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LP-Sasakian manifold equipped with quarter-symmetric non-metric connection

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Abstract

The object of the present paper is to study on an LP-Sasakian manifold with quarter-symmetric non-metric connection. In this paper, we consider some properties of the curvature tensor, projective curvature tensor, conharmonic curvature tensor, concircular curvature tensor and conformal curvature tensor with respect to the quarter-symmetric non-metric connection in an LP-Sasakian manifold, curvature tensor.

Keywords-LP-Sasakian manifold, curvature tensor, projective curvature tensor, conharmonic curvature tensor, concircular curvature tensor, conformal curvature tensor and quarter-symmetric non-metric connection.

Introduction

Let (M, g) be an n-dimensional connected semi-Riemannian manifold of class C^∞ and D be its Levi-Civita connection. The Riemannian-Christoffel curvature tensor R, projective curvature tensor P, Conharmonic curvature tensor H, Conformal curvature tensor C and concircular curvature tensor V of (M, g) are defined by Yano and Kon (1984)

$$R(X,Y)Z = D_X D_Y Z - D_Y D_X Z - D_{[X,Y]}Z, \quad (1.1)$$

$$P(X,Y)Z = R)X,Y)Z - \frac{1}{n-1}[Ric(Y,Z)X - Ric(X,Z)Y] \quad (1.2)$$

$$H(X,Y)Z = R(X,Y)Z - \frac{1}{n-2}[Ric(Y,Z)X - Ric(X,Z)Y + g(Y,Z)QX - g(X,Z)QY], \quad (1.3)$$

$$C(X,Y)Z = R(X,Y)Z - \frac{1}{n-2}Ric(Y,Z)X - Ric(X,Z)Y + g(Y,Z)QX - g(X,Z)QY + \frac{r}{(n-1)(n-2)}[g(Y,Z)X - g(X,Z)Y], \quad (1.4)$$

and

$$V(X,Y)Z = R(X,Y)Z - \frac{r}{n(n-1)}[g(Y,Z)X - g(X,Z)Y], \quad (1.5)$$

respectively, where r is the scalar curvature tensor, Ric is the Ricci tensor and Q is the Ricci operator such that $\mathrm{Ric}(X, Y) = g(QX, Y)$.

A linear connection $\overline{D}$ defined on (M, g) is said to be a quarter-symmetric connection Golab (1975) if its torsion tensor $\overline{T}$ is given by the equation

$$\overline{T}(X, Y) = \overline{D}_X Y - \overline{D}_Y Y - [X, Y], \tag{1.6}$$

satisfies

$$\overline{T}(X, Y) = \eta(Y)\phi X - \eta(X)\phi Y,$$

where η is a 1-form and ϕ is a (1, 1) tensor field. If moreover, a quarter-symmetric connection $\overline{D}$ satisfies the condition

$$(\overline{D}_X g)(Y, Z) = 0, \tag{1.7}$$

where X, Y, Z ∈ Ω(M) and Ω(M) is the set of all differentiable vector fields on M, then $\overline{D}$ is said to be quarter-symmetric metric connection Friedmenn and Schonton (1924). In particular, if $\phi X = X$, then the quarter-symmetric connection reduces to the semi-symmetric connetion. Thus the notion of quarter-symmetric connection generalizes the notion of semi-symmetric connection.

If moreover, a quarter-symmetric connection $\overline{D}$ satisfies the condition

$$(\overline{D}_X g)(Y, Z) \neq 0, \tag{1.8}$$

then $\overline{D}$ is said to be a quarter-symmetric non-metric connection.

These connections have been studied by many authors. For instance, we cite Berman (2015), De and Sengupta (2000), Prasad and Narain (2011), Hui (2013), Prasad and Haseeb (2016), Bahdir (2016)and references therin.

In the present paper, after introduction and preliminaries, in section 3 and section 4, we prove the existence of quarter-symmetric non-metric connection and then find its curvature tensor, Ricci tensor and scalar curvature of $\overline{D}$. In section 5, we prove that if LP-Sasakian manifold satisfying the condition $\overline{R}.\overline{\mathrm{Ric}} = 0$, then the manifold is η-Einstein manifold with $\overline{D}$. In section 6, 7 and 8 we consider Z-tensor, Schouten tensor and Projective Ricci tensor of LP-Sasakian manifold with respect to quarter-symmetric non-metric connection $\overline{D}$ respectively. ξ-projectively flat, ξ-conharmonically flat, ξ-conformally flat and ξ-concircularly flat of LP-Sasakian manifold with respect to quarter-symmetric non-metric connection $\overline{D}$ studied in section 9, 10, 11 and 12 respectively.

2. Preliminaries

Let M be a differentiable manifold of dimension n endowed with a (1, 1) tensor field ϕ, a contravariant vector field ξ, a 1-form η and a Lorentzian metric g, which satisfies

$$\phi^2 X = X + \eta(X)\xi, \eta(\xi) = -1, \tag{2.1}$$

$$g(\phi X, \phi Y) = g(X, Y) + \eta(X)\eta(Y), g(X, \xi) = \eta(X) \tag{2.2}$$

$$(D_X\phi)(Y) = g(X, Y)\xi + \eta(Y)X + 2\eta(X)\eta(Y)\xi, \qquad (2.3)a$$

$$D_X\xi = \phi X, \qquad (2.3)b$$

for all vector fields X and Y on M, where D is the Levi-Civita connection with respect to the Lorentzian metric g. Such manifold (M, ϕ, ξ, η, g) is called Lorentzian para-Sasakian (Shortly, LP-Sasakion) manifold Motsumoto (1989), Matsumoto and Mihai (1988). The following are also true in LP-Sasakian manifold

$$\phi\xi = 0, \eta(\phi X) = 0 \text{ and rank } \phi = n - 1. \tag{2.4}$$

Let us put

$$F(X, Y) = g(\phi X, Y), \tag{2.5}$$

for all vector fields X and Y on M, then the tensor field F is a symmetric (0, 2) tensor field, That is

$$F(X, Y) = F(Y, X). \tag{2.6}$$

Moreover, if η is closed on an LP-Sasakian manifold then we have

$$(D_X\eta)(Y) = F(X, Y) \text{ and } F(X, \xi) = 0, \tag{2.7}$$

for any vector field X and Y on M. An LP- Sasakian manifold provides the following results Matsumoto and Mihai (1988), Mihai and Rosca (1992) and Mihai, Shaikh and De (1999);

$$R(\xi, X)Y = g(X, Y)\xi - \eta(Y)X, \tag{2.8}$$

$$R(X, Y)\xi = \eta(Y)X - \eta(X)Y, \tag{2.9}$$

$$R(\xi, X)\xi = X + \eta(X)\xi, \tag{2.10}$$

$$g(R(X, Y)\, Z, \xi) = \eta(R(X, Y)Z) = g(Y, Z)\eta(X) - g(X, Z)\eta(Y), \tag{2.11}$$

$$Ric(X, \xi) = (n - 1)\eta(X), Ric(\xi, \xi) = -(n - 1), \tag{2.12}$$

$$Ric(\phi X, \phi Y) = Ric(X, Y) + (n - 1)\eta(X)\eta(Y), \tag{2.13}$$

for all vector fields X and Y on M.

An LP-Sasakian manifold M is said to be an η-Einstein manifold if its Ricci tensor is of the form

$$Ric(X, Y) = a\, g(X, Y) + b\eta(X)\eta(Y), \tag{2.14}$$

where a and b are scalar functions on M.

An LP-Sasakian function M is said to be a generalized η-Einstein manifold if its Ricci tensor Ric is of the following form Haseeb and Prasad (2018)

$$Ric(X, Y) = a\, g(X, Y) + b\, \eta(X)\eta(Y) + cF(X, Y), \tag{2.15}$$

where a, b and c are scalar functions.

$F(X, Y) = g(\phi X, Y)$. If c = 0 then the manifold reduces to an η-Einstein manifold.

3. Existence of quarter-symmetric non-metric connection in an LP-Sasakiam manifold

Let (M, g) be an LP-Sasakian manifold with Levi-Civita connection D. We define a linear connection $\overline{D}$ on M by

$$\overline{D}_X Y = D_X Y + \eta(Y)\phi X - \eta(X)\phi Y, \tag{3.1}$$

where η is 1-forms associated with vector field ξ on M given by

$$g(X, \xi) = \eta(X), \tag{3.2}$$

for all vector field X on M.

Using (3.1), the torsion tensor $\overline{T}$ of M with respect to the connection $\overline{D}$ is given by

$$\overline{T}(X,Y) = 2[\eta(Y)\phi X - \eta(X)\phi Y]. \qquad (3.3)$$

A linear connection satisfying (3.3) is called a quarter-symmetric connection. Further using (3.1), we get

$$(\overline{D}_X g)(Y,Z) = 2\eta(X)g(\phi Y,Z) - \eta(Y)g(\phi X,Z) - \eta(Z)g(Y,\phi X). \qquad (3.4)$$

A linear connection $\overline{D}$ defined by (3.1) satisfies (3.3) and (3.4) is called a quarter-symmetric non-metric connection.

Conversely, we show that a linear connection $\overline{D}$ defined on M satisfying (3.3) and (3.4) is given by (3.1).

Let H be a tensor field of type (1, 2) is given by

$$\overline{D}_X Y = D_X Y + H(X,Y). \qquad (3.5)$$

Thus we have

$$\overline{T}(X,Y) = H)X,Y) - H(Y,X). \qquad (3.6)$$

This implies that

$$g(H(X,Y)Z + g\big(Y,H(X,Z)\big) = \eta(Y)g(\phi X,Z) + \eta(Z)g(Y,\phi X) - 2\eta(X)g(\phi Y,Z). \qquad (3.7)$$

Moreover, we have

$$g(\overline{T}(X,Y),Z) + g(\overline{T}(Z,X),Y) + g(\overline{T}(Z,Y),X) = 2g(H(X,Y),Z - 4\eta(Z)g(\phi X,Y) + 2\eta(X)g(\phi Y,Z) + 2\eta(Y)g(\phi Z,X). \qquad (3.8)$$

Equation (3.8) can be put as

$$g(H(X,Y),Z) = \frac{1}{2}[g(\overline{T}(X,Y),Z) + g(\overline{T}(Z,X),Y) + g(\overline{T}(Z,Y),X)] + 2\eta(Z)g(\phi X,Y) - -\eta(X)g(\phi Y,Z) - \eta(Y)g(\phi Z,X). \qquad (3.9)$$

Let $\overline{T}'$ be a tensor field of type (1, 2) defined by

$$g(\overline{T}'(X,Y),Z) = g(\overline{T}(Z,X),Y). \qquad (3.10)$$

Thus in view of (3.3) and (3.10), we get

$$\overline{T}'(X,Y) = 2\eta(X)\phi Y - 2g(\phi X,Y)\xi. \qquad (3.11)$$

Hence from (3.9), (3.10) and (3.11), we get

$$H(X,Y) = \eta(Y)\phi X - \eta(X)\phi Y. \qquad (3.12)$$

Hence from (3.5) and (3.12), we get

$$\overline{D}_X Y = D_X Y + \eta(Y)\phi X - \eta(X)\phi Y. \qquad (3.13)$$

This proves the existence of quarter-symmetric non-metric connection in LP-Sasakian manifold.

4. Curature tensor of an LP-Sasakian manifold with respect to the quarter-symmetric non-metric connection $\overline{D}$

Let $\overline{R}$ and R be the curvature tensor of the connection $\overline{D}$ and D respectively then

$$\overline{R}(X,Y)Z = \overline{D}_X\overline{D}_Y Z - \overline{D}_Y\overline{D}_X Z - \overline{D}_{[X,Y]} Z. \qquad (4.1)$$

From (3.1) and (4.1), we get

$$\bar{R}(X,Y)Z = R(X,Y)Z + [(D_X\eta)(Z)\phi Y - (D_Y\eta)(Z)\phi X] + \eta(Z)[(D_X\phi)(Y) - (D_Y\phi)(X)] - [(D_X\eta)(Y)\phi Z - (D_Y\eta)(X)\phi Z + \eta(Y)(D_X\phi)(Z) - \eta(X)(D_Y\phi)(Z)] - [\eta(X)\eta(Z)\phi^2 Y - \eta(Z)\eta(Y)\phi^2 X]. \quad (4.2)$$

In view of (2.1), (2.3) and (4.2), we get

$$\bar{R}(X,Y)Z = R(X,Y)Z + g(\phi X,Z)\phi Y - g(\phi Y,Z)\phi X - \eta(Y)g(X,Z)\xi + \eta(X)g(Y,Z)\xi. \quad (4.3)$$

From (4.3), we see that

$$\bar{R}(X,Y)Z = -\bar{R}(Y,X)Z, \quad (4.4)$$

and

$$\bar{R}(X,Y)Z + \bar{R}(Y,Z)X + \bar{R}(Z,X)Y = 0. \quad (4.5)$$

From (4.3), we get

$$'\bar{R}(X,Y,Z,W) = 'R(X,Y,Z,W) + g(\phi X,Z)g(\phi Y,W) - g(\phi Y,Z)g(\phi X,W) - \eta(Y)\eta(W)g(X,Z) + \eta(X)\eta(W)g(Y,Z), \quad (4.6)$$

where $\bar{R}(X,Y,Z,W) = g(\bar{R}(X,Y)Z,W)$

and

$$'R(X,Y,Z,W) = g(R(X,Y)Z,W).$$

Hence in view of (4.6), we get

$$'\bar{R}(X,Y,Z,W) + '\bar{R}(X,Y,W,Z) \neq 0, \quad (4.7)$$

and

$$'\bar{R}(X,Y,Z,W) - '\bar{R}(X,Y,W,Z) \neq 0, \quad (4.8)$$

Contracting (4.3) with respect to X, we get

$$\bar{R}ic(Y,Z) = Ric(Y,Z) - g(\phi Y,Z)\psi, \quad (4.9)$$

where $\bar{R}ic$ is the Ricci tensor of $\bar{D}$ and trace $\phi = \psi$.

Again contracting (4.9), we get

$$\bar{r} = r - \psi^2, \quad (4.10)$$

where $\bar{r}$ is the scalar curvature with respect to $\bar{D}$.

Putting ξ for Z in (4.3) and (4.9), we get

$$\bar{R}(X,Y)\xi = \eta(Y)X - \eta(X)Y, \quad (4.11)$$

$$\bar{R}ic(Y,\xi) = (n-1)\eta(Y) \quad (4.12)$$

and

$$\bar{Q}\xi = Q\xi \quad (4.13)$$

From the above discussion, we can state the following theorem as follows:

Theorem (4.1) : For an LP-Sasakian manifold M with respect to the quarter-symmetric non-metric connection $\bar{D}$

(i) The curvature tensor $\bar{R}$ is given by (4.3),

(ii) $'\bar{R}(X,Y,Z,W) + '\bar{R}(Y,X,Z,W) = 0$,

(iii) $'\bar{R}(X,Y,Z,W) + '\bar{R}(X,Y,W,Z) \neq 0$,

(iv) $'\bar{R}(X,Y,Z,W) - '\bar{R}(Z,W,X,Y) \neq 0$,

(v) $'\bar{R}(X,Y,Z,W) + '\bar{R}(Y,Z,X,W) + '\bar{R}(Z,X,Y,W) = 0$,

(vi) The Ricci tensor $\overline{Ric}$ is given by (4.9),

(vii) The scalar curvature $\bar{r}$ is given by (4.10),

(viii) $\bar{R}(X,Y)\xi = \eta(Y)X - \eta(X)Y$,

(ix) $\overline{Ric}(Y,\xi) = (n-1)\eta(Y)$,

(x) $\bar{Q}\xi = Q\xi$.

Theorem (4.2): If an LP-Sasakian manifold admits a quarter-symmetric non-metric connection $\bar{D}$, then the Ricci tensor $\overline{Ric}$ of the $\bar{D}$ is equal to Ric tensor Ric of D if and only if $\psi = 0$.

Theorem (4.3): If an LP-Sasakian manifold admits a quarter-symmetric non-metric connection $\bar{D}$, then the scalar curvature $\bar{r}$ of $\bar{D}$ is equal to the scalar curvature r of D if and only if $\psi = 0$.

5. LP-Sasakian manifold with respect to quarter-symmetric non-metric connection satisfying $\bar{R}.\overline{Ric} = 0$.

In this section we consider a LP-Sasakian with respect to the quarter symmetric non-metric connection $\bar{D}$ satisfying the condition

$$\bar{R}(X,Y).\overline{Ric} = 0. \tag{5.1}$$

From (5.1), we get

$$\overline{Ric}(\bar{R}(X,Y)Z,W) + \overline{Ric}(Z,\bar{R}(X,Y)W) = 0. \tag{5.2}$$

Putting ξ for Z and X in (5.2), we get

$$\overline{Ric}(\bar{R}(\xi,Y)\xi,W) + \overline{Ric}(\xi,\bar{R}(\xi,Y)W = 0. \tag{5.3}$$

In view of (4.11), (4.12), (4.13) and (5.3), we get

$$\overline{Ric}(Y,W) = (n-1)\eta(Y)\eta(W). \tag{5.4}$$

Hence we can state the following theorem:

Theorem (5.1) : For an LP-Sasakian manifold with respect to the quarter-symmetric non-metric connection $\bar{D}$ satisfying the condition $\bar{R}.\overline{Ric} = 0$, then the Ricci tensor is η-Einstein manifold where a = 0 and b = $(n-1)$ with respect to quarter-symmetric non-metric connection $\bar{D}$.

6. Z-tensor with respect to the quarter-symmetric non-metric connection $\bar{D}$ in an LP-Sasakian manifold

Z-tensor in a Riemannian manifold $Z(X,Y)$ is defined by Mantica and Molinari (2012) as

$$Z(X,Y) = Ric(X,Y) + a.g(X,Y), \tag{6.1}$$

where a is smooth function.

Analogous to this definition, we define Z-tensor with respect to quarter-symmetric non-metric connection $\bar{D}$ given by

$$\bar{Z}(X,Y) = \overline{Ric}(X,Y) + a.g(X,Y). \tag{6.2}$$

In view of (4.9), (6.1) and (6.2), we get

$$\bar{Z}(X,Y) = Z(X,Y) - g(\phi X,Y)\psi. \tag{6.3}$$

In view of (6.3), we can state the following theorem:

Theorem (6.1) : The Z-tensor $\overline{Z}(X,Y)$ of the manifold with respect to quarter-symmetric non-metric connection $\overline{D}$ in an LP-Sasakian manifold is equal to Z-tensor $Z(X,Y)$ of the manifold with respect to Levi-Civita connection D if and only if $\psi = 0$.

7. Schouten tensor $\overline{P}_S(X,Y)$ with respect to quarter-symmetric non-metric connection $\overline{D}$ in an LP-Sasakian manifold

Schouten tensor $P_S(X,Y)$ in a Riemannian manifold is defined by Chen and Yano (1972)

$$P_S(X,Y) = \frac{1}{n-2}\left[Ric(X,Y) - \frac{r}{2(n-1)}g(X,Y)\right]. \tag{7.1}$$

Analogous to this definition, we define Schouten tensor with respect to quarter-symmetric non-metric connection $\overline{D}$ as

$$\overline{P}_S(X,Y) = \frac{1}{n-2}\left[\overline{Ric}(X,Y) - \frac{\overline{r}}{2(n-1)}g(X,Y)\right]. \tag{7.2}$$

In view of (4.9), (4.10), (7.1) and (7.2), we get

$$\overline{P}_S(X,Y) = P_S(X,Y) - \frac{\psi}{n-2}\left[g(\phi X,Y) - \frac{\psi}{2(n-1)}g(X,Y)\right]. \tag{7.3}$$

In view of (7.3), we have the following theorem:

Theroem (7.1) : The Schouton tensor $\overline{P}_S(X,Y)$ of the manifold with respect to quarter-symmetric non-metric connection $\overline{D}$ in an LP-Sasakian manifold is equal to Schouten tensor $P_S(X,Y)$ of the manifold with respect to Levi-Civita connection D if and only if $\psi = 0$.

8. Projective Ricci tensor $\overline{P}_R(X,Y)$ with respect to quarter-symmetric non-metric connection $\overline{D}$ in an LP-Sasakian manifold

Projective Ricci tensor $P_R(X,Y)$ in a Riemannian manifold is defiend by Chaki and Saha (1995)

$$P_R(X,Y) = \frac{n}{n-1}\left[Ric(X,Y) - \frac{r}{n}g(X,Y)\right]. \tag{8.1}$$

Analogous to this definition, we define projective Ricci tensor with respect to quarter-symmetric non-metric connection $\overline{D}$ by

$$\overline{P}_R(X,Y) = \frac{n}{n-1}\left[\overline{Ric}(X,Y) - \frac{\overline{r}}{n}g(X,Y)\right]. \tag{8.2}$$

In view of (4.9), (4.10) and (8.2), we get

$$\overline{P}_R(X,Y) = \frac{n}{n-1}Ric(X,Y) - \frac{n}{n-1}g(\phi X,Y)\psi - \frac{1}{n-1}(r - \psi^2)g(X,Y). \tag{8.3}$$

Form (8.3), we get

$$\overline{P}_R(Y,X) = \frac{n}{n-1}Ric(Y,X) - \frac{n}{n-1}g(\phi Y,X)\psi - \frac{1}{n-1}(r - \psi^2)g(X,Y). \tag{8.4}$$

From equation (8.3) and (8.4), we get

$$\overline{P}_R(x,Y) + \overline{P}_R(Y,X) = \frac{2n}{n-1}Ric(X,Y) - \frac{2n}{n-1}g(\phi X,Y)\psi - \frac{2}{n-1}(r - \psi^2)g(X,Y). \tag{8.5}$$

If $\overline{P}_R(X,Y)$ is skew-symmetric than the L.H.S. of (8.5) vanishes and hence, we get

$$Ric(X,Y) = g(\phi X,Y)\psi + \frac{1}{n}(r - \psi^2)g(X,Y) \tag{8.6}$$

Moreover if Ric(X, Y) is given by (8.6), then from (8.5), we get

$\overline{P}_R(X, Y) + \overline{P}_R(Y, X) = 0,$

i.e. projective Ricci tensor of $\overline{D}$ is skew-symmetric.

Hence we can state the following theorem:

Theorem (8.1) If an LP-Sasakian manifold admits a quarter-symmetric non-metric connection $\overline{D}$ than a necessary and sufficient condition for the projective Ricci tensor $\overline{P}_R(X, Y)$ of $\overline{D}$ to be skew-symmetric is that the Ricci tensor of the Levi-Civita connection D is given by (8.6).

9. Projective curvature tensor $\overline{P}$ with respect to quarter-symmetric non-metric connection in an LP-Sasakian manifold

Projective curvature tensor $\overline{P}$ of type (1, 3) of M with respect to quarter-symmetric non-metric connection $\overline{D}$ is defined by

$$\overline{P}(X, Y)Z = \overline{R}(X, Y)Z - \frac{1}{n-1}[\overline{Ric}(Y, Z)X - \overline{Ric}(X, Z)Y]. \tag{9.1}$$

In view of (1.2), (4.3) and (4.9), we get

$$\overline{P}(X, Y)Z = R(X, Y)Z + g(\phi X, Z)\phi Y - g(\phi Y, Z)\phi X - \eta(Y)g(X, Z)\xi + \eta(X)g(Y, Z)\xi - \frac{1}{n-1}[Ric(Y, Z)X - Ric(X, Z)Y - g(\phi Y, Z)X.\psi + g(\phi X, Z)Y.\psi]. \tag{9.2}$$

Put ξ for Z in (9.2) and then using (4.11) and (4.12), we get

$$\overline{P}(X, Y)\xi = 0. \tag{9.3}$$

Again equation (9.2) can be written as

$$\overline{P}(X, Y)\xi = P(X, Y)\xi. \tag{9.4}$$

Form (9.3) and (9.4), we get

$$P(X, Y)\xi = 0. \tag{9.5}$$

From (9.3) and (9.4), we can state the following theorem :

Theorem (9.1) : Let M be an n-dimensional LP-Sasakian manifold. Then M is ξ-projectively flat with respect to quarter-symmetric non-metric connection $\overline{D}$ and hence manifold is also ξ-protectively flat with respect to Levi-Civita connection D.

10. Conharmonic curvature tensor $\overline{H}$ with respect to quarter-symmetric non-metric connection in an LP-Sasakian manifold

Definition (10.1) : An LP-Sasakian manifold M with respect to the quarter-symmetric non-metric connection is said to be ξ-conharmonically flat if

$$\overline{H}(X, Y)\xi = 0, \tag{10.1}$$

for all vector fields X and Y of M.

Conharmonic curvature tensor $\overline{H}$ with respect to quarter-symmetric non-metric connection $\overline{D}$ is given by

$$\overline{H}(X, Y)Z = \overline{R}(X, Y)Z - \frac{1}{n-2}[\overline{Ric}(Y, Z)X - \overline{Ric}(X, Z)Y + g(Y, Z)\overline{Q}X - g(X, Z)\overline{Q}Y]. \tag{10.2}$$

In view of (1.3), (4.3), (4.9) and (10.2), we get

$$\bar{H}(X,Y)Z = H(X,Y)Z + g(\phi X,Z)\phi Y - g(\phi Y,Z)\phi X - \eta(Y)g(X,Z)\xi + \eta(X)\, g(Y,Z)\xi - \frac{\psi}{n-2}[-g(\phi Y,Z)X + g(\phi X,Z) - g(Y,Z)\phi X + g(X,Z)\phi Y]. \quad (10.3)$$

Put ξ for Z in (10.3), we get

$$\bar{H}(X,Y)\xi = H(X,Y)\xi + \frac{\psi}{n-2}[\eta(Y)\phi X - \eta(X)\phi Y]. \quad (10.4)$$

In view of (10.4), we can state the fallowing theorem:

Theorem (10.1) A n-dimensional LP-Sasakian manifold with respect to quarter-symmetric non-metric connection is ξ-conharmonically flat if and only if the manifold with respect to the Levi-civita connection is also ξ-conharmonically flat, provided $\psi = 0$.

11. Conformal curvature tensor $\bar{C}$ with respect to the quarter-symmetric non-metric connection in an LP-Sasakian manifold

Definition (11.1) : An LP-Sasakian manifold M with respect to the quarter-Symmetric non-metric connection is said to be ξ-conformally flat if

$$\bar{C}(X,Y)\xi = 0, \quad (11.1)$$

for all vector fields X and Y on M.

In view of (1.4), (4.3), (4.9), (4.10), we get

$$\bar{C}(X,Y)Z = C(X,Y)Z + g(\phi X,Z)\phi Y - g(\phi Y,Z)\phi X - \eta(Y)g(X,Z)\xi + \eta(X)g(Y,Z)\xi + \frac{\psi}{n-2}[g(\phi Y,Z)X - g(\phi X,Z)\, Y + g(Y,Z)\phi X - g(X,Z)\phi Y + \frac{\psi^2}{(n-1)(n-2)}[g(Y,Z)X - g(X,Z)Y], \quad (11.2)$$

where $\bar{C}(X,Y)Z$ is the conformal curvature tensor of an LP-Sasakian manifold with respect to $\bar{D}$.

Putting ξ for Z in (11.2), we get

$$\bar{C}(X,Y)\xi = C(X,Y)\xi + \frac{\psi}{n-2}[\eta(Y)\phi X - \eta(X)\phi Y] + \frac{\psi^2}{(n-1)(n-2)}[\eta(Y) - \eta(X)Y]. \quad (11.3)$$

In view of (11.3), we can state the following theorem:

Theorem (11.1) A n-dimensional LP-Sasakiam manifold with respect to $\bar{D}$ is ξ-conformally flat if and only if the manifold with respect to D is also ξ-conformally flat provided $\psi = 0$.

12. Concircular curvature tensor $\bar{V}$ with respect to quarter-symmetric non-metric connection $\bar{D}$ in an LP-Sasakian manifold

Form (1.5), we can define concircular curvature tensor $\bar{V}$ with respect to $\bar{D}$ as follows

$$\bar{V}(X,Y)Z = \bar{R}(X,Y)Z - \frac{\bar{r}}{n(n-1)}[g(Y,Z)X - g(X,Z)Y]. \quad (12.1)$$

In view of (4.3), (4.10) and (12.1), we get

$$\bar{V}(X,Y)Z = R(X,Y)Z + g(\phi X,Z)\phi Y - g(\phi Y,Z)\phi X - \eta(Y)g(X,Z)\xi + \eta(X)g(Y,Z)\xi - \left(\frac{r-\psi^2}{n(n-1)}\right).[g(Y,Z)X - g(X,Z)Y]. \quad (12.2)$$

Putting ξ for Z in (12.2), we get

$$\bar{V}(X,Y)\xi = [\eta(Y)X - \eta(X)Y]\left[1 - \frac{r-\psi^2}{n(n-1)}\right] \qquad (12.3)$$

Definition (12.1) : For an LP-Sasakian manifold, manifold is ξ-concircularly flat with respect to quarter symmetric non-metric connection $\bar{D}$ if

$$\bar{V}(X,Y)\xi = 0.$$

Hence in view of (12.3) and definition (12.1), we can state the following theorem :

Theorem (12.1) For an LP-Sasakian manifold M is ξ-concircularly flat with respect $\bar{D}$ if and only if the scalar curvature tensor with D is equal to $\psi^2 + n(n-1)$.

Definition (12.2) Let M be an n-dimensional LP-Sasakian manifold. Then M is ϕ-concircularly flat with respect to quarter-symmetric non-metric connection $\bar{D}$. if $'\bar{V}(\phi X,\phi Y,\phi Z,\phi W) = 0$.

Now (12.1) can be written as

$$'\bar{V}(X,Y,Z,W) = {}'\bar{R}(X,Y,Z,W) - \frac{\bar{r}}{n(n-1)}[g(Y,Z)g(X,W) - g(X,Z)g(Y,W)], \qquad (12.4)$$

where $g(\bar{V}(X,Y)Z,W) = {}'\bar{V}(X,Y,Z,W)$.

Operating ϕ on X, Y, Z, and W in equation (12.4), get

$$'\bar{V}(\phi X,\phi Y,\phi Z,\phi W) = {}'\bar{R}(\phi X,\phi Y,\phi Z,\phi W) - \frac{\bar{r}}{n(n-1)}.[g(\phi Y,\phi Z)g(\phi X,\phi W) - g(\phi X,\phi Z)g(\phi Y,\phi W)]. \qquad (12.5)$$

For ϕ-concircularly flatness, we have from (12.5),

$$'\bar{R}(\phi X,\phi Y,\phi Z,\phi W) = \frac{\bar{r}}{n\,(n-1)}[g(\phi Y,\phi Z)g(\phi X,\phi W) - g(\phi X,\phi Z)g(\phi Y,\phi W)]. \qquad (12.6)$$

Let $\{e_1, e_2, \ldots \ldots e_{n-1}, \xi\}$ be a local orthogonal basis of the vector fields in M and using the fact $\{\phi e_1, e_2 \ldots \ldots \phi e_{n-1}, \xi\}$ is also a local orthogonal basis, putting e_i for X and W and summing up with respect to $i = 1, 2, \ldots (n-1)$, we get

$$\sum_{i=1}^{n-1} {}'\bar{R}(\phi e_i,\phi Y,\phi Z,\phi e_i) = \frac{\bar{r}}{n(n-1)}\sum_{i=1}^{n-1}\{g(\phi Y,\phi Z)\,g(\phi e_i,\phi e_i) - g(\phi e_i,\phi Z)g(\phi Y,\phi e_i)]. \qquad (12.7)$$

From (12.7), we get

$$\overline{Ric}(\phi Y,\phi Z) = \frac{\bar{r}(n-2)}{n(n-1)}.g(\phi Y,\phi Z). \qquad (12.8)$$

From (2.2), (2.13), (4.9), (4.10) and (12.8), we get

$$Ric(Y,Z) = (n-1)\eta(Y)\eta(Z) + g(\phi Y,Z)\psi + \frac{(r-\psi^2)(n-2)}{n(n-1)}[g(Y,Z) + \eta(Y)\eta(Z)] . \qquad (12.9)$$

Again contracting (12.9) over Y and Z, we get

$$r = \psi^2 - \frac{n(n-1)}{2}. \qquad (12.10)$$

Then from (12.10), we get the following theorem:

Theorem (12.2): If an LP-Sasakian manifold is ϕ-concircularly flat with respect to quarter-symmetric non-metric connection $\overline{D}$, then the scalar curvature with respect to Levi-Civita connection D is given by (12.10).

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