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Decomposition of curvature tensor fields in a tachibana first order recurrent space

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Abstract

Takano (1967), have been studied decomposition of curvature tensor in a recurrent space. Sinha and Singh (1970), have been studied and defined decomposition of recurrent curvature tensor field in a Finsler space. Singh and Negi (1985), studied on decomposition of recurrent curvature tensor fields in a Kaehlerian space. Further, Negi and Rawat (1995), studied decomposition in a Kaehlerian recurrent space. Rawat and Silswal (2007), studied decomposition of recurrent curvature fields in a Tachibana space. In the present paper, we considered the decomposition of curvature tensor fields R_{ijk}^{h} in terms of four non-zero vectors in a Tachibana first order recurrent space. Several theorems also have been established and proved.

Key Words: Recurrent space Curvature tensor, Tachibana Space, First Order.

1.Introduction. An almost Tachibana space is an Almost Hermite space (F_i^h, g_{ij}), where F_i^h is an Almost
complex structure and g_{ij} is the Hermite metric such that

$$F_{i,j}^{h} + F_{j,i}^{h} =, \qquad \dots (1.1)$$

where the comma (,) followed by an index denotes the operator of covariant differentiation w. r. t. to the Riemannian metric tensor g_{ij}.

In an Almost Tachibana space [10], we have

$$N_{j,i}^{h} = -4\left(F_{i,j}^{a}\right)F_{a}^{h}, \qquad \dots (1.2)$$

where $F_{i,j}^{h}$ is pure in I and j and N_{j}^{h} is Nijenhuis tensor (Yano, 1965), when the Nijenhuis tensor vanishes, the almost Tachibana space is called a Tachibana space and denoted by T_n. A Tachibana space is called Tachibana recurrent space of first order by Lal and Singh,1971, if its curvature tensor field satisfy the condition

$$R_{ijk,a}^{h} = \lambda_a R_{ijk}^{h}$$

For some non-zero vector λ_a and called the vector of recurrence.

2. DECOMPOSITION OF RECURRENT CURVATURE TENSOR FIELD R_{ijk}^{h}

We consider the decomposition of recurrent curvature tensor field R_{ijk}^{h} in the following form

$$R_{ijk}^{h} = v^h X_{,i} \phi_j \psi_k \,, \qquad ...(2.1)$$

Where the vectors v^h , $X_{,i}$, ϕ_j and ψ_k are such that

$$\lambda_h v^h = 1 \,. \qquad \ldots (2.2)$$

Theorem 2.1 Under the decomposition (2.1), the Bianchi identity for R_{ijk}^{h} takes the forms

$$X_{,i}\phi_j\psi_k + X_{,j}\phi_k\psi_i + X_{,k}\phi_i\psi_j = 0, \qquad \ldots (2.3)$$

and

$$\lambda_a\phi_j\psi_k + \lambda_j\phi_k\psi_a + \lambda_k\phi_a\psi_j = 0. \qquad \ldots (2.4)$$

Proof: From (2.1), we have

$$X_{,i}\phi_j\psi_k + X_{,j}\phi_k\psi_i + X_{,k}\phi_i\psi_j = 0 \qquad \ldots (2.5)$$

(since $v^h \neq 0$)

From (1.3) and (2.1), we have

$$v^h X_{,i}[\lambda_a\phi_j\psi_k + \lambda_j\phi_k\psi_a + \lambda_k\phi_a\psi_j] = 0, \qquad \ldots (2.6)$$

Multiplying (2.6) by λ_k and using (2.2), we get

$$X_{,i}[\lambda_a\phi_j\psi_k + \lambda_j\phi_k\psi_a + \lambda_k\phi_a\psi_j] = 0 \qquad \ldots (2.7)$$

(since $X_{,i} \neq 0$)

Or

$$\lambda_a\phi_j\psi_k + \lambda_j\phi_k\psi_a + \lambda_k\phi_a\psi_j = 0$$

This completes the proof of the theorem.

Theorem 2.2 Under the decomposition (2.1), the tensor fields R_{ijk}^{h}, R_{ij} and vectors $X_{,i}$, ϕ_j and $\psi_k$satisfies the relations

$$\lambda_a\ R_{ijk}^{a} = \lambda_i R_{jk} - \lambda_j R_{ik} = X_{,i}\phi_j\psi_k \qquad \ldots (2.8)$$

Proof: With the help of Bianchi Identity, equation (1.3) yields

$$\lambda_a\, R^a_{ijk} = \lambda_i R_{jk} - \lambda_j R_{ik} \qquad \ldots (2.9)$$

Multiplying (2.1) by λ_h and using relation (2.2), we get

$$\lambda_h R^a_{ijk} = X_{,i}\phi_j\psi_k \qquad \ldots (2.10)$$

From equation (2.9) and (2.10) we get the required relation (2.8).

Theorem 2.3 Under the decomposition (2.1), the quantities λ_a and v^h behave like the recurrent vectors, the recurrent form of these quantities are given by

$$\lambda_{a,m} = \mu_m\lambda_a \qquad \ldots (2.11)$$

And $v^h_{,m} = -\mu_m v^h$ ... (2.12)

Proof: Differentiating (2.9) covariantly w. r. to x^m and using (2.1) and (2.8), we have

$$\lambda_{a,m}v^a X_{,i}\phi_j\psi_k = \lambda_{i,m}R_{jk} - \lambda_{j,m}R_{ik} \qquad \ldots (2.13)$$

Multiplying (2,13) by λ_a and using (2.1) and (2.9), we obtain

$$\lambda_{a,m}\left(\lambda_i R_{jk} - \lambda_j R_{ik}\right) = \lambda_a(\lambda_{i,m}R_{jk} - \lambda_{j,m}R_{ik}) \qquad \ldots (2.14)$$

Now, multiplying equation (2.14) by λ_h, we get

$$\lambda_{a,m}\left(\lambda_i R_{jk} - \lambda_j R_{ik}\right)\lambda_h = \lambda_a\lambda_h(\lambda_{i,m}R_{jk} - \lambda_{j,m}R_{ik}) \qquad \ldots (2.15)$$

Since the expression on the R.H.S. of the above equation is symmetric in a and h, therefore

$$\lambda_{a,m}\lambda_h = \lambda_{h,m}\lambda_a \qquad \ldots (2.16)$$

Provided that

$$\lambda_i R_{jk} - \lambda_j R_{ik} \neq 0$$

The vector field λ_a being non-zero, we can have a proportion vector μ_m such that

$$\lambda_{a,m} = \mu_m\lambda_a \qquad \ldots (2.17)$$

Further, differentiating the equation (2.2) w. r. to x^m and using relation (2.11), we get

$$v^h_{,m} = -\mu_m v^h \ \ (\text{since } \lambda_h \neq 0) \qquad \ldots (2.18)$$

This proves the theorem

Theorem 2.4 Under the decomposition (2.1), the vectors $X_{,i}, \phi_j$ and $\psi_k$satisfies the relations

$$(\lambda_h + \mu_m)X_{,i}\phi_j\psi_k = X_{,im}\phi_j\psi_k + X_{,i}\phi_{j,m}\psi_k + X_{,i}\phi_j\psi_{k,m} \qquad \ldots (2.19)$$

Proof: Differentiating (2.1) covariantly w. r. to x^m and using (1.3), (2.1) and (2.12), we obtain the required
result (2.19).

Theorem 2.5 Under the decomposition (2.1), the curvature tensor and holomorphically projective curvature tensor are equal if

$$\phi_k\psi_m\left\{\left(X_{,i}\delta^h_j - X_{,j}\delta^h_i\right) + X_{,l}\left(F^l_i F^h_j - F^l_j F^h_i\right)\right\} + 2F^l_i F^h_k X_{,l}\phi_j\psi_k = 0 \qquad \ldots (2.20)$$

Proof: The holomorphically projective curvature tensor field P^h_{ijk} is defined by

$$P^h_{ijk} = R^h_{ijk} + \frac{1}{(n+2)}\left(R_{ik}\delta^h_j - R_{jk}\delta^h_i + S_{ik}F^h_j - S_{jk}F^h_i + 2S_{ij}F^h_k\right), \quad \ldots (2.21)$$

which may be written in the form

$$P^h_{ijk} = R^h_{ijk} + D^h_{ijk}, \quad \ldots (2.22)$$

where

$$D^h_{ijk} = \frac{1}{(n+2)}\left(R_{ik}\delta^h_j - R_{jk}\delta^h_i + S_{ik}F^h_j - S_{jk}F^h_i + 2S_{ij}F^h_k\right) \quad \ldots (2.23)$$

Contracting indices h and k in (2.1), we have

$$R_{ij} = v^k X_{,i}\phi_j\psi_k \quad \ldots (2.24)$$

In view of (2.24), we have

$$S_{ij} = F^l_i v^m X_{,i}\phi_j\psi_k \quad \ldots (2.25)$$

Making use of (2.24) and (2.25) in equation (2.23), we have

$$D^h_{ijk} = \frac{1}{(n+2)}[v^m\phi_k\psi_m\left(X_{,i}\delta^h_j - X_{,j}\delta^h_i\right) + v^m X_{,l}\phi_k\psi_m\left(F^l_i F^h_j - F^l_j F^h_i\right) + 2F^l_i F^h_k X_{,l} m\phi_j\psi_m$$

… (2.26)

From equation (2.22), it is clear that

$P^h_{ijk} = R^h_{ijk}$ if $D^h_{ijk} = 0$, which in view of equation (2.26) becomes

$$v^m\phi_k\psi_m\left(X_{,i}\delta^h_j - X_{,j}\delta^h_i\right) + v^m X_{,l}\phi_k\psi_m\left(F^l_i F^h_j - F^l_j F^h_i\right) + 2F^l_i F^h_k X_{,l} m\phi_j\psi_m = 0 \ldots (2.27)$$

Multiplying the above equation by λ_m and using relation (2.2), we obtain the required condition (2.20).

Theorem 2.6 Under the decomposition (2.1), the scalar curvature R, satisfy the relation

$$\lambda_k R = g^{ij} X_{,i}\phi_j\psi_k \quad \ldots (2.28)$$

Proof: Contracting indices h and k in (2.1), we get

$$R_{ij} = v^k X_{,i}\phi_j\psi_k \quad \ldots (2.29)$$

Multiplying (2.29) by g^{ij} on both sides, we get

$$g^{ij} R_{ij} = g^{ij} v^k X_{,i}\phi_j\psi_k \quad \ldots (2.30)$$

Or

$$R = g^{ij} v^k X_{,i}\phi_j\psi_k \quad (R=g^{ij}R_{ij}) \quad \ldots (2.31)$$

Now, multiplying (2.31) by λ_k and using (2.2) , we get

$$\lambda_k R = g^{ij} X_{,i}\phi_j\psi_k,$$

which completes the proof of the theorem.

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