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Pseudo W_8–flat Lorentzian α–para Kenmotsu manifold

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Abstract

The notion of Lorentzian α–para Kenmotsu manifold has been introduced by Prasad, Verma and Yadav (2023). In this paper we study some properties of pseudo W_8–flat Lorentzian α–para Kenmotsu manifold.

Keywords: Lorentzian α–para Kenmotsu manifold, Pseudo W_8–curvature tensor, Manifold of constant curvature.

1. Introduction

Pokharial (1982) defined W_8–curvature tensor. Further, Prasad, Yadav and Pandey (2018) generalized this concept and introduced the notion of pseudo W_8–curvature tensor $\widetilde{W}_8$ (Prasad, Yadav and Pandey, 2018). Prasad, verma and Yadav (2023) defined Lorentzian α–para Kenmotsu manifold and studied some properties of it. In 1989, Matsumoto introduced the notion of LP–Sasakian manifolds. De, Shaikh and Sengupta (2002) introduced the notion of LP–Sasakian manifold with a coefficient α which generalized the notion of LP–Sasakian manifold. In 2007, Bagewadi, Prakasha and Venkatesha studied the pseudo projectively flat LP–Sasakian manifold with a coefficient α. Singh and Maurya (2022) studied quasi conformally flat LP–Sasakian manifold with a coefficient α. In this paper we study some properties of pseudo W_8–flat Lorentzian α–para Kenmotsu manifold. We prove that a pseudo W_8–flat Lorentzian α–para Kenmotsu manifold is always a η–Einstein manifold, provided α and σ are constants. Futher we prove that of $a-(n-1)b \neq 0$ and scalar curvature r is constant, then a pseudo W_8–flat Lorentzian α–para Kenmotsu manifold is of constant curvature, provided α and σ are constants.

2. Preliminaries

An n–dimensional smooth manifold M is said to be Lorentzian almost paracontact manifold, provided M is equipped with a (1,1)–tensor field ϕ, a covariant vector field ξ, a covariant vector field η and a (0,2) type Lorentzian metric g. Let $g_m : T_m M \times T_m M \rightarrow R$ be an inner product of signature (- ,+ , + , …… , +), here m is a point in M, $T_m M$ represents tangent space of smooth manifold M at m and R is real number space. Some basic results, given below hold:

$$\phi^2 (X) = X + \eta(X)\xi , \quad \eta(\xi) = -1, \tag{2.1}$$

$$g(X,\xi) = \eta(X), g(\phi X, \phi Y) = g(X,Y) + \eta(X)\,\eta(Y), \tag{2.2}$$

$\forall\ X, Y$ on M, and structure (ϕ, ξ, η, g) is said to be Lorentzian almost paracontact structure. An n– dimensional smooth manifold M endowed with structure (ϕ, ξ, η, g) is said to be Lorentzian almost paracontact manifold (De, 2009, Motsumoto, 1989). Results given below hold (Motsumoto, 1989) for Lorentzian almost paracontact manifold,

$$\phi\xi = 0\,,\ \eta(\phi X) = 0,\ \Omega(X, Y) = \Omega(Y, X), \tag{2.3}$$

where,

$$\Omega(X, Y) = g(\,X, \phi Y).$$

Definition 2.1: A Lorentzian almost paracontact manifold M is said to be Lorentzian para–Kenmotsu manifold, provided

$$(D_X\, \phi)(Y) = -\,g(\phi X, Y)\xi - \eta(Y)\,\phi X,\ \forall\ \ X, Y \quad \text{(Haseeb, 2020, 2021, Pankaj, 2021)}$$

Hence, we have the following:

Definition 2.2: A Lorentzian para–Kenmotsu manifold is said to be Lorentzian α–para Kenmotsu manifold, provided

$$(D_Z\, \Omega)(X, Y) + \alpha\, \eta(X)\, \Omega(Y, Z) + \alpha\, \eta(Y)\, \Omega(X, Z) = 0, \tag{2.4}$$

$\forall\ \ X, Y$ on M, where α is a non–zero smooth function and

$$\Omega(\phi X, Y) = -\,\frac{1}{\alpha}\,(D_X\, \eta)(Y).$$

We defined,

$$\bar{\Omega}(X, Y)\ =\ \Omega(\phi X, Y),$$

Then, we have

$$\bar{\Omega}(X, Y) = -\,\frac{1}{\alpha}\,(D_X\, \eta)(Y), \tag{2.5}$$

and

$$\bar{\Omega}(X, Y) = \bar{\Omega}(Y, X),$$

where, D is covariant differential operator.

From equation (2.4), we get,

$$(D_X\, \phi)(Y) = -\,\alpha\, g(\phi X, Y)\xi - \alpha\, \eta(Y)\phi X. \tag{2.6}$$

Putting $Y = \xi$ in the above equation, we get,

$$(D_X\, \phi)(\xi) = -\,\alpha\, g(\phi X, \xi)\, \xi - \alpha\, \eta(\xi)\phi X.$$

Using equation (2.1) and (2.3), we obtain,

$$-\,\phi(D_X\, \xi) = \alpha\, \phi X.$$

Operating ϕ on both sides of the above relation and using relation (2.1), it yields

$$D_X\, \xi + \eta\, (D_X\, \xi)\, \xi = -\,\alpha\, (X + \eta(X)\, \xi).$$

Relation (2.1) implies $\eta\, (D_X\, \xi) = 0$. Using this relation in the above equation, we get

$$D_X\, \xi = -\, \alpha X - \alpha\eta(X)\, \xi. \tag{2.7}$$

Also,

$$(D_X\, \eta)(Y) = D_X\, \eta(Y) - \eta\, (D_X\, Y) = g(Y, D_X\, \xi). \tag{2.8}$$

Relation (2.7) and (2.8) together yield

$$(D_X\, \eta)(Y) = -(\alpha)[g(X,Y) + \eta(X)\eta(Y)]. \tag{2.9}$$

In particular, if α satisfies (2.9) together with the following relation

$$D_X\, \alpha = d\alpha(X) \;=\; \sigma\, \eta(X), \tag{2.10}$$

Then ξ is said to be concircular vector field. Here, σ is smooth function and η is 1-form.

For Lorentzian α–para Kenmotsu manifold $M\ (\,\phi, \xi, \eta, g\,)$, following results hold good (Kachar, 1982)

$$\eta(R(X,Y)\, Z) = (\alpha^2 + \sigma)\, [g(Y,Z)\, \eta(X) - g(X,Z)\, \eta(Y)], \tag{2.11}$$

$$Ric\, (X, \xi) = (n-1)(\alpha^2\, +\, \sigma)\, \eta(X), \tag{2.12}$$

$$R(X,Y)\, \xi = (\alpha^2 + \sigma)\, [\eta(Y)X - \eta(X)Y], \tag{2.13}$$

$$R(\xi, Y)\, X = (\alpha^2 + \sigma)\, [g(X,Y)\xi - \eta(X)Y], \tag{2.14}$$

$$(D_X\, \phi)(Y) = -\, \alpha\, g(\phi X, Y)\, \xi - \alpha\, \eta(Y)\, \phi X, \tag{2.15}$$

$$Ric\, (\phi X, \phi Y) = Ric\, (X,Y) + (n-1)(\alpha^2 + \sigma)\, \eta(X)\, \eta(Y). \tag{2.16}$$

The notion of pseudo W_8–curvature tensor $\widetilde{W}_8$ was given by Prasad, Yadav and Pandey (2018) as follows:

$$\widetilde{W}_8\, (X,Y)Z = a\, R(X,Y)Z + b\Big[\, Ric\, (X,Y)Z - Ric\, (Y,Z)X\Big] - \frac{r}{n}\Big[\, \frac{a}{n-1} - b\Big]\Big[g(X,Y)Z - g(Y,Z)X\Big]. \qquad a, b \neq 0. \tag{2.17}$$

If $\widetilde{W}_8(X,Y)Z = 0$ and $a - (n-1)b \neq 0$, then Ricci tensor is given by (Prasad, Yadav & Pandey, 2018)

$$Ric\, (Y,Z) = -\, \frac{r}{n}\, g(Y,Z). \tag{2.18}$$

If $= 1\,, b = \frac{1}{n-1}$, then from (2.17), we get,

$$\widetilde{W}_8\, (X,Y)Z = R(X,Y)Z + \frac{1}{(n-1)}[\, Ric\, (X,Y)Z - Ric\, (Y,Z)X].$$

$= W_8$–curvature tensor (Pokhariyal, 1982). (2.19)

3. Pseudo $\widetilde{W}_8$–flat Lorentzian α–para Kenmotsu manifold

Let us consider a Pseudo W_8–flat Lorentzian α–para Kenmotsu manifold. Then $\widetilde{W}_8 = 0$.

Then form (2.17), we get

$$R(X,Y)Z = \frac{b}{a}\, [\, Ric\, (Y,Z)X - Ric\, (X,Y)Z] + \frac{r}{an}\, \Big[\frac{a}{n-1} - b\Big]\, [g(X,Y)\, Z - g(Y,Z)X]. \tag{3.1}$$

Implies that,

$$'R(X,Y,Z,W) = \frac{b}{a}[\, Ric\, (Y,Z)g(X,W\,) - Ric\, (X,Y)g(Z,W)] +$$

$$\frac{r}{an}\left[\frac{a}{(n-1)}-b\right]\ [g(X,Y)g(Z,W)-g(Y,Z)g(X,W)]. \qquad (3.2)$$

where

$$'R\ (X,Y,Z,W)=g\ (R(X,Y)Z,W).$$

Putting $W=\xi$ in (3.2) and using (2.2) and (2.11), we get

$$\eta(R(X,Y)Z)=\frac{b}{a}\ [\,Ric\ (Y,Z)\eta(X)-Ric\ (X,Y)\eta(Z)]+$$
$$\frac{r}{an}\left[\frac{a}{(n-1)}-b\right][g(X,Y)\ \ \eta(Z)-g(Y,Z)\ \eta(X)].$$

This gives

$$(\alpha^2+\sigma)[g(Y,Z)\eta(X)-g(X,Z)\eta(Y)]=\frac{b}{a}[\,Ric\ (Y,Z)\ \eta(X)-Ric\ (X,Y)\ \eta(Z)]+$$
$$\frac{r}{a}\left[\frac{a}{(n-1)}-b\right][g(X,Y)\eta(Z)-g(Y,Z)\ \eta(X)]. \qquad (3.3)$$

Putting $\xi=X$ in (3.3) and using (2.1), (2.2) and (2.12), we get

$$-(\alpha^2+\sigma)\ g(Y,Z)-(\alpha^2+\sigma)\ \eta(Y)\ \eta(Z)\ =-\frac{b}{a}\ Ric\ (Y,Z)-\frac{b}{a}(n-1)(\alpha^2+\sigma)\ \eta(Y)\ \eta(Z)$$
$$+\frac{r}{an}\left[\frac{a}{n-1}-b\right][g(X,Y)\eta(Z)-g(Y,Z)\ \eta(X)].$$

From above, we get

$$Ric\ (Y,Z)=\left[\frac{a}{b}(\alpha^2+\sigma)+\frac{r}{bn}\left(\frac{a}{n-1}-b\right)g(Y,Z)\right]+$$
$$\left[\left\{\frac{a-b(n-1)}{b}\right\}(\alpha^2+\sigma)+\frac{r}{bn}\left(\frac{a}{n-1}-b\right)\right]\eta(Y)\eta(Z). \qquad (3.4)$$

From (3.4), we can state the following theorem:

Theorem 3.1: A pseudo W_8–flat Lorentzian α – para Kenmotsu manifold is always an η–Einstein manifold, provided α and σ are constants.

Differentiating (3.4) along X and using (2.2) and (2.3), we get

$$(D_X\ Ric)(Y,Z)=\frac{dr(X)}{bn}\left[\frac{a}{(n-1)}-b\right]\ [g(Y,Z)+\eta(Y)\eta(Z)]+$$
$$\left[\left\{\frac{a-(n-1)b}{b}\right\}(\alpha^2+\sigma)+\frac{r}{bn}\left(\frac{a}{(n-1)}-b\right)\right]\times$$
$$[(D_X\ \eta)Y\ \eta(Z)+\ \eta(Y)(D_X\ \eta)Z].$$

$$(D_X\ Ric)(Y,Z)=\frac{dr(X)}{bn}\left[\frac{a}{(n-1)}-b\right][g(Y,Z)+\eta(Y)\ \eta(Z)]+$$
$$\left[\left\{\frac{a-(n-1)b}{b}\right\}(\alpha^2+\sigma)+\ \frac{r}{bn}\left(\frac{a}{(n-1)}-b\right)\right]\times$$
$$\left[-\alpha\ \Omega(\phi X,Y)\eta(Z)-\ \alpha\ \eta(Y)\Omega\ (\phi X,Z)\right].$$

$$(D_X\ Ric)(Y,Z)=\frac{dr(X)}{bn}\left[\frac{a}{(n-1)}-b\right][g(Y,Z)+\eta(Y)\ \eta(Z)]+$$
$$\left[\left\{\frac{a-(n-1)b}{b}\right\}(\alpha^2+\sigma)+\ \frac{r}{bn}\left(\frac{a}{(n-1)}-b\right)\right]\times$$

$$[-\alpha\,\{\,g(X,Y)+\eta(X)\eta(Y)\}\,\eta(Z)- \alpha\,\eta(Y)\{\,g(X,Z)+\eta(X)\,\eta(Z)\}].$$

$$(D_X\,Ric)(Y,Z)=\frac{dr(X)}{bn}\left[\frac{a}{(n-1)}-b\right][g(Y,Z)+\eta(Y)\,\eta(Z)]- \alpha\left[\left\{\frac{a-(n-1)b}{b}\right\}(\alpha^2+\sigma)+\frac{r}{bn}\left(\frac{a}{(n-1)}-b\right)\right]\times [g(X,Y)\eta(Z)+g(X,Z)\eta(Y)+2\,\eta(X)\eta(Y)\eta(Z)], \tag{3.5}$$

where α and are σ constant.

Using (3.5), we get

$$(D_X\,Ric)(Y,Z)-(D_Y\,Ric)(X,Z)=\frac{dr(X)}{bn}\left[\frac{a}{(n-1)}-b\right][g(Y,Z)+\eta(Y)\,\eta(Z)]- \frac{dr(Y)}{bn}\left[\frac{a}{(n-1)}-b\right][\,g(X,Z)+\,\eta(X)\eta(Z)]- \alpha\left[\left\{\frac{a-(n-1)b}{b}\right\}(\alpha^2+\sigma)+\frac{r}{bn}\left(\frac{a}{(n-1)}-b\right)\right]\times [\,g(X,Z)\eta(Y)-\,g(Y,Z)\eta(X)]. \tag{3.6}$$

Differentiating (2.18) covariantly along X, we get

$$(D_X\,Ric)(Y,Z)=-\frac{dr(X)}{n}\,g(Y,Z),\quad \text{provided } a-(n-1)b\neq 0. \tag{3.7}$$

Using (3.7), we get

$$(D_X\,Ric)(Y,Z)-(D_Y\,Ric)\,(X,Z)=\frac{dr(Y)}{n}\,g(X,Z)-\frac{dr(X)}{n}\,g(Y,Z). \tag{3.8}$$

From (3.6) and (3.8), we get

$$\frac{dr(Y)}{n}g(X,Z)-\frac{dr(X)}{n}g(Y,Z)=\frac{dr(X)}{bn}\left[\frac{a}{(n-1)}-b\right][g(Y,Z)+\eta(Y)\,\eta(Z)]- \frac{dr(Y)}{bn}\left[\frac{a}{(n-1)}-b\right][\,g(X,Z)+\,\eta(X)\eta(Z)]- \alpha\left[\left\{\frac{a-(n-1)b}{b}\right\}(\alpha^2+\sigma)+\frac{r}{bn}\left(\frac{a}{(n-1)}-b\right)\right]\times [\,g(X,Z)\,\eta(Y)\,-\,g(Y,Z)\,\eta(X)].$$

If r is constant from above, we get

$$-\alpha\left[\left\{\frac{a-(n-1)b}{b}\right\}(\alpha^2+\sigma)+\frac{r}{bn}\left(\frac{a}{(n-1)}-b\right)\right][\,g(X,Z)\,\eta(Y)-g(Y,Z)\,\eta(X)]=0,$$

or,

$$\left\{\frac{a-(n-1)b}{b}\right\}\left[\,(\alpha^2+\sigma)+\frac{r}{n(n-1)}\,\right]=0,$$

or,

$$r=-\,n\,(n-1)(\alpha^2+\sigma), \tag{3.9}$$

provided $a-(n-1)b\neq 0$ and α and σ are constants.

From (3.4) and (3.9), we get

$$Ric\ (Y,Z) = (\alpha^2 + \sigma)\ (n-1)\ g(Y,Z). \tag{3.10}$$

Using (3.9) and (3.10) in (3.2), we get,

$$'R\ (X,Y,Z,W) = \frac{b}{a}\ [(\alpha^2+\sigma)\ (n-1)\ g(Y,Z)\ g(X,W) -$$
$$(\alpha^2+\sigma)\ (n-1)\ g(X,Y)\ g(Z,W)] -$$
$$\frac{n\ (n-1)(\alpha^2+\sigma)}{an}\ \left(\frac{a}{n-1} - b\right)\ [g(X,Y)g(Z,W) - g(Y,Z)g(X,W)].$$

$$'R\ (X,Y,Z,W) = (\alpha^2+\sigma)\ [\ g(Y,Z)\ g(X,W) - g(X,Y)\ g(Z,W)\ ]. \tag{3.11}$$

From (3.11), we can state the following theorem:

Theorem 3.2: In a pseudo W_8–flat Lorentzian α–para Kenmotsu manifold, if the scalar curvature r is constant, then manifold is of constant curvature, provided α, σ are constants and $a - (n-1)b \neq 0$.

From (3.11), we get

$$R\ (X,Y)Z = (\alpha^2+\sigma)[\ g(Y,Z)X - g(X,Y)Z\ ]. \tag{3.12}$$

Using (2.1), (2.2), (2.3), (3.10) and (3.12), we can state the following theorem:

Theorem 3.3: In a pseudo W_8–flat Lorentzian α–para Kenmotsu manifold, if the scalar curvature r is constant, then

(i) $R(X,\xi)\ Y = (\alpha^2+\sigma)\ [\eta(Y)X - \eta(X)Y]$,

(ii) $R(\xi,X)\ Y = (\alpha^2+\sigma)\ [g(X,Y)\xi - \eta(X)Y]$,

(iii) $R(X,Y)\ \xi = (\alpha^2+\sigma)\ [\eta(Y)X - g(X,Y)\xi]$,

(iv) $Ric\ (X,\xi) = (n-1)(\alpha^2+\sigma)\ \eta(X)$,

(v) $Ric\ (\xi,X) = (n-1)(\alpha^2+\sigma)\ \eta(X)$, and

(vi) $Ric\ (\phi X,\phi Y) = Ric\ (X,Y) + (n-1)(\alpha^2+\sigma)\ \eta(X)\ \eta(Y)$,

provided $a - (n-1)b \neq 0$, α and σ are constants.

From (3.4), we get

$$QY = \left[\frac{a}{b}\ (\alpha^2+\sigma) + \frac{r}{bn}\left(\frac{a}{n-1} - b\right)\ Y\right] +$$
$$\left[\left\{\frac{a-(n-1)b}{b}\right\}(\alpha^2+\sigma)\ + \frac{r}{bn}\left(\frac{a}{n-1} - b\right)\right]\ \eta(Y)\xi.$$

Contracting above with respect to Y and using (2.1), we get

$$r\ = \left[\frac{a}{b}\ (\alpha^2+\sigma) + \frac{r}{bn}\left(\frac{a}{n-1} - b\right)\right]\ n +$$
$$\left[\left\{\frac{a-b(n-1)}{b}\right\}(\alpha^2+\sigma) + \frac{r}{bn}\ \left(\frac{a}{n-1} - b\right)\right],$$

which gives

$$r = \frac{1}{[(2n-1)b-a]} n(n-1)(a+b)(\alpha^2+\sigma), \text{ provided } (2n-1)b - a \neq 0. \quad (3.13)$$

Using (3.13) in (3.4), we get

$$Ric(Y,Z) = \frac{(\alpha^2+\sigma)}{[(2n-1)b-a]} \left[\{(n+1)a - (n-1)b\} g(Y,Z) + 2n\{a - (n-1)b\} \eta(Y)\eta(Z)\right], \text{ provided } (2n-1)b - a \neq 0, \quad (3.14)$$

Using (3.13) and (3.14) in (3.1), we get,

$$R(X,Y)Z = (\alpha^2+\sigma)[g(Y,Z)X - g(X,Y)Z] + \frac{2n(\alpha^2+\sigma)\{a-(n-1)b\}}{a\{(2n-1)b-a\}}[\eta(Y)\eta(Z)X - \eta(X)\eta(Y)Z], \text{provided } (2n-1)b - a \neq 0. \quad (3.15)$$

Since $\alpha \neq 0$, From (3.15), we can state the following theorem:

Theorem 3.4: In a pseudo W_8–flat Lorentzian α – para Kenmotsu manifold, if $a(n-1)b \neq 0$ and $(2n-1)b - a \neq 0$, then the manifold can not be of constant curvature, provided α and σ are zero constants.

If $a = 1$ and $b = \frac{1}{n-1}$, then from (2.19), we get $\widetilde{W}_8 = W_8$. Also from (3.15), we get

$$R(X,Y)Z = (\alpha^2+\sigma)[g(Y,Z)X - g(X,Y)Z], \text{ provided } (2n-1)b - a \neq 0. \quad (3.16)$$

From (3.16) we can state the following theorem:

Theorem 3.5: In a W_8–flat Lorentzian α–para Kenmotsu manifold, if $(2n-1)b - a \neq 0$, then manifold is of constant curvature, provided α and σ are constants.

Using (2.1), (2.2), (2.3), (3.14) and (3.15), we can state the following theorem:

Theorem 3.6: In a pseudo W_8–flat Lorentzian α–para Kenmotsu manifold, if $(2n-1)b - a \neq 0$, the following relations hold:

(i) $$R(X,\xi)Y = (\alpha^2+\sigma)\left[1 - \frac{2n\{a-(n-1)b\}}{a\{(2n-1)b-a\}}\right][\eta(Y)X - \eta(X)Y], \quad (3.17)$$

(ii) $$R(\xi,X)Y = (\alpha^2+\sigma)g(X,Y)\xi - (\alpha^2+\sigma)\left[1 - \frac{2n\{a-(n-1)b\}}{a\{(2n-1)b-a\}}\right]\eta(X) + \left[\frac{2n(\alpha^2+\sigma)\{a-(n-1)b\}}{a\{(2n-1)b-a\}}\right]\eta(X)\eta(Y)\xi, \quad (3.18)$$

(iii) $$R(X,Y)\xi = (\alpha^2+\sigma)\left[1 - \frac{2n\{a-(n-1)b\}}{a\{(2n-1)b-a\}}\right]\eta(Y)X - (\alpha^2+\sigma)g(X,Y)\xi - \left[\frac{2n(\alpha^2+\sigma)\{a-(n-1)b\}}{a\{(2n-1)b-a\}}\right]\eta(X)\eta(Y)\xi, \quad (3.19)$$

(iv) $$Ric(X,\xi) = (\alpha^2+\sigma)(n-1)\eta(X), \quad (3.20)$$

(v) $$Ric(\xi,X) = (\alpha^2+\sigma)(n-1)\eta(X), \quad (3.21)$$

(vi) $$Ric(\phi X,\phi Y) = \frac{(\alpha^2+\sigma)}{[(2n-1)b-a]}\left[\{(n+1)a - (n-1)b\}\{g(X,Y) + \eta(X)\eta(Y)\}\right], \quad (3.22)$$

provided α and σ are constants.

If $a = 1$ and $b = \frac{1}{n-1}$, then from (3.14), we get

$$Ric\,(Y,Z) = (\alpha^2 + \sigma)(n-1)g(Y,Z). \tag{3.23}$$

Using (2.1), (2.2), (2.3), (3.16) and (3.23), we can state the following theorems:

Theorem 3.7: In a W_8 –flat Lorentzian α–para Kenmotsu manifold the following relations hold:

(i) $R(X,\xi)\,Y = (\alpha^2 + \sigma)\,[\eta(Y)X - \eta(X)Y],$ (3.24)

(ii) $R(\xi,X)Y = (\alpha^2 + \sigma)[g(X,Y)\xi - \eta(X)Y],$ (3.25)

(iii) $R(X,Y)\xi = (\alpha^2 + \sigma)\,[\eta(Y)X - g(X,Y)\xi],$ (3.26)

(iv) $Ric\,(X,\xi) = (\alpha^2 + \sigma)\,(n-1)\,\eta(X),$ (3.27)

(v) $Ric\,(\xi,X) = (\alpha^2 + \sigma)\,(n-1)\,\eta(X),$ (3.28)

(vi) $Ric\,(\phi X,\phi Y) = Ric\,(X,Y) + (\alpha^2 + \sigma)(n-1)\eta(X)\eta(Y),$ (3.29)

provided $(2n-1)b - a \neq 0$, α and σ are constants.

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