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Quarter-symmetric non-metric connection in a Cosymplectic manifold

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Abstract

The object of present paper is to study a quarter-symmetric non-metric connection in a cosymplectic manifold.

Keywords and Phrases- Cosymplectic manifold, quarter-symmetric non-metric connection, Bianchi first identity, generalized η-Einstein manifold, projective curvature tensor and concircular curvature tensor and quasi-constant curvature tensor.

Introduction

The product of an almost contact manifold M and the real line $\dot{R}$ caries a natural almost complex structure. However if one takes M to be an almost contact metric manifold and suppose that the product metric G and $M \times \dot{R}$ is Kahelerian, then the structure on M is cosymplectic and not Sasakian.

A linear connection $\overline{D}$ on a Riemannian manifold M is said to be symmetric if the torsion tensor $\overline{T}$ of $\overline{D}$ vanishes identically, otherwise it is non-symmetric. In particular, if $\overline{T}(X, Y) = \eta(Y) X - \eta(X)Y, \forall X, Y \in \chi(M)$, then it is known as a semi-symmetric connection, where η is a 1-form associated with the Riemannian metric g on $\eta(X) = g(X, \xi), \xi$ is a vector field of type (1, 0) and $\chi(M)$ denotes the set of differentiable vector fields of M. Also if

$$\overline{T}(X, Y) = \eta(Y)\phi X - \eta(X)\phi Y, \forall X, Y \in \chi(M), \tag{1.1}$$

then the linear connection $\overline{D}$ is said to be quarter-symmetric connection Golab (1975). A semi-symmetric (quarter-symmetric) connection is said to be metric if $\overline{D}g = 0$, otherwise it is not-metric, i.e., $\overline{D}g \neq 0$ Hayden (1932). Since then, the properties of the quarter-symmetric metric (non-metric) connections on a different structures have been studied by many geometers. For more details, we refer Berman (2015), Chaubey and Ojha (2008), Mishra and Pandey (1980), Rastogi (1978), Prasad and Haseeb (2016) and the references their in.

The concircular curvature tensor C with respect to Levi-Civita connection D is given by Yano (1940) as follows

$$C(X, Y)Z = R(X, Y)Z - \frac{r}{n(n-1)}[g(Y, Z)X - g(X, Z) Y], \tag{1.2}$$

where $X, Y, Z \in \chi$ (M), R and r are the curvature tensor and the scalar curvature tensor with D.

The concircular curvature tensor $\bar{C}$ with respect to quarter-symmetric non-metric connection $\bar{D}$ is defined by

$$\bar{C}(X,Y)Z = \bar{R}(X,Y)Z - \frac{\bar{r}}{n(n-1)}\left[g(Y,Z)\,X - g(X,Z)\,Y\right], \tag{1.3}$$

where $\bar{R}$ and $\bar{r}$ are the curvature tensor and the scalar curvature with respect to $\bar{D}$. Recently Barmann(2012) defined and studied a type of quarter-symmetric non metric ϕ connection on a Kanmotsu manifold and he showed that if a Kenmotsu manifold are semi symmetric and Ricci-semi symmetric with respect to the quarter-symmetric non-metric ϕ connection, then the manifold are semi symmetric and Ricci-semi symmetric with respect to Levi-Civita connection. In this paper, we study the quarter-symmetric non-metric connection in a cosymplectic manifold. After introduction in section 1, section 2 is devoted to the preliminaries. In section 3, we find the expression for the curvature tensor, Ricci tensor and scalar curvature of cosymplectic manifold with respect to the quarter-symmetric non-metric connection $\bar{D}$ and investigate relation between curvature tensor, Ricci tensor and scalar curvature with respect to Levi-Civita connection D. Skew-symmetric property of projective Ricci tensor with respect to the quarter-symmetric non metric connection $\bar{D}$ of cosymplectic manifold is obtained in section 4. Section 5 is devoted to obtain ξ-concircularly flat cosymplectic manifold with respect to the quarter-symmetric non-metric connection $\bar{D}$.

1. Preliminaries

Let M = 2n + 1 be an odd dimensional differentiable manifold on which there are defined a real vector valued linear function ϕ, 1-form η and a vector field ξ satisfying

$$\phi^2 X = -X + \eta(X)\xi \text{ and } \eta(\xi) = 1, \tag{2.1)a$$

$$\phi\xi = 0;\ \eta(\phi X) = 0 \text{ and rank } (\phi) = n - 1, \tag{2.1)b$$

for arbitrary vector fields X, Y and Z is called an almost contact manifold Sasaki (1965, 66, 68) and the structure (ϕ, ξ, η) is called an almost contact structure on M Blair (1976). An almost contact manifold M on which there is a metric g on M satisfying

$$g(\phi X, \phi Y) = g(X,Y) = \eta(X)\eta(Y), \tag{2.2)a$$

and

$$g(X,\xi) = \eta(X), \tag{2.2)b$$

is called an almost contact metric manifold and the structure (ϕ, ξ, η, g) is called an almost contact metric structure Blair (1976).

Let us put

$$F(X,Y) = g(\phi X, Y) = -g(X, \phi Y) = -F(Y,X). \tag{2.3}$$

Thus the tensor F is skew symmetric i.e.

$$F(X,Y) + F(Y,X) = 0.$$

An almost contact metric manifold on which

$$D_X\phi = 0 \Leftrightarrow (D_XF)(Y,Z) = 0, \tag{2.4}$$

is said to be cosymplectic manifold Goldberg (1963) defined a cosymplectic manifold on which $(D_X\phi)(Y) = 0$ and $(D_X\eta)(Y) = 0$.
(2.5)

But the second equation is not necessarybecause the first equation implies the second equation as follow:

$$(D_XF)(Y,Z) = 0 \Rightarrow (D_XF)(Y,\xi) = 0 \Rightarrow (D_X\eta)(Y) = 0.$$

It is easy to sec that a cosymplictic manifold is normal.

Definition (2.1) : A cosympectic manifold (M, g) is said to be of quasi-constant curvature Yano and Chen (1972) if the curvature tensor R of the Levi-Civita connection is given by

$$R(X,Y)Z = a[g(Y,Z)X - g(X,Z)Y] + b[\eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y + g(Y,Z)\eta(X)\xi - g(X,Z)\eta(Y)\xi], \tag{2.6}$$

where a and b are scalar function. If b = 0 then the manifold reduces to a manifold of constant curvature Kabayashi and Nomizu (1963).

Definition (2.2): A cosymplectic manifold (M, g) is said to be an η-Einstein manifold if its Ricci tensor Ric is of the form

$$Ric(X,Y) = ag(X,Y) + b\eta(X)\eta(Y). \tag{2.7}$$

Definition (2.3): A cosymplectic manifold is said to be generalized η-Einstein manifold if its Ricci tensor Ric is of the form Haseeb and Prasad (2018)

$$Ric(X,Y) = ag(X,Y) + b\eta(X)\eta(Y) + cF(X,Y),$$

where a, b and c are scalar function and $F(X,Y) = g(\phi X,Y)$. In particular if c = 0 then generalized η-Einstein manifold reduced to η-Einstein manifold.

3.Quarter-symmetric non-metric connection in a cosympectic manifold

Let (M, g) be a cosymplectic manifold with the Levi-Civita connection D. We define a linear connection $\overline{D}$ on M Berman (2012) by

$$\overline{D}_XY = D_XY - \eta(X)\phi Y + g(X,Y)\xi - \eta(Y)X - \eta(X)Y. \tag{3.1}$$

Torsion tensor $\overline{T}$ of M with respect to $\overline{D}$ is

$$\overline{T}(X,Y) = \eta(Y)\phi X - \eta(X)\phi Y. \tag{3.2}$$

A linear connection satisfying (3.2) is called quarter-symmetric connection. Further from (3.1)

$$(\overline{D}_Xg)(Y,Z) = 2\eta(X)g(Y,Z) \neq 0. \tag{3.3}$$

A linear connection $\overline{D}$ satisfying (3.2) and (3.3) is called a quarter-symmetric non metric connection. Here the connection defined by equation (3.1) is not a ϕ-connection in a cosymplectic manifold. Analogous to the definition of curvature tensor of the manifold with respect to the Levi-Civita connection D, we define the curvature tensor of M with respect to the quarter-symmetric non-metric connection $\overline{D}$ by

$$\overline{R}(X, Y) Z = D_X D_Y Z - D_Y D_X Z - D_{[X, Y]} Z, \quad (3.4)$$

where $\overline{R}$ is the curvature tensor with respect to the quarter-symmetric non metric connection $\overline{D}$.

In view of (3.1) and (3.4), we get

$$\overline{R}(X,Y)Z = R(X,Y)Z + \eta(X)(D_Y\phi)(Z) - \eta(Y)(D_X\phi)(Z) + \eta(X)\eta(Z)\phi Y - \eta(Y)\eta(Z)\phi X + \eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y - \eta(Y)g(X,\phi Z)\xi + \eta(X)g(Y,\phi Z)\xi + g(Y,Z)(D_X\xi) - g(X,Z)(D_Y\xi) + g(Y,Z)\eta(X)\xi - g(X,Z)\eta(Y)\xi - g(Y,Z)X + g(X,Z)Y - (D_X\eta)(Y)Z + (D_Y\eta)(X)Z + (D_Y\eta)(X) - (D_X\eta)(Y) - (D_X\eta)(Y)\phi Z + (D_Y\eta)(X)\phi Z. \quad (3.5)$$

From (2.5) and (3.5), we get

$$\overline{R}(X,Y)Z = R(X,Y)Z + \eta(X)\eta(Z)\phi Y - \eta(Y)\eta(Z)\phi X + \eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y + \eta((Y)g(\phi X,Z)\xi - \eta(X)g(\phi Y,X)\xi + g(Y,Z)\eta(X)\xi - g(X,Z)\eta(Y)\xi - g(Y,Z)X + g(X,Z)Y. \quad (3.6)$$

Let us assume that if the curvature tensor of $\overline{D}$ is of the following form

$$\overline{R}(X,Y)Z = \eta(X)\eta(Y)\phi Y - \eta(Y)\eta(Z)\phi X + \eta(Y)g(\phi X,Z)\xi - \eta(X)g(\phi Y,X)\xi. \quad (3.7)$$

Then by virtue of (3.6) and (3.7), we get

$$R(X,Y)Z = [g(Y,Z)X - g(X,Z)Y] - [\eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y + g(Y,Z)\eta(X)\xi - g(X,Z)\eta(Y).\xi]. \quad (3.8)$$

Hence in view of (2.6) and (3.8), we can state the following theorem:

Theorem (3.1): If a cosymplectic manifold M admits a quarter-symmetric non-metric connection $\overline{D}$ whose curvature tensor is of the from (3.7), then manifold is quasi constant curvature.

Now taking the inner product of (3.6) with W, it follows that

$$\overline{R}(X,Y,Z,W) = {}'R(X,Y,Z,W) + \eta(Y)\eta(Z)g(X,W) - \eta(X)\eta(Z)g(Y,W) - \eta(Y)\eta(Z)g(\phi X,W) + \eta(X)\eta(Z)g(\phi Y,W) + g(Y,Z)\eta(X)\eta(W) - g(X,Z)\eta(Y)\eta(W) - g(Y,Z)g(X,W) + g(X,Z)g(Y,W) + \eta(Y)g(\phi X,Z)\eta(W) - \eta(X)\eta(W)g(\phi Y,Z), \quad (3.9)$$

where $g(\overline{R}(X,Y)Z,W) = {}'\overline{R}(X,Y,Z,W)$ and $g(R(X,Y)Z,W) = {}'R(X,Y,Z,W)$.

From (3.9), we get

$$'\overline{R}(X,Y,Z,W) + {}'\overline{R}(Y,X,Z,W) = 0, \quad (3.10)$$

$$'\overline{R}(X,Y,Z,W) + {}'\overline{R}(X,Y,W,Z) = 0, \quad (3.11)$$

$$'\bar{R}(X,Y,Z,W) + '\bar{R}(Z,W,X,Y) = 2[\eta(Y)\eta(Z)g(\phi W,X) + \eta(Z)\eta(X)g(\phi Y,W) + \eta(X)\eta(W)g(\phi Z,Y) + \eta(W)\eta(Y)g(\phi X,Z)], \quad (3.12)$$

and

$$\bar{R}(X,Y,Z,W) + '\bar{R}(Y,Z,X,W) + '\bar{R}(Z,X,Y,W) = 2[\eta(X)g(\phi Z,Y) + \eta(Y)g(\phi X,Z) + \eta(Z)g(\phi Y,X)\eta(W). \quad (3.13)$$

Hence, we can state the following theorem

Theorem (3.2): The curvature tensor $'\bar{R}$ of cosymplectic manifold with respect to quarter-symmetric non-metric connection $\bar{D}$ satisfies the following :

(i) It is skew symmetric in first two slots,

(ii) It is skew symmetric in last two slots,

(iii) $'\bar{R}(X,Y,Z,W) - '\bar{R}(Z,W,X,Y) = 2[\eta(Y)\eta(Z)g(\phi W,X) + \eta(Z)\eta(X)g(\phi Y,W) + \eta(X)\eta(W)g(\phi Z,Y) + \eta(W)\eta(Y)g(\phi X,Z)]$,

(iv) $'\bar{R}(X,Y,Z,W) + '\bar{R}(Y,Z,X,W) + '\bar{R}(Z,X,Y,W) = 2[\eta(X)g(\phi Z,Y) + \eta(Y)g(\phi X,Z) + \eta(Z)g(\phi Y,X)]\eta(W)$.

Let $\{E_1, \ldots E_{2n+1}\}$ be a local orthonormal basis of the tangent space at a paint of the manifold. Putting $E_i$for X and W in (3.9) and taking summation over E_i, $1 \leq E_i \leq 2n + 1$ and also using (2.1), we get

$$\bar{R}ic(Y,Z) = Ric(Y,Z) - \eta(Y)\eta(Z)(\text{trace}\,\phi - n + 2) - (n-2)g(Y,Z) - g(\phi Y,Z). \quad (3.14)$$

From (3.14), we get

$$\bar{R}ic(Y,Z) - \bar{R}ic(Z,Y) = 2g(\phi Z,Y), \quad (3.15)$$

and

$$\bar{R}ic(Y,Z) + \bar{R}ic(Z,Y) = 2Ric(Y,Z) - 2\eta(Y)\eta(Z)(\text{trace}\,\phi - n + 2) - 2(n-2)g(Y,Z). \quad (3.16)$$

If $\bar{R}ic\,(Y,Z) + \bar{R}ic\,(Z,Y) = 0$ then

From (3.16), we get

$$Ric(Y,Z) = \eta(Y)\eta(Z)\,(\text{trace}\,\phi - n + 2) + (n-2)\,g(Y,Z), \quad (3.17)$$

where $\bar{R}ic$ and Ric denote the Ricce tensor of M with respect to $\bar{D}$ and D respectively.

Let $\bar{r}$ and r denote the scalar curvature of M with respect to $\bar{D}$ and D. respectively, i.e .$\bar{r} = \sum_{i=1}^{2n+1} \bar{R}ic(E_i,E_i)$and $R = \sum_{i=1}^{2n+1} Ric\,(E_i,E_i)$.

Again, putting $Y = Z = E_i$ in (3.14) and taking the summation over E_i $1 \leq 2n + 1$, we get

$$\bar{r} = r - 2\,\text{trace}\,\phi - (n-1)\,(n-2). \quad (3.18)$$

Putting ξ for Z in (3.14), we get

$$\bar{R}ic\,(Y,\xi) = Ric(Y,\xi) - \eta(Y)\,\text{trace}\,\phi. \quad (3.19)$$

Summing up the above discussion, we can state the following theorems:

Theroem (3.3): Ricci tensor $\overline{\mathrm{Ric}}$ and scalar curvature $\bar{r}$ of cosymplectic manifold with quarter-symmetric non-metric connection $\overline{D}$ satisfiies :

(i) The Ricci tensor $\overline{\mathrm{Ric}}(Y,Z)$ is given by the equation (3.14);

(ii) If the $\overline{\mathrm{Ric}}(Y,Z) = 0$ then, from (3.14) Ricci tensor of cosymplectic manifold with Levi-civita connection D becomes generalizedη-Einstein manifold Haseeb and Prasad (2018);

(iii) $\overline{\mathrm{Ric}}(Y,\xi) = \mathrm{Ric}(Y,\xi) - \eta(Y)\,\mathrm{trace}\,\phi$;

(iv) Ricci tensor$\overline{\mathrm{Ric}}$ satisfies the following Ricci expression
$\overline{\mathrm{R}}(Y,Z) - \overline{\mathrm{Ric}}(Z,Y) = 2g(\phi Z,Y)$;

(v) If a cosymplectic manifold admits a quarter-symmetric non-metric connection $\overline{D}$ then a necessary and sufficient condition for the Ricci tensor of $\overline{D}$ to be skew-symmetric is that the Ricci tensor of the Levi-Civita connection D is given by
$\mathrm{Ric}(Y,Z) = \eta(Y)\eta(Z)\,(\mathrm{trace}\,\phi - n + 2) + (n-2)g(Y,Z)$;

(vi) The scalar curvature of cosymplectic manifold with respect to quarter-symmetric non-metric connection $\overline{D}$ is equal to the scalar curvature of the manifold with respect to Levi-Civita connection $\overline{D}$ if and only if $\mathrm{trace}\,\phi = \frac{(n-1)(n-2)}{2}$.

4. Skew symmetric properties of projective Ricci tensor with respect to the quarter-symmetric non-metric connection $\overline{D}$ in a cosymplectic manifold :

Projective Ricci tensor $P_R(X,Y)$ in a Riemannian manifold D is defined as follows Chaki and Saha (1994)

$$P_R(X,Y) = \frac{n}{n-1}\left[\mathrm{Ric}(X,Y) - \frac{r}{n}g(X,Y)\right]. \tag{4.1}$$

Analogous to this definition, we define projective Ricci tensor with respect to quarter-symmetric non-metric connection $\overline{D}$ is by the equation

$$P_R\,(X,Y) = \frac{n}{n-1}\left[\overline{\mathrm{Ric}}(X,Y) - \frac{\bar{r}}{n}g(X,Y)\right]. \tag{4.2}$$

In view of (3.14), (3.18), (4.1) and (4.2), we get

$$\overline{P}_R(X,Y) = \frac{n}{n-1}\Big[\mathrm{Ric}(X,Y) - \eta(X)\eta(Y)(\mathrm{trace}\,\phi - n + 2) - g(\phi X,Y) - \frac{r-2\,\mathrm{trace}\,\phi+n-2}{n}.\,g(X,Y)\Big]. \tag{4.3}$$

In view of (4.3), we get

$$\overline{P}_R(X,Y) + \bar{P}_R(Y,X) = \frac{n}{n-1}\Big[2Ric(X,Y) - 2\eta(X)\eta(Y)(trace\,\phi - n + 2) - 2.\frac{r-2\,trace\,\phi+n-2}{n}.\,g(X,Y)\Big]. \tag{4.4}$$

If $\overline{P}_R(X,Y) + \overline{P}_R(Y,X) = 0$, then from (4.4), we get

$$\mathrm{Ric}(X,Y) = (\mathrm{trace}\,\phi - n + 2)[\eta(X)\eta(Y)) + g(X,Y)] - \frac{r}{n}g(X,Y). \tag{4.5}$$

Conversely if Ric(X, Y) is given by (4.5), then we get from (4.4)

$$\bar{P}_R(X,Y) + \bar{P}_R(Y,X) = 0. \tag{4.6}$$

That is, projective Ricci tensor of $\bar{D}$ is skew-symmetric. Hence we have the following theorem:

Theorem (4.1) : If a cosymplectic manifold admits a quarter-symmetric non-metric connection $\bar{D}$ then a necessary and sufficient condition for the projective Ricci tensor of $\bar{D}$ to be skew-symmetric is that the Ricci tensor of the Levi-Civita connection D is given by

$$Ric(X,Y) = (\text{trace}\,\phi - n + 2)[\eta(X)\eta(Y) + g(X,Y)] - \frac{r}{n}.g(X,Y).$$

5. ξ-concircularly flat cosymplectic manifolds with respect to the quarter-symmetric non-metric connection $\bar{D}$

Definition (5.1) Acosymplectic manifold M with respect to quarter-symmetric non-metric connection $\bar{D}$ is said to be ξ-concircularly flat if

$$\bar{C}(X,Y)\,\xi = 0, \tag{5.1}$$

for all vector fields X, Y $\in\chi(M)$ is the set of all differentiable vector fields on manifold. In consequences of (1.2), (1.3), (3.6) and (3.18), we get

$$\begin{aligned}\bar{C}(X,Y)Z = {} & C(X,Y)Z + \eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y - \eta(Y)\eta(Z)\phi X + \\ & \eta(X)\eta(Z)\phi Y + g(Y,Z)\eta(X)\xi - g(X,Z)\eta(Y)\xi - g(Y,Z)X + \\ & g(X,Z)Y + \eta(Y)g(\phi X,Z)\xi - \eta(X)g(\phi Y,Z)\xi + \\ & \frac{2\,\text{trace}\,\phi+(n-1)(n-2)}{n}\big[g(Y,Z)X - g(X,Z)Y\big].\end{aligned} \tag{5.2}$$

Putting ξ for Z in (5.2) and using (2.1), we get

$$\begin{aligned}\bar{C}(X,Y)\xi = {} & \\ & C(X,Y)\xi + [\eta(X)\phi Y - \eta(Y)\phi X] + \\ & \frac{2\,\text{trace}\,\phi+(n-1)(n-2)}{n}[\eta(Y)X - \eta(X)Y].\end{aligned} \tag{5.3}$$

In view of (5.1) and (5.3), we get

Theorem (5.1): A cosymplectic manifold with respect to quarter-symmetric non-metric connection$\bar{D}$ is ξ-concircularly flat if and only if the manifold with respect to the Levi-Civita connection is also ξ-concircularly flat provided that the vector fields X and Y orthogonal to ξ.

In view of (5.2), we can state the following theorem:

Theorem (5.2): The concircular curvature tensor of cosymplectic manifold with respect to the quarter-symmetric non-metric connection $\bar{D}$ satisfies the following algebraic relations:

$$\bar{C}(X,Y)Z + \bar{C}(Y,X)Z = 0$$

and

$$\begin{aligned}\bar{C}(X,Y)Z + \bar{C}(Y,Z)X + \bar{C}(Z,X)Y = {} & \\ & 2\eta(Y)g(\phi X,Z)\xi - 2\eta(X)g(\phi Y,Z)\xi - 2\eta(Z)g(\phi X,Y)\xi.\end{aligned}$$

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