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Analysis of non-exponential queueing networks with application

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Abstract

Blocking of queues is common feature in present day applications of Queueing Networks. Blocking Phenomenon of queues was studied under Non-exponential Queueing Networks. Study of blocking is done under two means one 'stop' and another 'repeat' i.e. when blocking occur, either service is slopped or repeated. Stop and repeat behaviours are equivalent for partial balance conditions. The present condition is correlated to product-form results. The illustration of results is in form of open and closed Networks applications.

Keywords- Repeat, Blocking, Stop, Job-Local- balance, Product form, Communication Network.

1- Introduction

Queueing theory is widely used and supporting study in operations research particularly in study of Networks. Blocking is daily routine problem of continuous flow system of service sector. In communication networks, for example, the Number of interconnecting links are limited, in computer systems processors are store and forward buffers are to be shared among users, while in manufacturing storage pools along assembly lines are finite. In the standard Blocking there are two types of situations, one is stop (Service or Interruption) and another is 'repeat or rejection' of the existing service process. In communication process, either message transmission is instantaneously stopped or respectively repeated upon blocking. Similar process to Blocking is Production (transfer or manufacturing) under which a job has to wait until deblocking Yao, and Buzacott, (1987). In the exponential queues, the 'stop' and 'repeat' behaviour can be taken as memoryless property and same with equivalance to production behaviour. In non-exponential queues, the equivalence of both the behaviours is not generally appear Nico (1991) has done effective work on this juncture.

In this paper effective attempt is done to show that 'stop' and 'repeat' phenomenon are same for non-exponential queues also if concept of partial balance is satisfied. This concept is responsible for insensitivity results and product- form expressions.Barbour (1976), Chandy and Muntz (1975), Chandy and Howard (1977), Whittle (1985). The equivalence of the both behaviour appears to be related to insensitive product-form results. Recently Van (1991) has shown a some what similar equivalence results between 'stop' and 'recirculate' activity for exponential Jacksonian networks with blocked system departures. Under recirculate process a job which is Blocked to leave the system is instantaneously rescheduled as a newly arriving job, e.g. from the end back to the beginning of an assembly line. This process has already been introduced by Jackson in his classical paper and investigated more extensively by retaining product-form results when total system size constraints are impose. Followed by Nico (1991) this work differs from past workers in the ways, taking Non-exponential services, job-local-balance and general framework of stochastic networks with blocking being studied. Insensitive product-form results for non-exponential queueing networks with Blocking have been widely reported in the literature under the condition of a reversible routing and the repeat (rejections or recirculate) product.

2-'Stop' and 'Repat' Equivalence Result

In a queuening Network a job mark l can be of the form: l= (r,s,p) which represents type r of job, station & at which it is present and the position p within the queve at this station that it occupies.

A state L= {l_1, l_2, l_3..., ln} denotes that currently n jobs are present with jobmarks l_1, l_2, l_3..., ln. The law of motion is determined by the following set of characteristics when the system is in state L:

(i) α: Poisson arrival rate

(ii)F_i :Distribution function of the amount of service that a job with jobmark $l \in$L requires.

(iii)β(l/L) : Probability that an arriving job is accepted and assigned jobmark l.

(iv)f(l/L): Probability that this request is granted and their the change of jobmark l in l' accepted. Where l'=0 wil indicate that the job leaves the system.

(v)b(l,i/L) : Probability that this request is granted and their the change of jobmark l in l' accepted, where l' =0 will indicate that the job leaves the system.

(vi)p(l,l'/L): Probability that upon completion of its service a job, with framework $l \in$L requests to change its jobmark in l; here l'=0 is used to denote that the job requests to leave the system and either one of the following behaviour.

S.(Stop behaviour): In state L, a job with jobmark $l \in$L is effectively provided an amount of service per unit of time:

$$f(l/L)[\Sigma_r p\left(l,\frac{l'}{L}\right) b(l,l'/L)] \quad (2.1)$$

upon completion of its service and provided (2.1) is positive it changes its job mark in l' for $l' \neq 0$ or leaves the system for l'=0 with probability

$$\frac{p(l,l'/L\ )b\ (l,l'/L)}{\Sigma_r p(l,l'/L\}b\ (l,l'/L)} \quad (2.2)$$

R (repeat behaviour): In state L a job with jobmark $l \in$L is always provided an amount of service per unit of time f(l/L). Upon completion of its service, however, it changes its jobmark in l' when $l' \neq 0$ or leaves the system when l'=0 with probability

$$p(l,l'/L)b\ (l,l'/L) \qquad l' \neq l$$

$$p\ (l,l'/L) + \underset{r\neq l}{\Sigma} p(l,l'/L)[1 - b(l,l'/L] \quad l' = L$$

Simply within the stop behaviour a job's servicing is delayed by the probability that it would be blocked upon service completion at that moment, whereas under the repeat process the servicing is not delayed but a job can be blocked to change its jobmark or to leave the system in which case it has to redo a complete new service.

Averaged delay: Note that (2.1) delays a job's servicing by the blocking probability 'averaged' over 'all' possible new jobmarks l', while the job will eventually change its jobmark is some specific jobmark l'.

Averaged Blocking: Note that with probability $1 - \Sigma_l \beta(l/L)$ an arriving job could blocked where blocked arrivals are implicitely assumed tobe rejected and lost. Here distinction in two blocking behaviour has omitted as both the 'stop' and 'repeat' ones in this case give the same discription of arrivals being lost upon blocking and an effective arrival rate $\alpha\beta(l/L)$.

Closed-Mixed Case- The closed case included by assuming

$$\alpha = 0\ or\ \beta(./.) = 0\ and$$

$$\underset{r\neq o}{\Sigma} p(l,l'/L) = 1\ for\ all\ l \in \mathrm{L}\ and\ l\ \in \mathrm{S} \quad (2.3)$$

Mixed situations could modeled where some jobs always remain within the system while others have arrived and will leave after some random time. Exponential case: First assume that the service requirements are exponential with parameter μ_i for jobmark l. The underlying process under process S or R then constituents a continuous-time Markov Chain. Let q_1(L,L') and q_2 (L,L') denote the corresponding transition rate for a transition from state L into L' under S and R respectively. Also for a state L, let L+l or L-l denote the same state with a job with jobmark l added or deleted. L-l+l' denotes the state L with one jobmark $l \in$ L changed in l', where for l'=0: L-l+l'=L-l. Thus in this exponential case one easily verifies.

$$q_p(L, L+l) = \alpha\beta(l/L)$$

$$q_p(L, L-l+l') = \mu_l f\left(\frac{l}{L}\right) p(l,l'/L)b(l,l'/L) \qquad (2.4)$$

$$q_p(L,L) = \sum_l \mu_l f(l/L)[p(l,l'/L)b(l,l'/L)] + \sum_l\sum_{l'} \mu_l f(l/L)p(l,l'/L)[1-b(l,l'/L)] + \alpha\sum_{l'} 1 - \beta(l'/L)]$$

while transition rates of any form, with exception of l'=l, or equal to o.

Assuring that these claim are irreducible with unique stationary distribution π_1 (.) and π_2 (.) at one and the same set w. these distributions are thus determined by the global balance equations with p=1 and p=2 respectively.

$$\pi_p(L)\sum_{l \in L}\left[\mu_l f(l/L)\sum_p p(l,l'/L)b(l,l'/L) + \alpha\sum_{l' \neq o}\beta(l'/L)\right] \qquad (2.5)$$

$$= \sum_{l \in L}\left[\sum_{l' \neq 0}\pi_p(L-l+l')\mu_p f(l'/L-l+l')p(i,l/L-l+l')b(l',l/L-l+l') + \right.$$
$$\pi_p(Ll)\alpha\beta(l/L-l] + \sum_{l' \neq 0}\pi(L+l')\mu_{l'} f(l'/L+l')p(l',0'/L+l')b(l',0'/L'+l')$$

and Normalization as a probability distribution. Here a transition from a state in itself due to blocking was deleted, as they would equally contribute to both the left and right-hand side. As the solution of this equation is assumed to be unique up to Normalization, we thus conclude that $\pi_1=\pi=\pi_2$ for some distribution π at T. When system is in equilibrium, in the exponential case the 'stop' and 'repeat' processes are equivalent. Van (1991) has shown that this holds also in non-exponential case provided a job-local-balance condition is satisfied. Job-Local-Balance

The clain said to satisfy job-local-balance (JLB) if for some distribution π at w any L∈W and l∈L;

$$\pi(L)\mu_l f(l/L)\sum_{l'} p(l,l'/L)b(l,l'/L)$$
$$= \sum_{l' \neq 0}\pi(L-l+l')\mu_r f(l'/L-l+l'x\, p(l',l/L-l+l')b(l',l/L-l+l')$$
$$+ \pi(L-l)\alpha\beta(l/L-l) \quad (2.6)$$

$$and\ \pi(L)\alpha\sum_{l' \neq 0}\beta(l'/L) = \sum_{l' \neq 0}\pi(l+l')\mu_r f(l'/L+l')p(l',0/L+l')b(l',0/L+l') - (2.7)$$

By substituting (2.4) in (2.5) and assuming over i, one directly concludes that any distribution π satisfying the JLB equation (2.6) and (2.7) also satisfy (2.5) and finally $\pi=\pi_1=\pi_2$.

Condition (2.7) requires that the total rate for a job to enter the system in a state L is equal to the total rate for a job to leave the system, which leads to state L. As such, similarly to (2.6) it has interpreted as a balance per job. In contrast with (2.6) though, in (2.7) the out and in rate are not necessarily due to the same job. Further (2.7) results from

(2.6) only when π(.) is already known to satisfy the global balance equation (2.5). The Nation of JLB directly related to the nation of Local balance as defined for generalized semi-Morkov processes given by Schassberger (1978) and has introduced in by Hordijk and Van (1983). Using appropriate substitutions it can be concluded that a distribution π satisfying JLB is insensitive under the 'Stop' behavior as (2.2) effectively does no Longer contain blocking (2.6) implies insensitivity under both the 'Stop' and 'repeat' behavior and that it thus guarantees stationary equivalance also under non-exponential services.

Non-exponential Case

Assume that the distribution function F_i are absolutely continuous with density function $q_i(.)$ denoted by

$$(L,T) = \{l, t_l)/t_l > 0, l_{\in} L\}$$

That the job with current jobmark l' has a residual service requirement t_l up to completion of its current service requirement. Let π_l (L,T) and $\pi_{l'}(L,T)$ be the unique stationary densities of the corresponding Markov process under the S (stop) and R (repeat) behavior respectively.

Equivalance Result

Nico (1991) has proved that if JLB is satisfied for all L∈W that is (2.6) for all l∈L and (2.7), then for all (L,T) with L∈W

$$\pi_1(L,T) = \pi_2(L,T) = \pi(L) \overset{\pi}{_{l_{\in}L}} ([t_l]^{-1} \quad (1 - F_l(t_l)]\} \qquad (2.8)$$

Insensitivity Result

Under the assumption of JLB we have

$$\pi_1(L) = \pi_2(L) = \pi(L) \qquad (2.9)$$

This result immediately occur by Integrating over all possible residual service requirements t_l for all l∈L and the identity

$$\int_0^{\varpi} [1 - F_l(t)]dt = \tau_1 \qquad (2.10)$$

General Distributions

Arbitrary distributions, e.g. for deterministic service requirements, can be approximated arbitrarily closely by absolute continuous distribution equivalence result and insensitively results remain valid for arbitrary service requirements. Jackson networks with finite stations and overall blocking.

We consider Jackson network with N service stations and a Poisson arrival input with parameter λ. An arriving job is routed to station J with probability p_{ij}, while upon completion at a station i a job routes to station J with probability p_{ij} or leaves the system with probability $p_{io} = 1 - [p_{i1} + p_{i2} + .. + p_{iN}]$. If we take finite capacity constraint N_i for station i, i=1,2...N, but without the seversible routing conditions of given Network. Instead, either one of the following two overall blocking process is in order.

Stop Process

As soon and long as one of the stations becomes and is saturated, say station i, i.e. $n_i=N_i$, servicing at all the other stations j≠i is stopped and arrivals are blocked and Lost.

Repeat Process

As soon and long as one of the stations becomes and is saturated, say station i, i.e. $n_i=N$, a job which completes a service at a stations j#i has to undergo a New service at station j, and arrivals are blocked and Lost.

Counter Example

In the cyclic three-stations network in Fig.1 services not only at station 2 but also at station 1 are blocked, or more precisely are to be slopped or repeated respectively, upon completion when station 3 is saturated. Similarly, service at stations 1 and 3 are to be blocked when 2 is saturated and at station 3 and 2 when 1 is saturated.

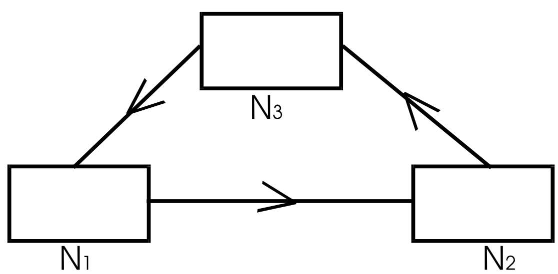

Fig.1

The open analogue of this example is a two-station tandem line (Fig.2) Where not only arrivals but also departures at station 2, either by stopping or repeating services, are to be blocked when station 1 is saturated, and arrivals as well as services at station 1 are to be blocked when station 2 is saturated.

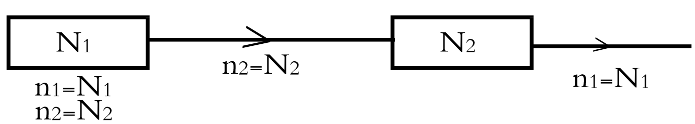

Fig.2

Note that under either behaviour no two stations can become saturated at the same time, so that the set of admissible states n is now restricted to:

$$V = \{n/n_i \leq N_i, \forall_i, n_i + n_j < N_i + N_j \forall_{j \neq i}\} \qquad (2.11)$$

again, with $\{\lambda_1, \ldots .. \lambda_n\}$ uniquely determined, up to normalization by the traffic equation.

$$\lambda_j = p_{oj} + \sum_{i=1}^{N} \lambda_i p_{ij} (j = 1, \dots N) \quad (2.12)$$

and C is normalizing constant, the following product from is to be expected -

$$\pi(n) = C \prod_{i=1}^{N} \frac{[\lambda_i \tau_s]^{ni}}{n_i!} \quad (n \in V)$$

For the exponential repeat case, this product form has presented by past researchers for the specific three-station example given above. For the non-exponential case, no such result has reported.

References

1. Nico M. Van Dijk (1991). On Stop=Repeat servicing for non-exponential queueing network with blocking, *J. Appl. Prob.* 28:159-173.
2. Barbour, A. (1976).Network of queues and the method of stages. Adv. Appl. Pub.8:584-591.
3. Baskett, F. Chandy, M; Muntz (1975). Open, closed and mixed network of Queues with different classes of customers, *J. Assoc. Comput. Mach*. 22:248-260.
4. Burman, D.Y. Lehoczky, J.P. and Lin, Y. (1982). Insensitivily of blocking probabilities in a circuit switching network. *J. Appl. Prob.* 21:850-859.
5. Chandy, K.M.; Howard, J.H. and Towsley, D.F. (1977). Product form and local balance in Queueing Network. *J. Assoc. Comput. Mach*. 24:250-263.
6. Chandy, K.M. and Martin, A.J. (1983). A Characterization of Product- form queueing Network,J. Assoc. *Comput. Mach*, 30:286-299.
7. Cohen, J.W. (1979).The multiple phase service network with generalized processor sharing ACTA Informat. 12:245-284.
8. Whittle, P. (1985). Partial Balance and Insensitivity, *J. Appl. Prob.*22, 168-175.
9. Yao, P.D. and Buzacott, J.A. (1987). Modelling a class of flexible manufacturing systems with reversible routing, *Operat. Res.* 35:87-83.

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