'R(X,Y,Z,W) = g(R(X,Y)Z,W)$.

Putting ξ for W in (3.1) and then using (2.2), (2.10), and (2.11), we get

$$-\alpha(X)\omega(Y,Z) + \alpha(Y)\omega(X,Z) + \alpha^2[\eta(X)g(Y,Z) - \eta(Y)g(X,Z)] = -\frac{b}{a}[Ric(Y,Z)\,\eta(X) - Ric(X,Z)\,\eta(Y) + g(Y,Z)\{-\psi\,\alpha(X) + (n-1)\alpha^2\eta(X) + \alpha\,(\phi X)\} - g(X,Z)\{-\psi\,\alpha(Y) + (n-1)\alpha^2\,\eta(Y) + \alpha\,(\phi Y)] + \frac{r}{n}\left(\frac{1}{n-1} + \frac{2b}{a}\right)[g(Y,Z)\,\eta(X) - g(X,Z)\,\eta(Y)]. \quad (3.2)$$

Again if we put ξ for X in (3.2) and then using (2.3) and (2.11), we obtain

$$Ric\,(Y,Z) = \left[\frac{ar}{bn(n-1)} + \frac{2r}{n} - \psi P - (n-1)\alpha^2 - \frac{a}{b}\alpha^2\right]g(Y,Z) + \left[\frac{ar}{bn(n-1)} + \frac{2r}{n} - 2(n-1)\alpha^2 - \frac{a}{b}\,\alpha^2\right]\eta(Y)\eta(Z) + [\psi\,\alpha(Z) - \alpha\,(\phi Z)]\eta(Y) + [\psi\,\alpha(Y) - \alpha\,(\phi Y)]\eta(Z) - \frac{a}{b}P\omega(Y,Z). \quad (3.3)$$

Remark 1. If we put $a = 1$ and $b = -\frac{1}{n-2}$ in (3.3) we get

$$Ric\,(Y,Z) = \left[\frac{r}{(n-1)} - \psi P - \alpha^2\right]g(Y,Z) + \left[\frac{r}{(n-1)} - n\,\alpha^2\right]\eta(Y)\eta(Z) + [\psi\,\alpha(Z) - \alpha(\phi Z)]\eta(Y) + [\psi\,\alpha(Y) - \alpha(\phi Y)]\eta(Z) + (n-2)P\omega(Y,Z),$$

which is same as was obtained by De, Jun and Shaikh (2002).

We now suppose that M^n is η-Einstein. If an LP-Sasakian manifold M^n with a coefficient α satisfies the relation $Ric\,(X,Y) = \lambda\,g(X,Y) + \mu\,\eta(X)\,\eta(Y)$, where λ and μ are the associated function on the manifold, then the manifold M^n is called η −Einstein manifold. Then we have De, Shaikh and Sengupta (2002),

$$Ric(X,Y) = \left[\frac{r}{(n-1)} - \frac{\psi P}{n-1} - \alpha^2\right] g(X,Y) + \left[\frac{r}{(n-1)} - n\,\alpha^2 - \frac{n\psi P}{n-1}\right] \eta(X)\,\eta(Y). \qquad (3.4)$$

Putting e_i for X and Y in (3.4), where $\{e_i\}$ is an orthonormal basis of the tangent space at a point of the manifold and taking summation over $1 \leq i \leq n$, we get

$$r = n(n-1)\alpha^2 + n\psi P. \qquad (3.5)$$

By virtue of (3.3) and (3.4) we get

$$\left[\frac{r}{(n-1)} - \frac{\psi P}{n-1} - \alpha^2\right] g(Y,Z) + \left[\frac{r}{(n-1)} - n\,\alpha^2 - \frac{n\psi P}{n-1}\right] \eta(Y)\eta(Z) =$$
$$\left[\frac{ar}{bn(n-1)} + \frac{2r}{n} - \psi P - (n-1)\alpha^2 - \frac{a}{b}\alpha^2\right] g(Y,Z) +$$
$$\left[\frac{ar}{bn(n-1)} + \frac{2r}{n} - 2(n-1)\alpha^2 - \frac{a}{b}\,\alpha^2\right] \eta(Y)\eta(Z) + [\psi\,\alpha(Z) -$$
$$\alpha\,(\phi Z)]\eta(Y) + [\,\psi\,\alpha(Y) - \alpha\,(\phi Y)]\eta(Z) - \frac{a}{b} P\,\omega(Y,Z). \qquad (3.6)$$

Putting ξ for Y in (3.6) and replacing Z by Y in resulting equation, we get

$$\psi\,\alpha(Y) - \alpha\,(\phi Y) = -\psi P\,\eta(Y) \text{ for all } Y. \qquad (3.7)$$

Using (3.7) in (3.6) and then by virtue of (3.5), we get

$$\frac{ab}{P}\left[\frac{\psi}{n-1}\{\,g(Y,Z) + \eta(\mathrm{Y})\,\eta(\mathrm{Z})\} - \omega(\mathrm{Y},\mathrm{Z})\right] = 0\,. \qquad (3.8)$$

If $P = 0$, then from (3.7) we get $\psi\,\alpha(Y) = \alpha\,(\phi Y)$. Thus ψ is equal to ± 1 as ψ is eigen value of the matrix ϕ. Hence by virtue of Lemma 2, we get $\alpha(Y) = 0$ for all Y and so α is constant, which contradicts our assumption. Consequently we have $P \neq 0$ and hence from (3.8), we get

$$\frac{ab}{P}\left[\frac{\psi}{n-1}\{\,g(Y,Z) + \eta(\mathrm{Y})\,\eta(\mathrm{Z})\} - \omega(\mathrm{Y},\mathrm{Z})\right] = 0\,. \qquad (3.9)$$

Putting ϕY for Y in (3.9) and then (2.3), we obtain

$$\frac{a}{b}\left[\frac{\psi}{n-1}\,\omega(\mathrm{Y},\mathrm{Z}) - \{\,g(Y,Z) + \eta(\mathrm{Y})\,\eta(\mathrm{Z})\}\right] = 0.$$
(3.10)

Combing (3.9) and (3.10) we get

$$[\psi^2 - (n-1)^2][g(Y,Z) + \eta(\mathrm{Y})\,\eta(\mathrm{Z})] = 0.$$

This gives $[\psi^2 - (n-1)^2] = 0.$ (3.11)

Hence Lemma (2.2) proves that ξ is a torse forming.

Now we have $(D_X\,\eta)Y = \beta[g\,(X,Y) + \eta(X)\,\eta(Y)]$. Then from (2.6) we get

$$\omega(X,Y) = g\left(\frac{\beta}{\alpha}\,(X + \eta(X)\xi), Y\right) \text{ and } \omega(X,Y) = g\,(\phi X, Y).$$

Since g is non-singular, we have

$$\phi X = \frac{\beta}{\alpha}(X + \eta(X)\xi.$$

and

$$\phi^2\,(X) = \left(\frac{\beta}{\alpha}\right)^2 (X + \eta(X)\xi).$$

It follows from (2.1) that $\left(\frac{\beta}{\alpha}\right)^2 = 1$ and hence $\alpha = \pm \beta$. Thus we have

$$\phi X = \pm(X + \eta(X)\xi).$$

By virtue of (3.7), $\alpha(Y) = -P\,\eta(Y)$. Thus we conclude that ξ is a concircular vector field. Conversely we suppose that ξ is a concircular vector field. Then we have the equation of the form $(D_X\,\eta)Y = \beta[g\,(X,Y) + \eta(X)\,\eta(Y)]$, where β is a certain function and $D_X\,\beta = q\,\eta(X)$ for a certain scalar function q. Hence by virtue of (2.6) we have $\alpha = \pm$ β. Thus we have $(D_X\,\eta)Y = \varepsilon[g\,(X,Y) + \eta(X)\,\eta(Y)]$, $\varepsilon^2 = 1, \psi = \varepsilon(n-1), D_X\,\alpha = \alpha\,(X) = P\,\eta(Y), P = \varepsilon q$. Using these relations in (3.3) and (3.7), it follows that M^n is η −Einstein. Thus we have the following theorem:

Theorem (3.1) In a quasi-conformally flat LP-Sasakian manifold M^n with a non-constant coefficient α, the characteristic vector field ξ is a concircular vector field if and only if M^n is η −Einstein.

Now we suppose that α is constant. In this case, the following relations hold good Bagewadi, Prakasha and Venkatesha (2007):

$$\eta(R(X,Y)Z) = \alpha^2[\,\eta(X)\,g(Y,Z) - \eta(Y)\,g(X,Z)], \tag{3.12}$$

and

$$Ric\,(X,\xi) = (n-1)\alpha^2\eta(X). \tag{3.13}$$

Putting ξ for W in (3.1) and then using (2.2), (3.12) and (3.13) we get

$$\alpha^2[\,\eta(X)g(Y,Z) - \eta(Y)g(X,Z)] = -\frac{b}{a}[\,Ric(Y,Z)\,\eta(X) - Ric(X,Z)\,\eta(Y)$$
$$+g(Y,Z)(n-1)\alpha^2\,\eta(X) - g(X,Z)(n-1)\,\alpha^2\,\eta(Y)] +$$
$$\frac{r}{n}\left(\frac{1}{n-1} + \frac{2b}{a}\right)[g(Y,Z)\,\eta(X) - g(X,Z)\,\eta(Y)]. \tag{3.14}$$

Putting ξ for X in (3.14) and then using (3.13) we get,

$$Ric\,(Y,Z) = \left[\frac{ar}{bn(n-1)} + \frac{2r}{n} - (n-1)\alpha^2 - \frac{a}{b}\alpha^2\right]g(Y,Z) +$$
$$\left[\frac{ar}{bn(n-1)} + \frac{2r}{n} - 2(n-1)\alpha^2 - \frac{a}{b}\,\alpha^2\right]\eta(Y)\eta(Z). \tag{3.15}$$

Remark 2. If we put $a = 1$ and $b = -\frac{1}{n-2}$ in (3.15), we get

$$Ric\,(Y,Z) = \left[\frac{r}{(n-1)} - \alpha^2\right]g\,(Y,Z) + \left[\frac{r}{(n-1)} - n\,\alpha^2\right]\eta(Y)\,\eta(Z).$$

which is same as was obtained by De, Jun and Shaikh (2002). From (3.15) we can state the following theorem:

Theorem (3.2) A quasi-conformally flat LP-Sasakian manifold M^n with a constant coefficient α is always an η- Einstein manifold.

Differentiating (3.15) covariantly along X and making the use of (2.6), we get

$$(D_X Ric)(Y,Z) = \frac{dr(X)}{n}\left[\frac{a}{b(n-1)} + 2\right][g\,(Y,Z) + \eta(Y)\eta(Z)] +$$
$$\left[-2(n-1)\alpha^2 - \frac{a}{b}\alpha^2 + \frac{r}{n}\left\{\frac{a}{b(n-1)} + 2\right\}\right].$$
$$[\alpha\{\omega(X,\,Y\}\,\eta(Z) + \{\omega(X,\,Y\}\,\eta(Z).$$

This equation implies that

$$(D_X Ric)(Y,Z) - (D_Y\,Ric)(X,Z) = \frac{dr(X)}{n}\left[\frac{a}{b(n-1)} + 2\right][\,g\,(Y,Z) + \eta(Y)\eta(Z)]$$

$$-\frac{dr(Y)}{n}\left[\frac{a}{b(n-1)}+2\right][g(X,Z)+\eta(X)\eta(Z)]$$
$$+\left[-2(n-1)\alpha^2-\frac{a}{b}\alpha^2+\frac{r}{n}\left\{\frac{a}{b(n-1)}+2\right\}\right].$$
$$[\alpha\{\omega(X,Z\}\eta(Y)-\omega(X,Z)\,\eta(X)\}]\,. \tag{3.16}$$

On the other hand in our case, since we have $\left(D_W\,\tilde{C}\right)(X,Y)Z=0\Rightarrow divC$ where $\tilde{C}$ is quasi-conformal curvature tensor and "div" denotes divergence. So for $n>3$, $div\tilde{C}=0$ gives

$$(D_X Ric)(Y,Z)-(D_Y\,Ric)(X,Z)=\frac{1}{n(n-1)(a+b)}\,[a-(n-1)(n-2)b]$$
$$[g(Y,Z)\,dr(X)\,-g(X,Z)\,dr(Y)\,]. \tag{3.17}$$

From (3.16) and (3.17), we get

$$\frac{dr(X)}{n}\left[\frac{a}{b(n-1)}+2\right][g(Y,Z)+\eta(Y)\eta(Z)]-\frac{dr(Y)}{n}\left[\frac{a}{b(n-1)}+2\right].$$
$$[g(X,Z)+\eta(X)\eta(Z)]+\left[-2(n-1)\alpha^2-\frac{a}{b}\alpha^2+\frac{r}{n}\left\{\frac{a}{b(n-1)}+2\right\}\right]$$
$$[\alpha\{\omega(X,Z\}\eta(Y)-\omega(X,Z)\,\eta(X)\}]=\frac{1}{n(n-1)(a+b)}\,[a-(n-1)\,(n-2)\,b]\,.$$
$$[\,g(Y,Z)\,dr(X)-\,g(X,Z)\,dr(Y)]. \tag{3.18}$$

If r is constant then from (3.18), we get

$\left(\frac{a+2(n-1)b}{b}\right)\left(\frac{r}{n(n-1)}-\alpha^2\right)=0$. Hence from this, we can write,

$$r=n(n-1)\alpha^2,\,a+2(n-1)\,b\neq 0. \tag{3.19}$$

Using (3.15) and (3.19) in (3.1), we get

$$'R(X,Y,Z,W)=\alpha^2[g(Y,Z)\,g(X,W)-g(X,Z)g(Y,W)]. \tag{3.20}$$

Thus, we have the following theorem:

Theorem (3.3) In a quasi-conformally flat LP-Sasakian manifold M^n with a constant coefficient α, if the scalar curvature r is constant then manifold is of constant curvature, provided $a+2(n-1)\,b\neq 0$.

Contracting (3.15)**,** we get

$$r=n(n-1)\alpha^2 \text{ if } a+(n-2)\,b\neq 0. \tag{3.21}$$

Using (3.21) in (3.15) we get

$$Ric\,(Y,Z)=\alpha^2(n-1)g(Y,Z),\;\; a+(n-2)\,b\neq 0\,. \tag{3.22}$$

Using (3.22) in (3.1), we obtain

$$'R(X,Y,Z,W)=\alpha^2\,[\,g(Y,Z)\,g(X,W)-\,g(X,Z)\,g(Y,W)\,]. \tag{3.23}$$

Hence we can state the following theorem:

Theorem (3.4) In a quasi-conformally flat LP-Sasakian manifold M^n with a constant coefficient α, then manifold is of constant curvature, provided $a+(n-2)\,b\neq 0$.

From (2.2), (3.22) and (3.23), we can state the following theorem:

Theorem (3.4) In a quasi-conformally flat LP-Sasakian manifold M^n with a constant coefficient α, if $a+(n-2)\,b\neq 0$ the following relations hold:

(i) $R(X,\xi)Z=\alpha^2[\,\eta(Z)X-g(X,Z)\xi]$, (3.24)

(ii) $R(\xi,X)Y=\alpha^2[g(X,Y)\xi-\eta(Y)X\,]$, (3.25)

(iii) $R(X,Y)\xi = \alpha^2[\eta(Y)X - \eta(X)Y]$, (3.26)

(iv) $Ric\,(X,\xi) = \alpha^2\,(n-1)\eta(X)$, (3.27)

(v) $Ric\,(\xi,X) = \alpha^2\,(n-1)\eta(X)$, (3.28)

(vi) $Ric\,(\phi X,\phi Y) = Ric\,(X,Y) + \alpha^2\,(n-1)\eta(X)\eta(X)$. (3.29)

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Received on 28.07.2023 and accepted on 29.11.2023