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On semi-pseudo symmetric and semi-pseudo Ricci-symmetric lorentzian β −Kenmotsu manifold

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Abstract

The object of this paper is to study a type of Lorentzian β −Kenmotsu manifold called Lorentzian $\beta-$ Kenmotsu $(SPS)_n-$ manifold and Lorentzian $\beta-$ Kenmotsu $(SPRS)_n-$ manifold $(n \neq 3)$. An example of non-existence are also given of such manifolds. Finally, we derive an expression for pressure and density for a perfect flow in the Lorentzian β −Kenmotsu manifolds.

Key words and phrases

Semi-pseudo symmetric, semi-pseudo Ricci symmetric, Lorentzian β −Kenmotsu manifold, pseudo symmetric and pseudo Ricci-symmetric manifold.

Introduction

In 1969, Tanno classified connected almost contact metric manifolds whose automorphism groups pass the maximum dimension. For such a manifold, the sectional curvatures of plane sections containing ξ are a constant, say c. He showed that they can be divided into three classes:

(1) Homogeneous normal contact Riemannian manifolds with $c > 0$,

(2) Global Riemannian products of a line or a circle with a Kähler manifold of constant holomorphic sectional curvature if $c = 0$ and

(3) A warped product space $R \times_f C$ ifc < 0.

It is know that the manifolds of class (1) are characterized by admitting a Sasakian structure. The manifold of class (2) is characterized by a tensorial relation admitting a cosymplectic structure. Kenmotsu (1972) characterized the differential geometric properties of the manifolds of class (3); the structure so obtained is now known as Kenmotsu structure. In general, these structures are not Sasakian (1972). In the Gray-Hervellaclassifition of almost Hermition manifolds (1980), there appears a class W_4 of Hermitian manifolds, which are closely related to locally conformal Kaehler manifolds (1998). An almost contact metric structure on a manifold M is called a trans-Sasakian structure (1985) if the product manifold $M \times R$ belongs to the class W_4 . The class $C_6 \otimes C_5$ (1989) coincides with the class of the trans-Sasakian structures of type (α, β). In fact in (1989), local nature of the two subclasses C_5 and C_6structures of trans-Sasakian structures are characterized completely.

We note that trans-Sasakian structures of type $(0,0), (0, \beta\ )\ and\ (\alpha, 0)$are cosymplectic (1976), β-

Kenmotsu (1981) and α-Sasakian (1981) respectively. In (1999-2000) it is proved that trans-Sasakian structures are generalized quasi-Sasakian (1991). Thus, trans-Sasakian structures also provide a large class of generalized quasi-Sasakian structures.
An almost contact metric structure (ϕ, ξ, η, g) on M is called a trans-Sasakian structures (1985) if $(M \times R, J, G,)$ belongs to the class W_4 (1980), where J is the almost complex structure on $M \times R$ defined by

$$J(X, f d/dt) = (\phi X - f\, \xi, \eta(X) f d/dt) \tag{1.1}$$

for all vector fields X on M, smooth functions f on $M \times R$ and G is the product metric on $M \times R$. This may be expressed by the condition (1990)

$$(D_X \phi) = \alpha(g(X,Y)\xi - \eta(Y)X) + \beta(g(\phi X, Y)\xi - \eta(Y)\phi X) \tag{1.2}$$

for some smooth functions on M and we say that the trans-Sasakian structure is of type (α, β).
Theorem 1.1: A trans-Sasakian structure of type (α, β) with βa non-zero constant is always β-Kenmotsu.

In this case βbecomes a constant. If $\beta = 1$, then β-Kenmotsu manifold is Kenmotsu.

2. Preliminaries

A differentiable manifold M of dimension n is called Lorentzian β-Kenmotsu manifold if it admits a (1,1)- tensor field ϕ, a contravariant vector field ξ, a covariant vector field $\eta$and a Lorentzian metric g which satisfy

$$\eta(\xi) = -1, \phi\xi = 0, \eta(\phi X) = 0, \tag{2.1}$$

$$\phi^2 X = X + \eta(X)\xi,\ g(X,\xi) = \eta(X), \tag{2.2}$$

$$g(\phi X, \phi Y) = g(X,Y) + \eta(X)\eta(Y), \tag{2.3}$$

for all $X, Y \in \chi(M)$.
A Lorentzian β-Kenmotsu manifold M satisfies

$$D_X \xi = \beta[(X - \eta(X)\xi], \tag{2.4}$$

$$(D_X \eta)(Y) = \beta[g(X,Y) - \eta(X)\eta(Y)], \tag{2.5}$$

where D denotes the covariant differentiation with respect to the Lorentzian metric g.
Further, on a Lorentzian β-Kenmotsu manifold M the following relations hold (Bagewadi and Girish Kumar (2004),Bagewadi and Venkatesha (2007), Bagewadietal (2008), Prakashaetal (2008)),

$$\eta(R(X,Y)Z) = g(R(X,Y)Z,\xi) = \beta^2[g(X,Z)\eta(Y) - g(Y,Z)\eta(X)], \tag{2.6}$$

$$R(\xi, X)Y = \beta^2[\eta(Y)X - g(X,Y)\xi], \tag{2.7}$$

$$R(X,Y)\xi = \beta^2[\eta(X)Y - \eta(Y)X], \tag{2.8}$$

$$S(X,\xi) = -(n-1)\beta^2\eta(X), \tag{2.9}$$

$$Q\xi = -(n-1)\beta^2\xi, \tag{2.10}$$

$$S(\xi,\xi) = (n-1)\beta^2, \tag{2.11}$$

for any vector fields $X, Y$and Z, where $R(X,Y)Z$ is the Riemannian curvature tensor and Ric denotes the Ricci tensor.

A non- flat Riemannian manifold (M^n, g) $(n > 3)$ is said to be a pseudo-symmetric in the sense of Chaki (1987), if it satisfies the relation

$$(D_X R)(Y,Z,W,U) = 2A(X)R(Y,Z,W,U) + A(Y)R(X,Z,W,U) + A(Z)R(Y,X,W,U) + A(W)R(Y,Z,X,U) + A(U)R(Y,Z,W,X)$$

That is

$$(D_X R)(Y,Z,W) = 2A(X)R(Y,Z,W) + A(Y)R(X,Z,W) + A(Z)R(Y,X,W) + A(W)R(Y,Z,X) + g(R(Y,Z,W),X)\rho$$

for any vector field X, Y, Z, W and U,where R is the Riemannian curvature tensor of the manifold. A is non-zero 1-form such that $g(X,\rho) = A(X)$ for every vector field X. Such an n-dimensional manifold was denoted by $(PS)_n$. Pseudo symmetric manifolds in the sense of Chaki have been studied by Chaki and De (1989), De, Murathan and Özğur (2010), Özenand Altay (), Tarafdar(1991,1995) and many others.

A non-flatRiemaniann manifold $(M^n, g)(n > 3)$ is said to be pseudo-Ricci symmetric (1988) if its Ricci tensor Ric of type (0,2) is not identically zero and satisfies the condition,

$$(D_X Ric)(Y,Z) = 2A(X)Ric(Y,Z) + A(Y)Ric(X,Z) + A(Z)Ric(Y,Z)$$

for any vector field X, Y, Z,where $A$is a non-zero 1-form such that $g(X,\rho) = A(X)$ for every vector field X. Such an n-dimensional manifold is denoted by $(PRS)_n$. $(PRS)_n$ manifold also studied by Arslan(2001), Chaki and Saha(1994), De and Mazumdar(1998), Özen (2011) and many others.

In 1995, Tarafdar and Jawarneh (1995) introduced a type of non-flat Riemannian manifold $(M^n, g)(n > 3)$whose curvature tensor R satisfies the condition

$$(D_X R)(Y,Z)W = 2A(X)R(Y,Z)W + A(Y)R(X,Z)W + A(Z)R(Y,X)W + A(W)R(Y,Z)X, \tag{2.12}$$

where A is a non zero 1-form satisfying

$$g(X,\rho) = A(X) \tag{2.13}$$

for every vector field X and D denotes the covariant differentiation with respect to g. Such a manifold was called by them a semi-pseudo-symmetric manifold, A was called its associated 1-form and an n-dimensional manifold of this kind was denoted by$(SPS)_n$. In a subsequent paper Tarafdar and Jawarneh (1993), introduced another type of non-flat Riemannian manifolds $(M^n, g)(n > 3)$, whose Ricci tensor of type (0,2) satisfies the condition,

$$(D_X Ric)(Y,Z) = A(Y)Ric(X,Z) + A(Z)Ric(X,Y), \tag{2.14}$$

where symbols have their usual meanings. Such a manifold was called by them a semi-pseudo-Ricci-symmetric manifold and an n-dimensional manifold of this kind was denoted by $(SPRS)_n$.

Some contributions in this direction is due to Prasad, Trafdar&Jawarneh,they discussed some aspect in (1998), (1993), (1995), (2011).

In the present paper we proved that Lorentzian β-Kenmotsu manifolds essentially do not admit neither semi-pseudo-symmetric nor semi-pseudo Ricci-symmetric structures with non trivial example.

3. Lorentzian $\boldsymbol{\beta}$-Kenmotsu$(\boldsymbol{SPS})_{\boldsymbol{n}}$-manifold $(\boldsymbol{n} > 3)$

In this section, we assume that an n-dimensional $(SPS)_n(n > 3)$ is a Lorentzian β-Kenmotsu manifold. Now we have

$$(D_X Ric)(Y,\xi) = D_X Ric(Y,\xi) - Ric(D_X Y,\xi) - Ric(Y, D_X\xi). \tag{3.1}$$

Using (2.9) in (3.1), we get

$$(D_X Ric)(Y,\xi) = -(n-1)\beta^2 g(D_X\xi, Y) - Ric(Y, D_X\xi). \tag{3.2}$$

From (2.12), we have

$$(D_X Ric)(Y,Z) = 2A(X)Ric(Y,Z) + A(Y)Ric(X,Z) + A(Z)Ric(Y,X) + A(R(X,Y)Z). \tag{3.3}$$

Putting ξ for Z in (3.3), we get

$$(D_X Ric)(Y,\xi) = -2(n-1)\beta^2 A(X)\eta(Y) - \beta^2(n-1)A(Y)\eta(X) + A(\xi)Ric(Y,X) + A(R(X,Y)\xi). \tag{3.4}$$

In view of (2.6}), (3.4) reduces to

$$(D_X Ric)(Y,\xi) = -2n\beta^2 A(X)\eta(Y) - \beta^2(n-2)A(Y)\eta(X) + \beta^2 A(X)\eta(Y) + A(\xi)Ric(Y,X) \quad (3.5)$$

In view of (3.2) and (3.5), we have

$$-2n\beta^2 A(X)\eta(Y) - \beta^2(n-2)A(Y)\eta(X) + \beta^2 A(X)\eta(Y) + A(\xi)Ric(Y,X) = -(n-1)\beta^2 g(D_X\xi, Y) - Ric(Y, D_X\xi). \quad (3.6)$$

Putting ξ for Xin (3.6), we obtain

$$\beta^2[(3n-2)A(\xi)\eta(Y) - (n-2)A(Y)] = 0. \quad (3.7)$$

Again putting $\xi$for Yin (3.7), we obtain

$$\beta^2 A(\xi) = 0. \quad (3.8)$$

Hence, from (3.8) and (3.7), we get

$$\beta^2 A(Y)=0. \quad (3.9)$$

But $\beta^2 \neq 0$. Hence from (3.9), we obtain

$$A(Y) = 0,$$

which is inadmissible by the definition of $(SPS)_n$.\
Thus, we have the following theorem:

Theorem 3.1: A $(SPS)_n$ cannot be a Lorentzian β-Kenmotsu manifold, provided $\beta^2 \neq 0$.

4. Example:

Let us consider the 3-dimensional manifold $M = \{(x,y,z) \epsilon \mathbb{R}^3, z \neq 0\}$, where (x,y,z) are standard co-ordinate in $\mathbb{R}^3$.

We choose the vector fields

$$e_1 = e^{-\beta z}\frac{\partial}{\partial x}, \quad e_2 = e^{-\beta z}\frac{\partial}{\partial y}, \quad e_3 = \frac{\partial}{\partial z}$$

which is linearly independently at each point of M.

Let g be the Lorentzian metric defined by

$$g(e_i, e_j) = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$$

Let η be the 1-form which satisfies the relation

$$\eta(e_3) = -1$$

Let ϕ be the (1,1) tensor field defined by

$$\phi e_1 = -e_1, \ \phi e_2 = -e_2, \ \phi e_3 = 0.$$

Then, we have

$$\phi^2 U = U + \eta(U)e_3 \text{and} \quad g(\phi U, \phi W) = g(U,W) + \eta(U)\eta(W),$$

for any $U, W \epsilon \chi(M)$.

Thus for $e_3 = \xi$, (ϕ, ξ, η, g) defines an almost LP contact structure on $\chi(M)$.

Let D be the Levi-Civita connection with respect to the Riemannian metric g and R be the curvature tensor of g.

Then we have

$$[e_1, e_2] = 0, \ [e_1, e_3] = \beta e_1, \ [e_2, e_3] = \beta e_2.$$

The Riemannian connection \$D\$ of the metric is given by

$$2g(D_X Y, Z) = Xg(Y,Z) + Yg(X,Z) - Zg(X,Y) - g(X,[Y,Z]) - g(Y,[X,Z]) + g(Z,[X,Y]),$$

which is know as Koszul's formula.

Koszul's formula yields

$$D_{e_1}e_1 = \beta e_3, \quad D_{e_1}e_2 = 0, \quad D_{e_1}e_3 = \beta e_1,$$
$$D_{e_2}e_1 = 0, \quad D_{e_2}e_2 = \beta e_3, \quad D_{e_2}e_3 = \beta e_2$$
$$D_{e_3}e_1 = 0, \quad D_{e_3}e_2 = 0, \quad D_{e_3}e_3 = 0.$$

From above it can be easily seen that $M^3(\phi, \xi, \eta, g)$is a Lorentzian β-Kenmotsu manifold .

It is known that

$$R(X,Y)Z = D_X D_Y Z - D_Y D_X Z - D_{[X,Y]}Z. \quad (4.1)$$

With the help of the above results and using eq. (4.1), we can easily calculate the non-vanishing components of the curvature tensor as follows

$$R(e_1,e_2)e_1 = -\beta^2 e_2, \quad R(e_1,e_2)e_2 = \beta^2 e_1, \quad R(e_1,e_2)e_3 = 0,$$
$$R(e_2,e_3)e_1 = 0, \quad R(e_2,e_3)e_2 = -\beta^2 e_3, \quad R(e_2,e_3)e_3 = -\beta^2 e_2,$$
$$R(e_1,e_3)e_1 = -\beta^2 e_3, \quad R(e_1,e_3)e_2 = 0, \quad R(e_1,e_3)e_3 = -\beta^2 e_1,$$
$$R(e_1,e_1)e_1 = R(e_1,e_1)e_2 = R(e_1,e_1)e_3 = 0,$$
$$R(e_2,e_2)e_1 = R(e_2,e_2)e_2 = R(e_2,e_2)e_3 = 0,$$
$$R(e_3,e_3)e_1 = R(e_3,e_3)e_2 = R(e_3,e_3)e_3 = 0.$$

and their covariant derivative are given by

$$(D_{e_1}R)(e_1,e_2)e_1 = (D_{e_2}R)(e_1,e_2)e_1 = (D_{e_3}R)(e_1,e_2)e_1 = 0,$$
$$(D_{e_1}R)(e_2,e_3)e_1 = (D_{e_2}R)(e_2,e_3)e_2 = (D_{e_3}R)(e_2,e_3)e_3 = 0,$$
$$(D_{e_1}R)(e_1,e_3)e_1 = (D_{e_2}R)(e_1,e_3)e_2 = (D_{e_3}R)(e_1,e_3)e_3 = 0.$$

We now verify that 3-dimensional Lorentzian β-Kenmotsu manifold is not semi-pseudo symmetric i.e. it satisfies the relation (2.12).

Let us now consider

$$A(e_i) = 0 \text{ for } i = 1,2,3$$

at any point $X \in \chi(M)$. In our M^3, (2.12) reduces with these 1-form to the following equations,

$$(D_{e_i}R)(e_1,e_2)e_1 = 2A(e_i)\, R(e_1,e_2)e_3 + A(e_1)\, R(e_i,e_2)e_3 + A(e_2)R(e_1,e_i)e_3 + A(e_3)R(e_1,e_2)e_i$$
$$(D_{e_i}R)(e_2,e_3)e_1 = 2A(e_i)\, R(e_2,e_3)e_1 + A(e_2)\, R(e_i,e_3)e_1 + A(e_3)R(e_2,e_i)e_1 + A(e_1)R(e_2,e_3)e_i$$
$$(D_{e_i}R)(e_1,e_3)e_1 = 2A(e_i)\, R(e_1,e_3)e_1 + A(e_1)\, R(e_i,e_3)e_1 + A(e_3)R(e_1,e_i)e_1 + A(e_i)R(e_1,e_3)e_i$$

This implies that with respect to the 1-form under consideration the manifold is not semi-pseudo symmetric.

Thus, we have the following theorem:

Theorem4.1: A $(SPS)_n$cannot be a 3-dimensional Lorentzian β-Kenmotsu manifold.

5. Lorentzian $\boldsymbol{\beta}$-Kenmotsu$(\boldsymbol{SPRS})_{\boldsymbol{n}}$-manifold$(\boldsymbol{n} > 3)$:

In this section, we assume that a $(SPRS)_n$is a Lorentzian β-Kenmotsu manifold. From (2.9) and (2.14), we have the following expression

$$(D_X Ric)(Y,\xi) = -(n-1)\beta^2 A(Y)\eta(X) + A(\xi)Ric(Y,X). \quad (5.1)$$

From (3.2) and (5.1), we get

$$-(n-1)\beta^2 A(Y)\eta(X) + A(\xi)Ric(Y,X) = -(n-1)\beta^2 g(D_X\xi,Y) - Ric(Y,D_X\xi). \quad (5.2)$$

Putting ξ for X in (5.2), we get

$$\beta^2[A(Y) - A(\xi)\eta(Y)] = 0. \quad (5.3)$$

Again putting $\xi$for Y in (5.3), we get

$$\beta^2 A(\xi) = 0. \quad (5.4)$$

From (5.3) and (5.4), we have

$$\beta^2 A(Y) = 0. \quad (5.5)$$

But $\beta^2 \neq 0$. Hence from (5.5), we get

$$A(Y) = 0,$$

which is inadmissible by the definition of $(SPRS)_n$.

Thus, we can state that the fallowing theorem:

Theorem 5.1: A $(SPRS)_n (n \geq 3)$ cannot be a Lorentzian β-Kenmotsu manifold, provided $\beta^2 \neq 0$.

6. Example:

Let us consider the 3-dimensional manifold $M = \{(x, y, z) \epsilon \mathbb{R}^3, z \neq 0\}$, where (x, y, z) are standard co-ordinate in $\mathbb{R}^3$.

We choose the vector fields

$$e_1 = e^z \frac{\partial}{\partial x}, \quad e_2 = e^z \frac{\partial}{\partial y}, \quad e_3 = k \frac{\partial}{\partial z}$$

which is linearly independently at each point of M.

Let g be the Riemannian metric defined by

$$g(e_i, e_j) = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$$

Let ηbe the 1-form which satisfies the relation

$$\eta(e_3) = -1$$

Let ϕ be the (1,1) tensor field defined by

$$\phi e_1 = -e_1, \ \phi e_2 = -e_2, \ \phi e_3 = 0.$$

Then, we have

$$\phi^2 U = U + \eta(U) e_3 \text{and} \quad g(\phi U, \phi W) = g(U, W) + \eta(U)\eta(W),$$

for any $U, W \epsilon \chi(M)$.

Thus for $e_3 = \xi, (\phi, \xi, \eta, g)$ defines an almost LP contact structure on $\chi(M)$.

Now calculating, we have

$$[e_1, e_2] = 0, \ [e_1, e_3] = -ke_1, \ [e_2, e_3] = -ke_2.$$

By the Koszul's formula, we get

$$D_{e_1} e_1 = -ke_3, \quad D_{e_1} e_2 = 0, \quad D_{e_1} e_3 = -ke_1,$$
$$D_{e_2} e_1 = 0, \quad D_{e_2} e_2 = -ke_3, \ D_{e_2} e_3 = -ke_2$$
$$D_{e_3} e_1 = 0, \quad D_{e_3} e_2 = 0, \quad D_{e_3} e_3 = 0.$$

From above it can be easily seen that (ϕ, ξ, η, g) is a Lorentzian β-Kenmotsu structure on M. Consequently $M^3(\phi, \xi, \eta, g)$ is a Lorentzian β-Kenmotsu manifold with $\beta = -k$.

Using the above relation, we can easily calculate the curvature tensor as follows

$$R(e_1, e_2)e_1 = -k^2 e_2, \quad R(e_1, e_2)e_2 = k^2 e_1, \ R(e_1, e_2)e_3 = 0,$$
$$R(e_2, e_3)e_1 = 0, \quad R(e_2, e_3)e_2 = -k^2 e_3, \quad R(e_2, e_3)e_3 = -k^2 e_2,$$
$$R(e_1, e_3)e_1 = -k^2 e_3, \quad R(e_1, e_3)e_2 = 0, \quad R(e_1, e_3)e_3 = -k^2 e_1,$$
$$R(e_1, e_1)e_1 = R(e_1, e_1)e_2 = R(e_1, e_1)e_3 = 0,$$
$$R(e_2, e_2)e_1 = R(e_2, e_2)e_2 = R(e_2, e_2)e_3 = 0,$$
$$R(e_3, e_3)e_1 = R(e_3, e_3)e_2 = R(e_3, e_3)e_3 = 0.$$

Form above expression of the curvature tensor, we obtain

$$Ric(X, Y) = \sum_{i=1}^{3} g\,(R(X, e_i)e_i, Y) \text{ as}$$

$$Ric(e_1, e_1) = 0, \ Ric(e_2, e_2) = 0 \ \ Ric(e_3, e_3) = -2k^2.$$

Since$\{e_1, e_2, e_3\}$ form a basis of the Lorentzian β-Kenmotsu manifold any vector field Y, Z can be written as

$$Y = a_1 e_1 + b_1 e_2 + c_1 e_2, \ Z = a_2 e_1 + b_2 e_2 + c_2 e_2.$$

where $a_i, b_i, c_i \epsilon \mathbb{R}^+$ (the set of all positive real numbers), $i = 1, 2, 3$. This implies that

$$Ric(Y, Z) = -2\, c_1 c_2 k^2.$$

By above equation, we have

$$(D_{e_i} Ric)(Y, Z) = D_{e_i} Ric\,(Y, Z) - Ric(D_{e_i} Y, Z) - Ric(Y, D_{e_i} Z)$$

$$(D_1Ric)(Y,Z) = -2k^3(a_1c_2 + a_2c_1)$$
$$(D_{e_2}Ric)(Y,Z) = -2k^3(b_1c_2 + b_2c_1)$$
$$(D_{e_3}Ric)(Y,Z) = 0.$$

Let us now consider

$$a_1c_2 + a_2c_1 = 0 \; \& b_1c_2 + b_2c_1 = 0 \text{and } A(e_3) = 0. \tag{6.1}$$

at any point $X \epsilon M$.

From (2.14), We have

$$(D_{e_i}Ric)(Y,Z) = A(Y)Ric(e_i,Z) + A(Z)Ric(e_i,Y). \tag{6.2}$$

It can be easily shown that the manifold with (6.1) satisfies the relation (6.2).

Hence the manifold under consideration is not $(SPRS)_n$ Lorentzian β-Kenmotsu maifold.

Thus we can state that the fallowing theorem:

Theorem 6.1: A $(SPRS)_n$ $(n \geq 3)$ cannot be a 3-dimensional Lorentzian β-Kenmotsu manifold.

7. Application

A perfect flow on Riemannian manifold (Chaki and Barua,1999) is a triple $\phi = (\xi, p, \sigma)$ where

(i) ξ is non null vector field call the flow vector.

(ii) p and σ are scalar field such that $p + \sigma \neq 0$.

If $p + \sigma = 0$, we may called (ξ, p, σ) a trivial perfect flow and if p=0, it is called an incoherent flow.

A tensor field

$$T(X,Y) = (p+\sigma)\eta(X)\eta(Y) - pg(X,Y). \tag{7.1}$$

where $g(X,\xi) = \eta(X)$ is called the energy-momentum tensor of the perfect flow (ξ, p, σ) if $div(T) = 0$.

Let $G(X,Y) = Ric(X,Y) - \frac{r}{2}g(X,Y)$, be the Einstein tensor. Then we suppose

$$G(X,Y) = k_1T(X,Y), \tag{7.2}$$

where k_1 is constant.

Thus in view of (7.1) and (7.2), we find

$$Ric(X,Y) - \frac{r}{2}g(X,Y) = k_1[(p+\sigma)\eta(X)\eta(Y) - pg(X,Y)]. \tag{7.3}$$

Contraction of (7.3), we get

$$\left(\frac{n-2}{2}\right).r = k_1[(n+1)p + \sigma]. \tag{7.4}$$

Again putting ξ for X in (7.3) and using (2.1) and (2.9), we find

$$r = 2[k_1(2p+\sigma) - (n-1)\beta^2]. \tag{7.5}$$

because $\eta(Y)$ cannot vanish.

By virtue of (7.4) and (7.5), we obtain

$$p = \frac{(n-1)(n-2)\beta^2 - k_1\sigma(n-3)}{k_1(n-5)}. \tag{7.6}$$

From (7.3) and (7.6), we get

$$\sigma = \frac{1}{k_1}\left[(n+1)\beta^2 - \frac{(n-5)}{2(n-1)}r\right]. \tag{7.7}$$

From (7.6) and (7.7), we get

$$p = \frac{1}{k_1}\left[-\beta^2 - \frac{(n-3)}{2(n-1)}r\right]. \tag{7.8}$$

Thus we have

$$p + \sigma = \frac{1}{k_1}\left[n\beta^2 + \frac{r}{n-1}\right] \neq 0. \tag{7.9}$$

Thus, we can state that the fallowing theorem:

Theorem 5.1: In Lorentzian β-Kenmotsu manifold, the mass density and pressure density σ and p are given by (7.7) and (7.8) such that $p + \sigma \neq 0$.

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