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On nearly pseudo Ricci-symmetric manifold

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Abstract

In this paper, we study certain properties of a nearly pseudo Ricci symmetric manifold $N(PRS)_n, n > 2$. Some curvature relations and characterization results are obtained under nearly pseudo Ricci symmetric condition. Finally, a non-trivial example is provided to illustrate the existence of such manifolds.

Keywords Weakly Ricci symmetric manifolds, generalized pseudo Ricci symmetric manifolds, pseudo Ricci symmetric manifold, generalized almost pseudo-Ricci symmetric manifold, generalized semi-pseudo Ricci-symmetric manifold, extended pseudo Ricci symmetric manifolds.

1. Introduction

The notion of locally symmetric and Ricci symmetric Riemannian manifold were introduced by Carten and Eisenhart (1926, 1949), respectively, and constitute important classes of curvature-restricted manifold in differential geometry.

Let $(M^n, g), (n > 2)$ be an n-dimensional Riemannian manifold with the Levi-Civita connection D. The manifold (M^n, g) is said to be locally symmetric if its Riemannian curvature tensor R satisfies $D.R = 0$, that is, the curvature tensor is covariantly constant. Further, the manifold is said to be Ricci-symmetric if its Ricci tensor I, a symmetric tensor of the type (0,2) satisfies $D.I = 0$.

It is evident that every locally symmetric manifold is Ricci-symmetric however, the converse is not true in general. Motivated by these curvature conditions, several related geometric structures have been introduced and extensively studied in the literature. These include recurrent manifolds Chaki (1956), Prakash (1962), pseudo symmetric manifolds (1987), and pseudo Ricci-symmetric manifolds (1988), generalized Ricci recurrent (1995), nearly Ricci recurrent (2021) among others.

In 1993, Tamassy and Binh introduced the notion of weakly Ricci symmetric manifolds whose non-zero Ricci tensor I satisfies the condition:

$$(D_X I)(Y, Z) = A(X)I(Y, Z) + B(Y)I(X, Z) + C(Z)I(Y, X), \tag{1.1}$$

where A, B and C are three non-zero1-forms, called the associated 1-forms of the manifold. Such an n-dimensional manifold is denoted by $(WRS)_n$. As an equivalent notion of $(WRS)_n$, Chaki and Koley

(1994) introduced the notion of generalized pseudo Ricci symmetric manifolds $G(PRS)_n$. If in (1.1), the 1-form A is replaced by $2A$, then the definition of $(WRS)_n$ reduces to that of $G(PRS)_n$ and is given by

$$(D_X I)(Y,Z) = 2A(X)I(Y,Z) + B(Y)I(X,Z) + C(Z)I(Y,X). \tag{1.2}$$

If $A = B = C$ in (1.2), then it takes the form

$$(D_X I)(Y,Z) = 2A(X)I(Y,Z) + A(Y)I(X,Z) + A(Z)I(Y,X), \tag{1.3}$$

which is known as pseudo Ricci symmetric manifold is the sence of Chaki (1988) and denoted by $(PRS)_n$.

Again, semi-Ricci-symmetric $(SRS)_n, n > 2$, manifold was investigated by Mukhopadhyay and Barua (1990-91) by the expression:

$$(D_X I)(Y,Z) = 2A(X)I(Y,Z) - A(Y)I(X,Z) - A(Z)I(Y,X). \tag{1.4}$$

Tarafdar and Jawarneh (1993) introduced semi-pseudo Ricci-symmetric n-dimensional manifold with the notation $S(PRS)_n, (n > 2)$, whose equation is given by:

$$(D_X I)(Y,Z) = A(Y)I(X,Z) + A(Z)I(Y,X). \tag{1.5}$$

Generalizing the notion of $(PRS)_n$ manifold introduced by Chaki (1988), recently Chaki and Kawaguchi (2007) studied a type of non-flat Riemannian manifold $(M^n, g), (n > 2)$ called almost pseudo-Ricci symmetric manifold whose Ricci tensor I of the type (0,2) is not identically zero and satisfies the condition:

$$(D_X I)(Y,Z) = [A(X) + B(X)]I(Y,Z) + A(Y)I(X,Z) + A(Z)I(Y,X), \tag{1.6}$$

for all vector fields X, Y, Z. Such a manifold was denoted by $A(PRS)_n, (n > 2)$.

In the similar fashion, Chakrabarti and Saha (2011) defined another type of Ricci symmetric whose Ricci tensor satisfies the condition:

$$(D_X I)(Y,Z) = [A(X) + B(X)]I(Y,Z) + C(Y)I(X,Z) + D(Z)I(Y,X), \tag{1.7}$$

associated with four non-zero1-forms A, B, C and D. Such an n-dimensional manifold of this kind denoted by the symbols $(GAPRS)_n$. If $A = C = D$, then the manifold defined by (1.7) becomes (1.6) $A(PRS)_n$. This justifies the name generalized almost pseudo-Ricci symmetric manifold.

Continuing above study, Jawarneh and Tashtoush (2012) introduced a new type of Ricci-symmetric structure whose expression:

$$(D_X I)(Y,Z) = A(Y)I(X,Z) + B(Z)I(Y,X), \tag{1.8}$$

and called by them as generalized semi-pseudo Ricci-symmetric manifold and abbreviated by the notation $G(SPRS)_n$. Recently Halder and Bhattacharyya, (2017) studied this structure under the title “Some properties of generalized semi-pseudo-Ricci-symmetric manifold”.

Subsequently, in 2016, Prasad and Doulo investigated extended pseudo Ricci symmetric manifolds whose equation given by the following Ricci tensor:

$$(D_X I)(Y,Z) = 2A(X)I(Y,Z) + B(Y)I(X,Z) + B(Z)I(Y,X), \tag{1.9}$$

where A, B and C have meaning already mentioned. Such an n-dimensional manifold was denoted by $E(PRS)_n,\ n > 2$.

The geometric beauty of an $E(PRS)_n$ space is reflected through the following important particular cases:

S. No.	Condition Imposed	Reduced Manifold	References
1.	If $2A$ is replaced by A and $B(Z)$ is replaced by $C(Z)$	$E(PRS)_n$ reduces in $(WRS)_n$	Tamassy and Binh, (1993)
2.	If $A = B$	$E(PRS)_n$ reduces in $(PRS)_n$	Chaki (1988)
3.	If $A = 0$	$E(PRS)_n$ reduces in $S(PRS)_n$	Trafdar and Musa (1993)
4.	If B is replaced by $-A$	$E(PRS)_n$ reduces in $(SRS)_n$	Mukhopadhyay and Barua (1990-91)
5.	If $A = 0$ and $B(Z)$ is replaced by $C(Z)$	$E(PRS)_n$ reduces in $G(SPRS)_n$	Jawarneh and Tashtoush (2012)
6.	If $B(Z)$ is replaced by $C(Z)$	$E(PRS)_n$ reduces in $G(PRS)_n$	Chaki and Koley (1994)

This table clearly exhibits how $E(PRS)_n$ generalizes several well-known classes of recurrent type manifolds under suitable conditions. Such generalization have contributed substantially to the development of the theory of curvature-restricted Riemannian manifolds.

In continuation of the preceding concepts, we introduced a new of class of a non-flat Riemannian manifold termed as "nearly pseudo-Ricci-symmetric manifold" and satisfies

$$(D_X I)(Y,Z) = 2A(X)I(Y,Z) + [A(Y) + B(Y)]I(X,Z) + [A(Z) + B(Z)]I(Y,X), \tag{1.10}$$

for all vector fields $X, Y, Z \in \varkappa(M^n)$, where A, and B are two non-zero1-forms on M^n. Such a manifold will be denoted by $N(PRS)_n, n > 2$. The manifold is called "nearly pseudo Ricci symmetric" because its defining condition is very close to that of a pseudo Ricci symmetric $(PRS)_n$. Hence, we denote it by $N(PRS)_n$, where N denotes "nearly".

In this paper, the existence of such a structure on a Riemannian manifold is first established and further it is shown that on such $N(PRS)_n$. The study of $N(PRS)_n$ an important role is played by the one-form T defined by

$$E(X) = 4A(X) + 2B(X). \tag{1.11}$$

It is shown that if $E = 0$, then the Ricci tensor is a cyclic tensor on the other hand if $E \neq 0, r \neq 0$ and Ricci tensor, then the Ricci tensor is of the form

$$I(X,Y) = \sigma E(X)E(Y), \tag{1.12}$$

where σ is a non-zero scalar. Finally, a conformally flat $N(PRS)_n, n > 2$ of non-zero constant scalar curvature, it is shown that for such a manifold $E \neq 0$, then its curvature tensor $'R$ is of the following form

$$'R(X,Y,Z,W) = a[g(X,Z)g(Y,W) - g(Y,Z)g(X,W)] + b[g(X,Z)E(Y)E(W) + g(Y,W)T(X)T(Z) - g(X,W)E(Y)E(Z) - g(Y,Z)E(X)E(W)], \tag{1.13}$$

where a and b are scalars, and $'R(X,Y,Z,W) = g(R(X,Y)Z,W)$.

2. Existence of $N(PRS)_n$

For the existence of such structure, defined in (1.10), consider a Riemannian manifold M^n with non-metric tensor g which admits a linear connection $\bar{D}$ defined by

$$\bar{D}_X Y = D_X Y + A(X)Y + A(Y)X + B(Y)X, \tag{2.1}$$

where A and B are non-zero 1-forms such that

$$g(X,\rho) = A(X), \tag{2.2a}$$

and

$$g(X,\lambda) = B(X), \tag{2.2b}$$

with $\quad (\bar{D}_X I)(Y,Z) = 0.$ (2.3)

If (2.3) holds, then

$$(\bar{D}_X I)(Y,Z) = \bar{D}_X I(Y,Z) - I(\bar{D}_X Y,Z) - I(Y,\bar{D}_X Z) = 0.$$

Using (2.1) in above equation, we get

$$(D_X I)(Y,Z) = 2A(X)I(Y,Z) + [A(Y) + B(Y)]I(X,Z) + [A(Z) + B(Z)]I(Y,X).$$

The connection $\bar{D}$ is not identical with D. Hence $DI \neq 0$. Thus, if a Riemannian manifold $(M^n, g), n > 2$, admits a linear connection $\bar{D}$, which satisfies (2.1) and (2.3) and then the manifold is $N(PRS)_n$.

Illustration

Let us consider $M^4 = \{(x^1, x^2, x^3, x^4) \in \mathcal{R}^4\}$ be an open subset of $\mathcal{R}^4$ endowed with the metric g defined by Yadav, Prasad and Chaubey (2025)

$$ds^2 = g_{ij}dx^i dx^j = (x^4)^{\frac{1}{2}}[(dx^1)^2 + (dx^2)^2 + (dx^3)^2] + (dx^4)^2, \tag{2.4}$$

where $i,j = 1,2,3,4$.

Then the only non-vanishes components of the Christoffel symbols are

$$\Gamma^1_{14} = \Gamma^2_{24} = \Gamma^3_{34} = \frac{1}{4(x^4)^1},\ \Gamma^4_{11} = \Gamma^4_{22} = \Gamma^4_{33} = -\frac{1}{4(x^4)^{\frac{1}{2}}}. \tag{2.5}$$

The curvature tensor of type (1,3) is defined by

$$R^a_{ijk} = -\frac{\partial}{\partial x^k}\Gamma^a_{ij} + \frac{\partial}{\partial x^j}\Gamma^a_{ik} + \Gamma^a_{bj}\Gamma^b_{ik} - \Gamma^a_{bk}\Gamma^b_{ij}. \tag{2.6}$$

In view of (8.2) and (8.3), we get

$$R^1_{441} = R^2_{442} = R^3_{443} = -\frac{3}{16(x^4)^2}. \tag{2.7}$$

The curvature tensor of type (0,4) is defined by

$$R_{i44i} = g_{i\alpha}R^\alpha_{44i} = g_{i1}R^1_{44i} + g_{i2}R^2_{44i} + g_{i3}R^3_{44i} \; + g_{i4}R^4_{44i}. \tag{2.8}$$

In view of (8.4) and (8.5), we get

$$R_{1441} = R_{2442} = R_{3443} = -\frac{9}{16(x^4)^{\frac{3}{2}}}. \tag{2.9}$$

The non-vanishing components of the Ricci tensor are

$$R_{11} = R_{22} = R_{33} = -\frac{9}{16(x^4)^{\frac{3}{2}}}, \quad R_{44} = R^1_{441} + R^2_{442} + R^3_{443} = -\frac{27}{16(x^4)^2}, \tag{2.10}$$

and scalar curvature is

$$R = g^{ii}R_{ii} = -\frac{27}{8(x^4)^2},$$

which is non-vanishing and non-constant.

Taking covariant derivative of Ricci tensor R_{ij} is given by

$$R_{ij,k} = \frac{\partial R_{ij}}{\partial x^k} - R_{il}\Gamma^l_{jk} - R_{lj}\Gamma^l_{ik}. \tag{2.11}$$

In view of (2.5), (2.10) and (2.11), we get

$$R_{11,4} = \; R_{22,4} = R_{33,4} = \frac{16}{32(x^4)^{\frac{5}{2}}}, \quad R_{44,4} = \frac{27}{8(x^4)^3}. \tag{2.12}$$

Let us choose the associated 1-form as

$$A_i = \begin{cases} -\frac{4}{9(x^4)^1}, & i = 4 \\ 0, & \text{otherwise} \end{cases} \tag{2.13}$$

$$B_i = \begin{cases} -\frac{7}{9(x^4)^1}, & i = 4 \\ 0, & \text{otherwise} \end{cases} \tag{2.14}$$

for any point $x \in \mathcal{R}^4$.

One can verify the followings

$$R_{12,k} = 2A_kR_{12} + (A_1 + B_1)R_{k2} + (A_2 + B_2)R_{1k},$$

$$R_{13,k} = 2A_kR_{13} + (A_1 + B_1)R_{k3} + (A_3 + B_3)R_{1k},$$

$$R_{14,k} = 2A_kR_{14} + (A_1 + B_1)R_{k4} + (A_4 + B_4)R_{1k},$$

$$R_{23,k} = 2A_kR_{23} + (A_2 + B_2)R_{k3} + (A_3 + B_3)R_{2k},$$

$$R_{24,k} = 2A_kR_{24} + (A_2 + B_2)R_{k4} + (A_4 + B_4)R_{2k},$$

$$R_{34,k} = 2A_kR_{34} + (A_3 + B_3)R_{k4} + (A_4 + B_4)R_{3k},$$

$$R_{11,k} = 2A_kR_{11} + (A_1 + B_1)R_{k1} + (A_1 + B_1)R_{1k},$$

$$R_{22,k} = 2A_kR_{22} + (A_2 + B_2)R_{k2} + (A_2 + B_2)R_{2k},$$

$$R_{33,k} = 2A_kR_{33} + (A_1 + B_1)R_{k3} + (A_3 + B_3)R_{3k},$$

$$R_{44,k} = 2A_kR_{44} + (A_4 + B_4)R_{k4} + (A_4 + B_4)R_{4k},$$

where $k = 1,2,3,4$. Hence, this verify the $N(PRS)_n$ for 4-dimensional manifold.

3. Preliminaries for the study of $N(PRS)_n$

In this section, we obtain some formulae which will be used later on. Let L denotes the symmetric endomorphism of the tangent space of $N(PRS)_n$ at each point corresponding to the Ricci tensor I. Then

$$g(LX, Y) = I(X, Y), \tag{3.1}$$

for all vector fields X and Y. Further, let

$$g(X, \rho) = A(X),\ \ g(X, \lambda) = B(X) \tag{3.2}$$

and

$$A(LX) = \bar{A}(X) \text{ and } \ B(LX) = \bar{B}(X), \tag{3.3}$$

for every arbitrary vector field X. Then ρ, and λ shall be called the basic vector fields corresponding to the associated 1-forms A, and B.

Contracting (1.10), and using (3.1) and (3.2), we get

$$dr(X) = 2A(X)r + 2[\bar{A}(X) + \bar{B}(X)], \tag{3.4}$$

where r is the scalar curvature tensor.

From (1.10), we have

$$(D_XI)(Y, Z) - (D_ZI)(Y, X) = [A(X) - B(X)]I(Y, Z) - [A(Z) - B(Z)]I(Y, X). \tag{3.5}$$

These formulas will be used in sequel.

4. Scalar curvature of $N(PRS)_n$

Here assume that

$$(D_XI)(Y, Z) - (D_ZI)(Y, X) = 0. \tag{4.1}$$

From (3.5) and (4.1), we get

$$F(X)I(Y, Z) = F(Z)I(Y, X), \tag{4.2}$$

where

$$F(X) = A(X) - B(X). \tag{4.3}$$

Contracting (4.2), we get

$$F(X)r = F(LX). \tag{4.4}$$

Now, we define the 1-form F given by (4.3) as follows:

$$g(X,\psi)=F(X). \tag{4.5}$$

From (4.4) and (4.5), we get

$$r\psi=L\psi, \tag{4.6}$$

i.e. r is an eigen value of the of the Ricci tensor $L$of the type (1,1) corresponding to the eigen vector ψ defined by (4.3) and (4.5).

Hence, we can state the following theorem:

Theorem (4.1): In a $N(PRS)_n$, r is an eigen value of the Ricci tensor of type of (1,1) corresponding to the given vector ψ defined by (4.3) and (4.5).

Putting $X=\psi$ in (4.2) and using (3.1) and (4.5), we get

$$F(\psi)I(Y,Z)=F(Z)F(LY). \tag{4.6}$$

From (4.4) and (4.6), we get

$$I(Y,Z)=rG(Y)G(Z), \tag{4.7}$$

where

$$G(Y)=\frac{F(Y)}{\sqrt{F(X)}},g(Y,\rho)=G(Y), \tag{4.8}$$

ρ is a unit vector.

From (4.7), it follows that if $r=0$, then $I(Y,Z)=0$, which is inadmissible by definition of $N(PRS)_n$. This leads to the following theorem:

Theorem (4.2): In a $N(PRS)_n$, the scalar curvature r cannot vanish and the Ricci tensor is of the form $I(Y,Z)=rG(Y)G(Z)$, where the 1-form G is defined by (4.3), (4.5) and (4.8), respectively.

5. Cyclic Ricci tensor in $N(PRS)_n$

We shall enquire the effect of the non-effect of the non-parallel cyclic Ricci tensor on $N(PRS)_n$.

A Riemannian manifold is said to have cyclic Ricci tensor if its Ricci tensor I of the type (0,2) is non-zero and satisfies the condition:

$$(D_XI)(Y,Z)+(D_YI)(Z,X)+(D_ZI)(X,Y)=0. \tag{5.1}$$

In view of (1.10) and (5.1), we get

$$\begin{aligned}(D_XI)(Y,Z)+(D_YI)(Z,X)+(D_ZI)(X,Y)=&[4A(X)+2B(X)]I(Y,Z)+\\&[4A(Y)+2B(Y)]I(X,Z)+\\&[4A(Z)+2B(Z)]I(X,Y),\end{aligned} \tag{5.2}$$

Let

$$E(X)=4A(X)+2B(X). \tag{5.3}$$

Hence, in view of (5.2) and (5.3), we get

$$(D_X I)(Y,Z) + (D_Y I)(Z,X) + (D_Z I)(X,Y) = E(X)I(Y,Z) + E(Y)I(X,Z) + E(Z)I(X,Y). \tag{5.4}$$

From (5.1) and (5.4), we get

$$E(X)I(Y,Z) + E(Y)I(X,Z) + E(Z)I(X,Y) = 0. \tag{5.5}$$

Now, we have Walker's Lemma (1950) as follows:

Walker's Lemma : If $a_{ij}.b_i$ are numbers satisfying $a_{ij} = a_{ji}$ and $a_{ij}.b_k + a_{jk}.b_i + a_{ki}.b_j = 0$, for $i,j,k = 1,2,3,\dots,n$, then either all $a_{ij} = 0$ or $b_i = 0$.

Hence, by Walker's Lemma and (5.4), we see that either $E = 0$ or $I = 0$. But since $I \neq 0$, then by definition of $N(PRS)_n, n > 2$ we have $E = 0$ which is gives $[2A(X) + B(X)] = 0$.

Conversely, if $E = 0$, then from (5.2), we get

$$(D_X I)(Y,Z) + (D_Y I)(Z,X) + (D_Z I)(X,Y) = 0,$$

i.e. the Ricci tensor I is cyclic.

Hence, we have the following theorem:

Theorem (5.1): A $N(PRS)_n$ admits a cyclic Ricci tensor if and only if its associated 1-forms satisfy the relation $[2A(X) + B(X)] = 0$ for all vector fields X.

6. Conformally flat $N(PRS)_n, n > 2$ with non-zero constant scalar curvature

It is known that in a conformally flat Riemannian manifold $(M^n, g), (n > 3)$, (Eisenhart, 1926)

$$(D_X I)(Y,Z) - (D_Z I)(Y,X) = \frac{1}{2(n-1)}[dr(X)g(Y,Z) - dr(Z)g(Y,X)]. \tag{6.1}$$

If the scalar curvature is a non-zero constant, then

$$dr(X) = 0. \tag{6.2}$$

In view of (6.1)and (6.2), we get

$$(D_X I)(Y,Z) - (D_Z I)(X,Y) = 0. \tag{6.3}$$

From (3.5) and (6.3), we get

$$H(X)I(Y,Z) = H(Z)I(X,Y), \tag{6.4}$$

where $H(X) = A(X) - B(X)$.

Contracting (6.4), we get

$$H(X)r = H(LX). \tag{6.5}$$

If we take

$$H(X) = g(X,\xi).$$

Now, putting ξ for X in (6.4) and using (6.5), we get

$$I(Y,Z) = \frac{r}{H(\xi)}H(Y)H(Z). \tag{6.6}$$

It is known (Eisenhart, 1926) that in a conformally flat Riemannian $(M^n, g), (n > 3)$, the curvature tensor $'R$ of type (0,4) is given by

$$'R(X,Y,Z,W) = \frac{1}{n-2}[I(Y,Z)g(X,W) - I(X,Z)g(Y,W) + I(X,W)g(Y,Z) - I(Y,W)g(X,Z)] - \frac{r}{(n-1)(n-2)}[g(Y,Z)g(X,W) - g(X,Z)g(Y,W)]. \tag{6.7}$$

From (6.6) and (6.7), we get

$$'R(X,Y,Z,W) = a[g(Y,Z)g(X,W) - g(X,Z)g(Y,W)] + b[g(Y,Z)G(X)G(W) - g(X,Z)G(Y)G(W) + g(X,W)G(Y)G(Z) - g(Y,W)G(X)G(Z)], \tag{6.8}$$

where $a = -\frac{r}{(n-1)(n-2)}$ and $b = \frac{r}{(n-2)H(\xi)}$.

According to Chen and Yano (1973), a Riemannian manifold whose curvature tensor $'R$ is of the form (6.8) is said to be of quasi-constant curvature. Therefore, in view of this, we can state the following theorem:

Theorem (6.1): In a conformally flat $N(PRS)_n$, $(n > 3)$, with non-zero constant scalar curvature and $H \neq 0$, the manifold is of quasi-constant curvature.

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