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Quarter-symmetric non-metric connection in LP-cosymplectic manifold

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Abstract
The main object this paper is introduce and study a quarter-symmetric non-metric connection on a LP-cosymplectic manifold.

Keywords-LP- cosymplectic manifold, quarter-symmetric non-metric connection, special curvature tensor, Einstein manifold, quasi concircular curvature tensor, Bianchi first and second identity.

Introduction
In 1975, Golab defined and studied quarter-symmetric connection in a differentiable manifold with affine connection. A linear connection on an n-dimensional Riemannian manifold (M^n, g) is called a quarter symmetric connection if its torsion tensor $\overline{T}$ satisfies

$$\overline{T}(X,Y) = \eta(Y)\phi X - \eta(X)\phi Y, \qquad (1.1)$$

where η is a 1-form and ϕ is a (1, 1) tensor field. In particular, if $\phi X = X$, then the quarter-symmetric connection reduces to the semi-symmetric connection Friedmann and Schouten (1924). Thus the notion of the quarter-symmetric connection generalizes the notion of the semi-symmetric connection.

If moreover, a quarter-symmetric connection $\overline{D}$ satisfies the condition

$$(\overline{D}_X g)(Y,Z) \neq 0, \qquad (1.2)$$

then $\overline{D}$ is said to be a quarter-symmetric non-metric connection.

After Golab (1975) and Rastogi (1978 and 1987) continued systematic study of quarter-symmetric metric connection by Mishra and Pandey (1980), Yano and Imai (1982), Roy, Barua and Mukhopadhyay (1991), De, Özgür and Sular (2008), Barman (2012) and others.

The study of the quarter-symmetric non-metric connection in a LP-cosymplectic manifold. Section 2 is devoted tothe preliminaries. In section 3 and 4 we define and prove the existence of quarter symmetric non-metric connection in a LP-cosymplectic manifold. In section 5, we find the expression for the curvature tensor, Ricci tensor and scalar curvature with respect to the quarter-symmetric non-metric connection and investigate relation between curvature tensor, Ricci tensor and scalar curvature with respect to Levi-Civita of connection. In section 6 special curvature tensor of quarter-symmetric non-metric connection is studied. In final section it is shown that the quasi-concircular curvature tensor of the quarter-symmetric non metric connection is equal to the quasi-concircular curvature tensor of LP-cosymplectic manifold under certain condition.

1. Preliminaries

Let (M^n, g) be an n-dimensional differentiable manifold on which there are defined a tensor field ϕ of type (1, 1), a contravariant vector field ξ, a covariant vector field η and Lorentzian metric g which satisfy

$$\phi^2 X = X + \eta(X)\xi;\ \eta(\xi) = -1, \tag{2.1}$$

and

$$g(\phi X, \phi Y) = g(X, Y) + \eta(Y)\eta(X);\ g(X, \xi) = \eta(X), \tag{2.2}$$

then (M^n, g) is called a Lorentzian para-contact manifold (LP-contact manifold) and the structure (ϕ, ξ, η, g) is called an LP-contact structure Matsumoto (1989). In an LP-contact manifold, we have

$$\text{(a) } \phi(\xi) = 0 \quad \text{(b) } \eta(\phi X) = 0, \quad \text{(c) } \operatorname{rank}(\phi) = (n - 1). \tag{2.3}$$

Let us put

$$F(X, Y) = g(\phi X, Y) = g(X, \phi Y) = F(Y, X). \tag{2.4}$$

Then the tensor field F is symmetric (0, 2) tensor field.

An LP-contact manifold is said to be an LP-cosymplictic manifold if Prasad and Ojha (1994)

$$D_X\phi = 0 \Rightarrow (D_X F)(Y, Z) = 0. \tag{2.5}$$

On this manifold, we have

$$(D_X\eta)(Y) = 0 \text{ and } D_X\xi = 0, \tag{2.6}$$

for vector fields X, Y and Z, where D denotes the covariant differentiation with respect to g.

2. Quarter-symmetric non metric connection in an LP-cosymplectic manifold :

Let (M^n, g) be an LP-cosymplectic manifold with Levi-Civita connection D. we define a linear connection $\overline{D}$ on M^n by

$$\overline{D}_X Y = D_X Y + a.\eta(Y)\,\phi X + b.\eta(X)\,\phi Y. \tag{3.1}$$

where a and b non zero constants and η is 1-form associated with the vector field ξ on M^n given by

$$g(X,\xi) = \eta(X), \tag{3.2}$$

for all vector fields $X\in\chi(M^n)$, where $X\in\chi\ (M^n)$ is the set of all differentiable vector fields on M^n. Using (3.1), the torsion tensor $\overline{T}$ of M^n with respect to the connection $\overline{D}$ is given by

$$\overline{T}(X,Y) = (a - b)\,[\eta(Y)\,\phi X - \eta(X)\,\phi Y]. \tag{3.3}$$

called a quarter-symmetric.

A linear connection satisfying (3.3) is quarter-symmetric connection. Further using (3.1), we get

$$(\overline{D}_X g)\,(Y,Z) = -a[\eta(Y)g(\phi X,Z) + \eta(Z)g(\phi X,Y)] - 2b\eta(X)\,g(\phi Y,Z) \neq 0. \tag{3.4}$$

A linear connection $\overline{D}$ defined by (3.1) satisfies (3.3) and (3.4) is called a quarter-symmetric non-metric connection.

Conversely, we will show that a linear connection $\overline{D}$ defined on M^n satisfying (3.3) and (3.4) is given by (3.1).

Let $\overline{D}$ be a linear connection in M^n given by

$$\overline{D}_X Y = D_X Y + H(X,Y). \tag{3.5}$$

Now, we shall determine the tensor field H of the type (1, 2) such that $\overline{D}$ satisfies (3.3) and (3.4).

From (3.5), we have

$$\overline{T}(X,\ Y) = H(X,Y) - H(Y,X). \tag{3.6}$$

Denote

$$G(X,Y,Z) \overset{\text{def}}{=} (\overline{D}_X g)(Y,Z).$$

(3.7)

From (3.5) and (3.7), we get

$$g(H(X,Y),Z) + g(H(X,Z),Y) = -\,G(X,Y,Z). \tag{3.8}$$

In view of (3.5), (3.6), (3.8) and (3.4), we get

$$g(\overline{T}(X,Y),Z) + g(\overline{T}(Z,X),Y) + g(\overline{T}(Z,Y),X = g(H(X,Y),Z) - g(H(Y,X),Z) + g(H(Z,X),Y) - g(H(X,Z),Y + g(H(Z,Y),X) - g(H(Y,Z),X).$$

$$\Rightarrow g(\overline{T}(X,Y),Z) + g(\overline{T}(Z,X),Y) + g(\overline{T}(Z,Y),X) = 2g(H(X,Y),Z) - 2a\eta(Z)g(\phi X,Y) - 2b\eta(X)g(\phi Y,Z) - 2bg(\phi X,Z)\eta(Y) + 2b\eta(Z)\,g(\phi X,Y).$$

$$\Rightarrow H(X,Y) = \frac{1}{2}\{\overline{T}(X,Y) + {}'\overline{T}(X,Y) + {}'\overline{T}(Y,X)\} + b\eta(X)\,\phi Y + b\eta(Y)\,\phi X + ag(\phi X,Y)\xi - bg(\phi X,Y)\xi,$$

where $'\overline{T}$ be a tensor field of the type (1, 2) defined by $g('\overline{T}(X,Y),Z) = g(\overline{T}(Z,X),Y)$.

This implies that $H(X,Y) = a\eta(Y)\phi X + b\eta(X)\phi Y$.

Hence from (3.5), we have

$$\overline{D}_X Y = D_X Y + a\eta(Y)\phi X + b\eta(X)\phi Y.$$

Hence, we can state the following theorem:

Theorem (3.1): Let (M^n, g) be an LP-cosymplectic manifold with Lorentzian para contact metric structure (ϕ, ξ, η, g) admitting a quarter-symmetric non-metric connection $\bar{D}$ which satisfies (3.3) and (3.4). Then the quarter-symmetric non-metric is given by (3.1).

3. Existence of a quarter-symmetric non metric connection $\bar{D}$ on LP-cosymplectic manifold :

Let X, Y and Z be any three rector fields on LP-cosymplectic manifold with LP-contact metric structure (ϕ, ξ, η, g). We define Koszul formula for D Koszul (1950) as

$$2g(D_XY,Z) = Xg(Y,Z) + Yg(Z,X) - Z(X,Y) + g((X,Y),Z) - g((Y,Z),X) + g((Z,X),Y). \quad (4.1)$$

Analogous to this definition, we define a connection $\bar{D}$ by the following equation:

$$\begin{aligned} 2g(\bar{D}_XY,Z) = {} & Xg(Y,Z) + Yg(Z,X) - Z(X,Y) + g((X,Y),Z) - g((Y,Z),X) + \\ & g((Z,X),Y) + g(a\eta(Y)\phi X - a\eta(X)\phi Y + b\eta(X)\phi Y - \\ & b\eta(Y)\phi X, Z) + g(a\eta(Z)\phi Y + b\eta(Y)\phi Z - a\eta(Y)\phi Z - \\ & b\eta(Z)\phi Y, X) + g(a\eta(X)\phi Z + b\eta(Z)\phi X - a\eta(Z)\phi X - \\ & b\eta(X)\phi Z, Y), \end{aligned} \quad (4.2)$$

which holds for all vector fields X, Y and Z $\in \chi(M^n)$. It can be easily verified that the mapping $\bar{D} : (X,Y) \to (\bar{D}_XY)$,

satisfies the following equalities:

$$\bar{D}_X(Y + Z) = \bar{D}_XY + (\bar{D}_XZ), \quad (4.3)$$

$$\bar{D}_{X+Y}Z = \bar{D}_XZ + \bar{D}_YZ, \quad (4.4)$$

$$\bar{D}_{Xf}Y = f\bar{D}_XY, \quad (4.5)$$

and

$$\bar{D}_X(fY) = f\bar{D}_XY + (X f)\, Y, \quad (4.6)$$

for all X, Y, Z $\in \chi(M^n)$ and for all $f \in F(M^n)$, the set of all differentiable mapping over M^n. From (4.3), (4.4), (4.5) and (4.6), we can conclude that $\bar{D}$ determines a linear connection on M^n.

Now from (4.2), we get

$$\begin{aligned} 2g(\bar{D}_XY,Z) - 2g(\bar{D}YX,Z) = {} & 2g((X,Y),Z) + \\ & 2a\eta(Y)\, g(\phi X,Z) - 2a\eta(X)g(\phi Y,Z) + \\ & 2b\eta(X)g(\phi Y,Z) - 2b\eta(Y)g(\phi X,Z). \end{aligned}$$

Hence

$$\bar{T}(X,Y) = (a - b)\, [\eta(Y)\phi X - \eta(X)\phi Y]. \quad (4.7)$$

Also we have from (4.1),

$$\begin{aligned} 2g(\bar{D}_XY,Z) - 2g(\bar{D}_XZ,Z) = {} & 2Xg(Y,Z) + \\ & 2a\eta(Y)\, g(\phi X,Z) + 2a\eta(Z)g(\phi X,Y) + \\ & 4b\eta(X)g(\phi Y,Z). \end{aligned}$$

That is,

$$(\bar{D}_X g)(Y,Z) = -a\eta(Y)g(\phi X,Z) - a\eta(Z)g(\phi X,Y) - 2b\eta(X)g(\phi Y,Z). \tag{4.8}$$

From (4.7) and (4.8) it follows that $\bar{D}$ determines a quarter-symmetric non-metric connection on (M^n, g). This show that $\bar{D}$ determines a unique quarter-symmetric non-metric connection on (M^n, g).

Hence we can state the following theorem:

Theorem (4.1). Let (M^n, g) be a LP-cosymplectic manifold with a LP-contact metric structure (ϕ, ξ, η, g) on it. Then there exist a unique linear connection $\bar{D}$ satisfying (3.3) and (3.4).

4. Curvature tensor of an LP-cosymlectic manifold with respect to the quarter-symmetric non-metric connection $\bar{D}$:

Let $\bar{R}$ and R be the curvature tensor of the connection $\bar{D}$ and D respectively then we get

$$\bar{R}(X,Y)Z = \bar{D}_X\bar{D}_Y Z - \bar{D}_Y\bar{D}_X Z - \bar{D}_{[X,Y]}Z. \tag{5.1}$$

From (3.1) and (5.1), we get

$$\bar{R}(X,Y)Z = \bar{D}_x(D_Y Z + a.\eta(Z)\phi Y + b\eta(Y)\phi Z) - \bar{D}_Y(D_X Z + a.\eta(Z)\phi X + b\eta(X)\phi Z) - D_{[X,Y]}Z - a.\eta(Z)\phi[X,Y] - b\eta([X,Y])\phi Z,$$

which gives on simplification

$$\bar{R}(X,Y)Z = R(X,Y)Z + a\left[(D_X\eta)(Z)\phi Y - (D_Y\eta)(Z)\phi X\right] + \eta(Z)\left[(D_X\phi)Y - (D_Y\phi)X\right] + b[(D_X\eta)(Y)\phi Z - (D_Y\eta)(X)\phi Z + \eta(Y)(D_X\phi)(Z) - (D_Y\eta)(X)\phi Z + \eta(Y)(D_X\phi)(Z) - \eta(X)(D_Y\phi)(Z) + ab\left[\eta(Y)\eta(\phi Z)\phi X - \eta(X)\eta(\phi Z)\phi Y + \eta(X)\eta(Z)\phi^2 Y - \eta(Z)\eta(Y)\phi^2 X\right] + a^2[\eta(Z)\eta(\phi Y)\phi X - \eta(Z)\eta(\phi X)(\phi Y)]. \tag{5.2}$$

In view of (2.3), (2.5), (2.6) and (5.2), we get

$$\bar{R}(X,Y)Z = R(X,Y)Z - ab[\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X], \tag{5.3}$$

where $R(X,Y)Z = D_X D_Y Z - D_Y D_X Z - D_{[X,Y]}Z$,

is the curvature tensor of $\bar{D}$ with respect to the Riemannian connection D.

Contracting (5.3) with respect to X, we get

$$\bar{R}ic(Y,Z) = Ric(Y,Z) - ab(n-1)\eta(Y)\eta(Z), \tag{5.4}$$

where $\bar{R}ic(Y,Z)$ and Ric(Y, Z) are the Ricci tensor with respect to $\bar{D}$ and D and

$$\bar{r} = r + ab(n-1), \tag{5.5}$$

where $\bar{r}$ and r are the scalar curvature with respect to $\bar{D}$ and D.

Taking the inner product of (5.3) with W, it follows that

$$'\bar{R}(X,Y,Z,W) = {}'R(X,Y,Z,W) - ab[\eta(X)\eta(Z)g(Y,W) - \eta(Y)\eta(Z)g(X,W)], \tag{5.6}$$

where

$$\bar{R}(X,Y,Z,W) = g(\bar{R}(X,Y)Z,W\} \text{ and } 'R(X,Y,Z,W) = g(R(X,Y)Z,W).$$

From (5.6), we get

$$'\bar{R}(X,Y,Z,W) + '\bar{R}(Y,X,Z,W) = 0,$$

$$'\bar{R}(X,Y,Z,W) + '\bar{R}(X,Y,W,Z) = ab[\eta(Y)\eta(Z)g(X,W) - \eta(X)\eta(Z)g(Y,W) + \eta(Y)\eta(W)g(X,Z)] + \eta(X)\eta(W)g(Y,Z)], \tag{5.7}$$

$$'\bar{R}(X,Y,Z,W) - '\bar{R}(Z,W,X,Y) = ab[\eta(Y)\eta(Z)g(X,W) - \eta(X)\eta(W)g(Y,Z)], \tag{5.8}$$

$$'\bar{R}(X,Y,Z,W) + '\bar{R}(Y,Z,X,W) + '\bar{R}(Z,X,Y,W) = 0. \tag{5.9}$$

Differentiating equation (5.3) covariantly with respect to X, we get

$$(\bar{D}_x\bar{R})(Y,Z,U) = (D_XR)(Y,Z,U) - ab\left[(D_X\eta)(Y)\eta(U)Z - (D_X\eta)(Z)\eta(Y)U\right] \tag{5.10}$$

From (2.6) and (5.10), we get

$$(\bar{D}_x\bar{R})(Y,Z,U) = (D_XR)(Y,Z,U). \tag{5.11}$$

From (5.4), we get

$$\bar{R}ic(Y,Z) = \bar{R}ic\,(Z,Y). \tag{5.12}$$

From the above discussion, we have the following theorem

Theorem (5.1). For an LP-cosymplectic manifold (M^n, g) with respect to the quarter-symmetric non-metric connection $\bar{D}$, we have

(i) The curvature tensor $\bar{R}$ is given by (5.3);

(ii) The Ricci tensor $\bar{R}ic$ is given by (5.4);

(iii) The scalar curvature $\bar{r}$ is given by (5.5);

(iv) $'\bar{R}(X,Y,Z,W) + '\bar{R}(Y,X,Z,W) = 0$;

(v) $'\bar{R}(X,Y,Z,W) + 'R(X,Y,W,Z) = ab[\eta(Y)\eta(Z)g(X,W) - \eta(X)\eta(Z)g(Y,W) + \eta(Y)\eta(W)g(X,Z) - \eta(X)\eta(W)g(Y,Z)]$.

(vi) $'\bar{R}(X,Y,Z,W) - '\bar{R}(Z,W,X,Y) = ab[\eta(Y)\eta(Z)g(X,W) - \eta(X)\eta(W)g(Y,Z)]$,

(vii) $'\bar{R}(X,Y,Z,W) + '\bar{R}(Y,Z,X,W) + '\bar{R}(Z,X,Y,W) = 0$ i.e. Bianchci first identity;

(viii) $(\bar{D}_X\bar{R})(Y,Z,U) + (\bar{D}_Y\bar{R})(Z,X,U) + (\bar{D}_Z\bar{R})(X,Y,U) = 0$ i.e. Bianchci second identity;

(ix) The Ricci tensor $\bar{R}ic(Y, Z)$ is symmetric.

5. Special curvature tensor of an LP-cosymplectic manifold with respect to quarter-symmetric non-metric connection $\bar{D}$.

Special curvature tensor J on a Riemannian manifold (M^n, g) of the type (0, 4) defined by Singh and Khan (1998) as follows.

$$'J(X,Y,Z,W) = '\bar{R}(X,Y,Z,W) + 'R(X,Z,Y,W). \tag{6.1}$$

Or equivalently

$$g(J(X,Y)Z,W) = g(R(X,Y)Z,W) + g(R(X,Z)Y,W).$$

It is obvious that

$$'J(X,Y,Z,W) - 'J(X,Z,Y,W) = 0, \tag{6.2}$$

and

$$'J(X,Y,Z,W) + 'J(Y,Z,X,W) + 'J(Z,X,Y,W) = 0. \tag{6.3}$$

Analogous to the definition of (6.1), we define special curvature tensor of M^n with respect to quarter-symmetric non-metric connection $\bar{D}$ in an LP-cosymplectic manifold by the expression

$$'\bar{J}(X,Y)Z = \bar{R}(X,Y)Z + \bar{R}(X,Z)Y. \quad (6.4)$$

In view of (5.3), (6.1) and (6.4), we get

$$\bar{J}(X,Y)Z = J(X,Y)Z - ab[\eta(X)\eta(Z)Y + \eta(X)\eta(Y)Z - 2\eta(Y)\eta(Z)X]. \quad (6.5)$$

From (6.2) and (6.5), we get

$$\bar{J}(X,Y)Z - \bar{J}(X,Z)Y = 0.$$

Again from (6.5), we get

$$\bar{J}(X,Y)Z + \bar{J}(Y,Z)X + \bar{J}(Z,X)Y = 0. \quad (6.6)$$

Thus, we have the following theorem:

Theorem (6.1): The special curvature tensor of an LP-cosymplectic manifold with respect to $\bar{D}$ satisfies the relation

(i) $\bar{J}(X,Y)Z - \bar{J}(X,Z)Y = 0;$

(ii) $\bar{J}(X,Y)Z + \bar{J}(Y,Z)X + \bar{J}(Z,X)Y = 0.$

6. Quasi-concircular curvature tensor $\bar{V}$ with respect to quarter-symmetric non-metric connection $\bar{D}$ in a LP-Cosymplectic manifold.

The notion of quasi-concircular curvature tensor V was introduced by Prasad and Mrurya (2007). They defined quasi concircular curvature tensor by

$$V(X,Y)Z = AR(X,Y)Z + \frac{r}{n}\left[\frac{A}{n-1} + 2B\right][g(Y,Z)X - g(X,Z)Y], \quad (7.1)$$

where A and B an constants such that $A \neq 0$, $B \neq 0$. If $A = 1$ and $B = -\frac{1}{n-1}$ then (7.1) takes the form

$$V(X,Y)Z = R(X,Y)Z - \frac{r}{n(n-1)}[g(Y,Z)X - g(X,Z)Y] = \tilde{V}(X,Y)Z,$$

where $\tilde{V}$ is the concircular curvature tensor Mishra (1984). Hence the concircular curvature tensor $\tilde{V}$ is a particular case of the tensor V for these reason V is called quasi concircular curvature tensor. It can be easily verified that

$$\left.\begin{aligned} &'V(X,Y,Z,W) + {}'V(Y,X,Z,W) = 0,\\ &'V(X,Y,Z,W) + {}'V(X,Y,W,Z) = 0,\\ &'V(X,Y,Z,W) - {}'V(Z,W,X,Y) = 0, \text{ and}\\ &'V(X,Y,Z,W) + {}'V(Y,Z,X,W) + {}'V(Z,X,Y,W) = 0,\end{aligned}\right\} \quad (7.2)$$

where $g(V(X,Y)Z,W) = {}'V(X,Y,Z,W)$.

Quasi concircular curvature tensor $\bar{V}$ with respect to quarter-symmetric non-metric connection $\bar{D}$ in an LP-cosymplectic is defined by

$$\bar{V}(X,Y)Z = A\bar{R}(X,Y)Z = \frac{\bar{r}}{n}\left[\frac{A}{n-1} + 2B\right][g(Y,Z)X - g(X,Z)Y], \quad (7.3)$$

where $g(\bar{V}(X,Y)Z,W) = \bar{V}(X,Y,Z,W)$.

In view of (5.3), (5.5), (7.1) and (7.3), we get

$$\overline{V}(X,Y)\,Z = V(X,Y)\,Z - ab\left[\left\{A.\eta(X)\eta(Z) + \frac{A+2B(n-1)}{n}.g(X,Z)\right\}Y - \left\{A.\eta(Y)\eta(Z) + \frac{A+2\ \ (n-1)}{n}.g(Y,Z)\right\}X\right]. \quad (7.4)$$

From (7.4), we can state the following theorem

Theorem (7.1): The quasi concircular tensor of an LP-cosymplectic manifold with respect to the quarter-symmetric non-metric connection $\overline{D}$and Levi-Civita connection D is equal if and only if

$$\left\{A\eta(X)\eta(Z) + \frac{A+2B(n-1)}{n}.g(X,Z)\right\}Y = \left\{A\eta(Y)\eta(Z) + \frac{A+2B(n-1)}{n}.g(Y,Z)\right\}X \quad ,$$

$provided \quad A,B\neq 0.$

This proves the theorem.

Let $\overline{R}ic(Y,Z) = 0 \Rightarrow \overline{r} = 0.$ (7.5)

Then from (7.3) and (7.5), we get

$$\overline{V}(X,Y)\,Z = A\overline{R}(X,Y)\,Z. \quad (7.6)$$

Hence in view of (7.4) and (7.6), we get

$$A.\overline{R}(X,Y)Z = V(X,Y)Z - ab\left[\left\{A.\eta(X)\eta(Z) + \frac{A+2B(n-1)}{n}.g(X,Z)\right\}Y - \left\{A.\eta(Y)\eta(Z) + \frac{A+2B(n-1)}{n}.g(Y,Z)\right\}X\right]. \quad (7.7)$$

Here in view of (7.7), we have the following theorem.

Theorem (7.2): If the curvature tensor $\overline{R}$ of the quarter-symmetrci non-metric connection $\overline{D}$ in an LP-cosymplectic manifold varishes, then the manifold is quasi concircully flat if and only if

$$\left\{A.\eta(X)\eta(Z) + \frac{A+2B(n-1)}{n}.g(X,Z)\right\}Y = \left\{A.\eta(Y)\eta(Z) + \frac{A+2B(n-1)}{n}.g(Y,Z)\right\}X, ab\neq 0.$$

Now equation (7.4) can be put as

$$'\overline{V}(X,Y,Z,W) = {}'V(X,Y,Z,W) - ab\left[\left\{A.\eta(X)\eta(Z) + \frac{A+2B(n-1)}{n}.g(X,Z)\right\}.g(Y,W) - \left\{A.\eta(Y)\eta(Z) + \frac{A+2B(n-1)}{n}.g(Y,Z)\right\}g(X,W)\right]. \quad (7.8)$$

From (7.2) and (7.8), we can state the following theorem:

Theorem (7.5): The quasi concircular curvature tensor with respect to quarter-symmetric non-metric connection $\overline{D}$ of an LP-cosymplictic manifold satisfies the following algebraic properties:

$$'\overline{V}(X,Y,Z,W) + {}'\overline{V}(Y,X,Z,W) = 0,$$

$$'\overline{V}(X,Y,Z,W) + {}'\overline{V}(X,Y,W,Z) = ab[A\{\eta(Y)\eta(Z)g(X,W) - \eta(X)\eta(Z)g(Y,W) - \eta(X)\eta(W)g(Y,Z) + \eta(Y)\eta(W)g(X,Z)\} + 2.\frac{A+2B(n-1)}{n}\{g(Y,Z)g(X,W) - g(X,Z)g(Y,W)\},$$

$$'\overline{V}(X,Y,Z,W) - {}'\overline{V}(Z,W,X,Y) = abA[\eta(Y)\eta(Z)g(X,W) - \eta(X)\eta(W)g(Y,Z)],$$

and

$$'\overline{V}(X,Y,Z,W) + {}'\overline{V}(Z,W,X,Y) + {}'\overline{V}(Z,X,Y,W) = 0.$$

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