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Quasi-conhormonic curvature tensor on K-contact and Sasakian manifold

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Abstract

Some necessary and / or sufficient condition form K-contact and / or Sasakian manifolds to be quasi quasi-conhormonically flat, $\xi -$ quasi-conhormonically and $\phi -$ quasi-conhormonically flat are obtained.

Keywords- K-contact manifold, Sasakian manifold quasi-conhormonic curvature tensor, $\phi -$ quasi-conhormonically flat manifold, quasi quasi-conhormonically flat, $\xi -$ quasi-conhormonically and $\phi -$ quasi-conhormonically flat.

Introduction

Let M be almost contact metric manifold equipped with an almost contact structure(ϕ, ξ, η, g). At each point $p \in M$, decompose the tangent space T_pM into the direct sum $T_pM = \phi(T_pM) \otimes \{\xi_p\}$, where $\{\xi_p\}$ is the 1-dimensional linear subspace of T_pM generated by $\{\xi_p\}$. Thus the Conformal curvature C is a map $C: T_pM \times T_pM \rightarrow \phi(T_pM\ ) \oplus \{\xi_p\}$.

An almost contact metric manifold M is said to be

(1) Conformally symmetric Gao (1992) if the projective of the image of C in $\phi(T_pM\ )$ is zero.

(2) $\xi -$Conformally flat Zhen, Cabrerizo, Fernandez and Fernandez (1977) if the projective of the image of C in $\{\xi_p\}$ is zero, and

(3) $\phi -$Conformally flat Cabrerizo, Fernandez, Fernandez and Zhen (1999) if the projective of the image of C in $\phi(T_pM\ )$is zero.

In 1992 Gao proved that a conformally symmetric K-contact manifold is locally isometric to the unit sphere. Cabrerizo etal proved that a K-contact manifold ξ −Conformally flat if and only if it is an η −Einstein Sasakian manifold in 1997. In 2000 Arslan etal obtained some results for ϕ −Conformally flat and ξ −concercularly flat on (k,μ) −contact manifolds. Several authors studied conformal curvature tensor and projective curvature tensor on contact manifold, P-contact manifold and LP-contact manifold respectively.

A rank four H that remains invariant under conhormonic transformation for an m-dimensional $m \geq 3$ Riemannian manifold is given by

$$'H(X,Y,Z,W) = {'R}(X,Y,Z,W) - \frac{1}{m-2}[\,Ric(Y,Z)g(X,W) - Ric(X,Z)g(Y,W)$$

$$+g(Y,Z)Ric(X,W) - g(X,Z)Ric(Y,W)]$$

where $'R$ denotes the Riemannian curvature tensor of type (0,4) defined by

$$'R(X,Y,Z,W) = g(R(X,Y)Z,W)$$

where R denotes the Riemannian curvature tensor of type (1,3) and Ric denotes Ricci tensor of type (0,2) and Q is the Ricci operator defined by $g(QX,Y) = Ric(X,Y)$ respectively.

The curvature tensor defined by (1.1) is known as conhormonic curvature tensor. A manifold whose conhormonic curvature tensor vanishes at every point of the manifold is called conhormonically flat manifold Ghosh (2010). It satisfies all the symmetric properties of the Riemannian curvature tensor $'R$. There are many physical applications of the tensor H . For example, in 2009, Özgür considered some conditions on conhormonic curvature tensor H on hypersurface in the semi-Euclidean space. He proved that every conhormonically Ricci-symmetric hypersurface satisfying the condition $H.R = 0$ is pseudo symmetric.

Motivated by the studies of conformal curvature tensor in Cabrerizo *et al.* (1997), Guo (1992), Dwivedi *et al.* (2009) and (2011), Zhen *et al.* (1997), Ghosh (2010), and the studies of projective curvature tensor in K-contact and Sasakian manifold in Özgür.

A quasi-conhormonic curvature tensor tensor $\widetilde{H}$ of type (1,3) on Riemannian manifold $(n > 3)$ defined as:

$$\widetilde{H}(X,Y)Z = a\,R(X,Y)Z + b\,[\,Ric(Y,Z)X - Ric(X,Z)Y]$$
$$+ c[\,g(Y,Z)QX - g(X,Z)QY\,]$$

$$- \frac{r}{n}\left[\frac{2a}{n-2} + b + c\right]\left[g(Y,Z)X - g(X,Z)Y\right], \quad (1.1)$$

for all $X, Y, Z \in M$, where R is the Riemannian curvature tensor Ric is the Ricci tensor of the type (0,2), Q is the Ricci operator of the type (1,1) defined by $Ric(X,Y) = g(QX,Y), r$ is the scalar curvature tensor and a, b, c are constants. If $a = 1$ and $b = c = -\frac{1}{n-2}$ then (1.1) takes the form

$$\widetilde{H}(X,Y)Z = R(X,Y)Z - \frac{1}{n-2}[\, Ric(Y,Z)X - Ric(X,Z)Y + g(Y,Z)QX - g(X,Z)QY \,] = H(X,Y)Z,$$

where H is the Conharmonic curvature tensor Mishra (1984). Hence the Conhormonic curvature tensor H is a particular case of the tensor $\widetilde{H}$. For this reason $\widetilde{H}$ is called Quasi-Conhormonic curvature tensor (QCC).

Equation (1.1) can be written as of type (0, 4) as follows

$$'\widetilde{H}(X,Y,Z,W) = a\,'R(X,Y,Z,W) + b[\, Ric(Y,Z)g(X,W) - Ric(X,Z)g(Y,W)] + c[g(Y,Z)Ric(X,W) - g(X,Z)Ric(Y,W)] - \frac{r}{n}\left[\frac{2a}{n-2} + b + c\right]\left[g(Y,Z)\, g(X,W) - g(X,Z)g(Y,W)\right], \quad (1.2)$$

where

$$\left.\begin{aligned} '\widetilde{H}(X,Y,Z,W) &= g(\widetilde{H}(X,Y)Z,W) \\ 'R(X,Y,Z,W) &= g(R(X,Y)Z,W). \end{aligned}\right\} \quad (1.3)$$

QCC tensor $\widetilde{H}$ satisfies the following algebraic properties:

$$'\widetilde{H}(X,Y,Z,W) + {'\widetilde{H}}(Y,X,Z,W) = 0,$$

$$'\widetilde{H}(X,Y,Z,W) + {'\widetilde{H}}(X,Y,W,Z) = 0,$$

$$'\widetilde{H}(X,Y,Z,W) - {'\widetilde{H}}(Z,W,X,Y) \neq 0,$$

and

$$'\widetilde{H}(X,Y,Z,W) + {'\widetilde{H}}(Y,Z,X,W) + {'\widetilde{H}}(Z,X,Y,W) = 0.$$

Lal et. al (2013) proved that quasi-conhormonically flat manifold is of zero scalar curvature provided that $a(3n-4) + 2(n-1)(n-2)(b+c) \neq 0.$

Continuing this study in this paper we extended these results in K-contact and Sasakian manifold on quasi-conhormonic curvature tensor. The paper is organized as follows: Introduction is given in section 1. Section 2 contains some preliminaries. In section 3, in an almost contact metric manifold we consider three cases of quasi-conhormonic curvature tensor. Analogous to the definition of quasi-conhormonically flat, $\xi -$ quasi-conhormonically and $\phi -$ quasi-conhormonically flat almost contact metric manifolds. It is proved that if a K-contact manifold is quasi quasi-conhormonically flat then the scalar curvature tensor is a constant. We also proved that a Sasakian manifold is, $\xi -$ quasi-conhormonically flat if and only if it is $\eta -$Einstein. Provided condition for a K-contact manifold and Sasakian manifold to be $\phi -$ quasi-conhormonically flat are obtained.

Preliminaries

Let M^{2n+1} be an almost contact metric manifold equipped with an almost contact structure (ϕ, ξ, η, g) consisting of (1,1) tensor field ϕ, a vector field ξ, a 1-form η and a Riemannian metric g.Then

$$\phi^2 I = -I + \eta \otimes \xi, \quad \eta(\xi) = -1, \;\; \phi\xi = 0, \;\; \eta o\, \phi = 0, \tag{2.1}$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \tag{2.2}$$

for all X and $Y \epsilon TM$. In view of (2.1) and (2.2), we have

$$g(X, \phi Y) + g(\phi X, Y) = 0, \quad g(X, \xi) = \eta(X),$$

for all X and $Y \in TM$.

An almost contact metric manifold is :

(1) a contact metric manifold if $g(X, \phi Y) = d\eta(X, Y)$ for all X and $Y \epsilon TM$. $R(X, Y)\xi = \eta(Y)X - \eta(X)Y$;

(2) a K-contact manifold if $D\xi = -\phi$ where D is Levi-Civita connection;

(3) a Sasakian manifold if and only if

$$R(X, Y)\xi = \eta(Y)X - \eta(X)Y \;, \tag{2.3}$$

every Sasakian manifold is K-contact but the converse need not be true, except in dim 3 Jun and Kim (1994). K-contact metric manifold are not too well know, because there is no such a simple expression for the curvature tensor as in the case of Sasakian manifold. Beside the above relations in K-contact manifold the following relations hold Blair (1976), Jun and Kim (1994)

$$D_X \xi = -\phi X, \tag{2.4}$$

$$'R(\xi, X, Y, \xi) = g(R(\xi, X)Y, \xi) = \eta(R(\xi, X)Y) = g(X, Y) - \eta(X)\eta(Y), \tag{2.5}$$

$$R(\xi, X)\xi = -X + \eta(X)\xi, \tag{2.6}$$
$$Ric(X, \xi) = 2n.\eta(X), \tag{2.7}$$
$$(D_X\phi) = R(\xi, X)Y, \tag{2.8}$$

for any vector fields X and Y.

Again a K-contact manifold is called Einstein if its curvature tensor is of the form

$$Ric(X, Y) = \lambda_1 g(X, Y), \tag{2.9}$$

where λ_1 is a constant.

Similarly, a K-contact manifold is called η-Einstein manifold if its Ricci tensor Ric is of the form

$$Ric(X, Y) = \lambda_2 g(X, Y) + \lambda_3 \eta(X)\eta(Y), \tag{2.10}$$

where λ_2 and λ_3 are smooth function on M^{2n+1}.

It is well known Jun and Kim (1994) that in a K-contact manifold λ_2 and λ_3 are constant. Also it is known as Boyer and Galicki (2001) that a compact η-Einstein K-contact manifold is Sasakian manifold, provided $\lambda_2 \geq -2$.

The following equations of this equation are taken from Tripathi and Dwivedi (2008). In a (2n+1)-dimensional almost contact metric manifold, if $\{e_1, e_2, \dots, e_{2n}, \xi\}$ be a local orthognormal basis of vector fields, then $\{\phi e_1, \phi e_2, \dots, \phi e_{2n}, \xi\}$ is also a local orthonormal basis. It is easy to verify that

$$\sum_{i=1}^{2n} g\,(e_i, e_i) = \sum_{i=1}^{2n} g\,(\phi e_i, \phi e_i) = 2n, \tag{2.11}$$

$$\sum_{i=1}^{2n} g\,(e_i, Z)Ric(Y, e_i) = \sum_{i=1}^{2n} g\,(\phi e_i, Z)Ric(Y, \phi e_i) = Ric(Y, Z) - Ric(Y, \xi)\eta(Z), \tag{2.12}$$

for all X and $Y \in TM$. In view of , $\eta o\ \phi = 0$, we get

$$\sum_{i=1}^{2n} g\,(e_i, \phi Z)Ric(Y, e_i) = \sum_{i=1}^{2n} g\,(\phi e_i, \phi Z)Ric(Y, \phi e_i) = Ric(Y, \phi Z), \tag{2.13}$$

for all X and $Y \in TM$.

If manifold is a K-contact manifold, then it is known that

$$R(X, \xi)\xi = X - \eta(X)\xi, \tag{2.14}$$

and

$$Ric(\xi, \xi) = 2n. \tag{2.15}$$

Moreover, M is Einstein if and only if

$$Ric(X, Y) = 2ng(X, Y). \tag{2.16}$$

From (2.15), we get

$$\sum_{i=1}^{2n} Ric(e_i, e_i) = \sum_{i=1}^{2n} Ric(\phi e_i, \phi e_i) = r\ \ - 2n. \tag{2.17}$$

In a K-contact manifold, we also get

$$'R(\xi, X, Y, \xi) = g(\phi X, \phi Y), \tag{2.18}$$

for all X and $Y \in TM$.

$$\sum_{i=1}^{2n} {}'R\,(e_i, Y, Z, e_i) = \sum_{i=1}^{2n} {}'R\,(\phi e_i, Y, Z, \phi e_i) = Ric(Y, Z) - g(\phi Y, \phi Z), \qquad (2.19)$$

for all X and $Y \epsilon TM$.

3. Some structure theorem

Analogous to the consideration of Conformal curvature tensor Cabrerizo et. al (1999) and Zhen et. al (1997) and conhormonic curvature tensor Dwivedi and Kim (2011), we give the following definition:

Definition (3.1). An almost contact metric manifold M is said to be quasi quasi-conhormonically flat if

$$g(H(X, Y)Z, \phi W) = 0; \qquad (3.1)$$

ξ −quasi-conhormonically flat if

$$H(X, Y)\xi = 0, \qquad (3.2)$$

and ϕ −quasi-conhormonically flat if

$$g(H(\phi X, \phi Y)\phi Z, \phi W) = 0, \qquad (3.3)$$

for all X, Y, Z and $W \epsilon\, TM$.

We begin with the following theorem:

Theorem (3.1). If a $(2n + 1) -$ dimensional K-contact manifold is quasi quasi-conhormonically flat then scalar curvature is not zero, $a, b, c \neq 0$ and Ricci tensor is given by the expression

$$[a + (2n - 1)b - c]Ric(Y, Z) = \left[a - c(r - 2n) + \frac{r(2n-1)}{2n+1}\left(\frac{2a}{2n-1} + b + c\right)\right] g(Y, Z) -$$
$$\left[a - \frac{r}{2n+1}\left(\frac{2a}{2n-1} + b + c\right)\right]\eta(Y)\eta(Z) - bRic(Y, \xi)\eta(Z) + cRic(Z, \xi)\eta(Y), \quad (3.4)$$

for all X and $Y \epsilon\, TM$.

Proof: From (3.1), we get

$$g(H(X, Y)Z, \phi W) = ag(R(X, Y)Z, \phi W) +$$
$$b[Ric(Y, Z)g(X, \phi W) - Ric(X, Z)g(Y, \phi W)]$$
$$+ c[g(Y, Z)Ric(X, \phi W) - g(X, Z)Ric(Y, \phi W)] -$$
$$\frac{r}{2n+1}\left(\frac{2a}{2n-1} + b + c\right)[g(Y, Z)g(X, \phi W) - g(X, Z)g(Y, \phi W)], \quad (3.5)$$

for all X, Y, Z and $W \epsilon\, TM$. For a local orthognormal basis $\{e_1, e_2, \ldots, e_{n-1}, \xi\}$ of vector fields in M, putting $X = \phi e_i$ and $W = e_i$ in (3.5), we get

$$\sum_{i=1}^{2n} g\,(H(\phi e_i, Y)Z, \phi e_i) = a \sum_{i=1}^{2n} {}'R\,(\phi e_i, Y, Z, \phi e_i) + b \sum_{i=1}^{2n} [\,Ric(Y, Z)g(\phi e_i, \phi e_i)$$
$$-Ric(\phi e_i, Z)g(Y, \phi e_i)] + c \sum_{i=1}^{2n} [\,g(Y, Z)Ric(\phi e_i, \phi W)$$
$$-g(\phi e_i, Z)Ric(Y, \phi W)] - \frac{r}{2n+1}\left(\frac{2a}{2n-1} + b + c\right).$$

$$b\sum_{i=1}^{2n}[g(Y,Z)g(\phi e_i,\phi e_i) - g(\phi e_i,Z)g(Y,\phi e_i)], \quad (3.6)$$

Using (2.11), (2.12), (2.13), (2.17) in (3.6), we get

$$\sum_{i=1}^{2n} g(H(\phi e_i,Y)Z,\phi e_i) = a[Ric(Y,Z) - g(\phi Y,\phi Z)] + b[(2n-1)Ric(Y,Z) + Ric(Y,\xi)\eta(Z)] + c[(r-2n)g(Y,Z) - Ric(Y,Z) + Ric(Z,\xi)\eta(Y)] - \frac{r}{2n+1}\left(\frac{2a}{2n-1}+b+c\right)[2ng(Y,Z) - g(\phi Y,\phi Z)]. \quad (3.7)$$

Here we assume that M is quasi quasi-conhormonically flat then (3.7) reduces to

$$[a+(2n-1)b-c]Ric(Y,Z) = \left[a - c(r-2n) + \frac{r(2n-1)}{2n+1}\left(\frac{2a}{2n-1}+b+c\right)\right]g(Y,Z) - \left[a - \frac{r}{2n+1}\left(\frac{2a}{2n-1}+b+c\right)\right]\eta(Y)\eta(Z) - bRic(Y,\xi)\eta(Z) + cRic(Z,\xi)\eta(Y), \quad (3.8)$$

This proves equation (3.4).

Putting $Z=\xi$ in (3.8) and using (2.15), we get

$$r = \frac{(a+2nb-c).2n}{\frac{2n}{2n+1}\left(\frac{2a}{2n-1}+b+c\right)-c} \neq 0. \quad (3.9)$$

Equation (3.9) shows that scalar curvature is not equal to zero, provided $a \neq b \neq c \neq 0$.

Again from (2.7) and (3.8), we get

$$Ric(Y,Z) = \frac{a-c(r-2n)+\frac{r(2n-1)}{2n+1}\left(\frac{2a}{2n-1}+b+c\right)}{a+(2n-1)b-c}g(Y,Z) - \frac{a+2(b+c)n-\frac{r}{2n+1}\left(\frac{2a}{2n-1}+b+c\right)}{a+(2n-1)b-c}\eta(Y)\eta(Z) \quad (3.10)$$

In view of (3.10), we state the following theorem:

Theorem (3.2): A $(2n+1)$ dimensional quasi quasi-conhormonically flat Sasakian manifold M is η −Einstein, provided $a+(2n-1)b-c \neq 0$, where

$A = \frac{a-c(r-2n)+\frac{r(2n-1)}{2n+1}\left(\frac{2a}{2n-1}+b+c\right)}{a+(2n-1)b-c}$ and $B = -\frac{a+2(b+c)n-\frac{r}{2n+1}\left(\frac{2a}{2n-1}+b+c\right)}{a+(2n-1)b-c}$.

Here we assume that M^{2n+1} is quasi quasi-conhormonically flat, then

In view of (3.1) and (3.5), we get

$$g(R(X,Y)Z,\phi^2W) = \frac{b}{a}[-Ric(Y,Z)g(X,\phi^2W) + Ric(X,Z)g(Y,\phi^2W)] + \frac{c}{a}[-g(Y,Z)Ric(X,\phi^2W) + g(X,Z)Ric(Y,\phi^2W)] + \frac{r}{a(2n+1)}\left(\frac{2a}{2n-1}+b+c\right)[g(Y,Z)g(X,\phi^2W) - g(X,Z)g(Y,\phi^2W)] \quad (3.11)$$

In view of (2.1), (2.2) and (3.11), we get

$$R(X,Y)Z = -\left[\left(\frac{b}{a}+\frac{c}{a}\right)\lambda_2 + \frac{r}{a(2n+1)}\left(\frac{2a}{2n-1}+b+c\right)\right][(g(Y,Z)X - g(X,Z)Y] - \frac{b}{a}\lambda_3[\eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y]. \quad (3.12)$$

Hence we have the following theorem:

Theorem (3.3). A Sasakian manifold M is quasi quasi-conhormonically flat if and only if (3.12) exist.

Next, we consider M^{2n+1} is $\xi -$ quasi-conhormonically flat, then from (3.1) and (3.2), we get

$$a\,R(X,Y)\xi + b\,[\,Ric(Y,\xi)X - Ric(X,\xi)Y] + c[\,\eta(Y)QX - \eta(X)QY\,]$$
$$- \frac{r}{2n+1}\left[\frac{2a}{n2n-1} + b + c\right]\left[\eta(Y)X - \eta(X)Y\right] = 0. \quad (3.13)$$

Using (2.3), (2.7) in (3.13), we get

$$\left[a + 2nb - \frac{r}{2n+1}\left(\frac{2a}{n2n-1} + b + c\right)\right]\left[\eta(Y)X - \eta(X)Y\right]$$
$$+ c\left[\,\eta(Y)QX - \eta(X)QY\,\right] = 0. \quad (3.14)$$

Taking the inner product with W in (3.14) and then putting ξ for Y, we get

$$\left[a + 2nb - \frac{r}{2n+1}\left(\frac{2a}{n2n-1} + b + c\right)\right]\left[g(X,W) - \eta(X)\eta(W)\right]$$
$$+ c\left[Ric(X,W) - \eta(X)Ric(\xi,W))\,\right] = 0. \quad (3.15)$$

From (2.17) and (3.15), we get

$$Ric(X,W) = -\frac{1}{c}\left[a + 2nb - \frac{r}{2n+1}\left(\frac{2a}{n2n-1} + b + c\right)\right]g(X,W) +$$
$$\frac{1}{c}\left[a + 2n(b+c) - \frac{r}{2n+1}\left(\frac{2a}{n2n-1} + b + c\right)\right]\eta(X)\eta(W), \quad (3.15)$$

In view of (3.16), we have the following theorem:

Theorem (3.4). A $(2n+1)$ dimensional Sasakian manifold M is $\xi -$ quasi-conhormonically flat then it is η −Einstein, provided a, b and c not equal to zero, where

$$A_1 = -\frac{1}{c}\left[a + 2nb - \frac{r}{2n+1}\left(\frac{2a}{n2n-1} + b + c\right)\right] \text{ and}$$
$$B_1 = \frac{1}{c}\left[a + 2n(b+c) - \frac{r}{2n+1}\left(\frac{2a}{n2n-1} + b + c\right)\right].$$

4. $\boldsymbol{\phi}$ −quasi-conhormonic flatness condition.

For a K-contact, we have from (3.1)

$$g(H(\phi X,\phi Y)\phi Z,\phi W) = ag(R(\phi X,\phi Y)\phi Z,\phi W) + b[Ric(\phi Y,\phi Z)g(\phi X,\phi W)$$
$$-Ric(\phi X,\phi Z)g(\phi Y,\phi W)] + c[g(\phi Y,\phi Z)Ric(\phi X,\phi W)$$
$$+ c[-g(\phi X,\phi Z)Ric(\phi Y,\phi W)] - \frac{r}{2n+1}\left(\frac{2a}{2n-1} + b + c\right)$$
$$[g(\phi Y,\phi Z)g(\phi X,\phi W) - g(\phi X,\phi Z)g(\phi Y,\phi W)]. \quad (4.1)$$

Let $\{e_1, e_2, \ldots, e_{2n}, \xi\}$ be a local orthognormal basis then $\{\phi e_1, \phi e_2, \ldots, \phi e_{2n}, \xi\}$ is also an orthonormal basis. Putting $X = W = e_i$ and taking summation over i in (4.1), we get

$$g(H(\phi e_i,\phi Y)\phi Z,\phi e_i) = ag(R(\phi e_i,\phi Y)\phi Z,\phi e_i) + b[Ric(\phi Y,\phi Z)g(\phi e_i,\phi e_i)$$
$$-Ric(\phi e_i,\phi Z)g(\phi Y,\phi W)] + c[g(\phi Y,\phi Z)Ric(\phi e_i,\phi e_i)$$
$$+ c[-g(\phi e_i,\phi Z)Ric(\phi Y,\phi e_i)] - \frac{r}{2n+1}\left(\frac{2a}{2n-1} + b + c\right)$$
$$[g(\phi Y,\phi Z)g(\phi e_i,\phi e_i) - g(\phi e_i,\phi Z)g(\phi Y,\phi e_i)]. \quad (4.2)$$

Here we assume that M is ϕ −quasi-conhormonically flat. Then using (3.3), (2.17), (2.19) and (4.2), we get

$$Ric(\phi Y, \phi Z) = \lambda_4 \, g(\phi Y, \phi Z), \tag{4.3}$$

where $\lambda_4 = \frac{\left[a-(r-2n)c+\frac{r}{2n+1}\{2a+(b+c)(2n-1)\}\right]}{a+(2n-1)b-c}$.

From (4.1) and (4.3), we get

$$g(R(\phi X, \phi Y)\phi Z, \phi W) = -\frac{1}{a}\left[\lambda_4(b+c) - \frac{r}{2n+1}\left(\frac{2a}{2n-1} + b + c\right)\right].$$

$$[g(\phi Y, \phi Z)g(\phi X, \phi W) - g(\phi X, \phi Z)g(\phi Y, \phi W)]. \tag{4.4}$$

In view of (4.4), we can state the following theorem:

Theorem (4.1). A $(2n+1)$ dimensional K-contact manifold M is $\phi -$ quasi-conhormonically flat if and only if

$$g(R(\phi X, \phi Y)\phi Z, \phi W) = -\frac{1}{a}\left[\lambda_4(b+c) - \frac{r}{2n+1}\left(\frac{2a}{2n-1} + b + c\right)\right].$$

$$[g(\phi Y, \phi Z)g(\phi X, \phi W) - g(\phi X, \phi Z)g(\phi Y, \phi W)].$$

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