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Pseudo W_8 –curvature tensor $\widetilde{W}_8$ on a Riemannian manifold

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Abstract

In this paper we introduce a new curvature tensor named as Pseudo W_8 –curvature tensor $\widetilde{W}_8$. Some algebraic properties of Pseudo W_8 –curvature tensor $\widetilde{W}_8$ are obtained. Further Quadratic killing tensor $\tilde{A}$ and Quadratic conformal killing tensor $\tilde{B}$ are considered with respect to Pseudo W_8 –curvature tensor $\widetilde{W}_8$. Finally it is proved that Pseudo W_8 –curvature tensor $\widetilde{W}_8$ is conservative if the scalar curvature tensor is constant under the certain condition.

Key words and Phrases: Pseudo W_8 –curvature tensor, Pseudo W_8 flat, Quadratic killing tensor, Quadratic conformal killing tensor and divergence.

Introduction

The notion of quasi conformal curvature tensor was given by Yano and Sawaki (1968) as follows

$$\begin{aligned}\tilde{C}(X,Y)Z = & a R(X,Y)Z + b[Ric(Y,Z)X - Ric(X,Z)Y + g(Y,Z)QX - g(X,Z)QY] \\ & - \frac{r}{n} \left[\frac{a}{n-1} + 2b \right] [g(Y,Z)X - g(X,Z)Y],\end{aligned}\quad (1.1)$$

where a and b are constants such that $a, b \neq 0$, g is Riemannian metric, X, Y and Z are arbitrary vector fields, R is the Riemannian curvature tensor of the type (1,3), Ric is the Ricci tensor of the type (0,2), Q is the Ricci operator defined by $g(QX, Y) = Ric(X, Y)$ and r is the scalar curvature of the manifold. If $a = 1$ and $b = -\frac{1}{n-2}$ then (1.1) becomes a conformal curvature tensor C , given by

$$\begin{aligned} C(X,Y)Z &= R(X,Y)Z - \frac{1}{n-2} [Ric(Y,Z)X - Ric(X,Z)Y \\ &\quad + g(Y,Z)QX - g(X,Z)Y] \\ &\quad + \frac{r}{(n-1)(n-2)} [g(Y,Z)QX - g(X,Z)Y], \end{aligned}$$

where C is conformal curvature tensor Mishra (1984). For this reason $\tilde{C}$ is called quasi conformal curvature tensor. The quasi conformal curvature tensor have been studied by several authors in various ways such as Amur and Maralabhavi (1977), Chaki and Ghosh (1977), Prasad and Verma(2009), Prakash and Singh(2009), Shukla and Singh(2011), Mallick, Zhao and De(2016), Venketasha and Kumar(2016) and many others.

In 2002, Prasad introduced Pseudo projective curvature tensor in a Riemannian manifold ($n > 2$) by the following expression

$$\begin{aligned} \tilde{P}(X,Y)Z &= a R(X,Y)Z + b[Ric(Y,Z)X - Ric(X,Z)Y] \\ &\quad - \frac{r}{n} \left[\frac{a}{n-1} + b \right] [g(Y,Z)X - g(X,Z)Y], \end{aligned} \quad (1.2)$$

where a and b are constants such that $a, b \neq 0$. If $a = 1$ and $b = -\frac{1}{n-1}$ then (1.2) takes the form of projective curvature tensor P , given by

$$P(X,Y)Z = R(X,Y)Z - \frac{1}{n-1} [Ric(Y,Z)X - Ric(X,Z)Y].$$

where P is projective curvature tensor Mishra (1984). In a recent papers Bagewadi, Prakash and Venkatesha (2007), Narain, Prakash and Prasad(2009), Jaisawal and Ojha (2010), Gular and Demirabag (2016), Kumar (2019) etc. explored various geometrical properties by using this curvature tensor on LP-Sasakian manifold, K-contact manifold, trans-Sasakian manifold and weakly symmetric Riemannian manifold.

Further in 2004 and 2007 Prasad and Maurya investigated Pseudo W_2 –curvature tensor $\tilde{W}_2$ and Quasi-concircular curvature tensor $\tilde{V}$ on a Riemannian manifold as follows respectively:

$$\begin{aligned} \tilde{W}_2(X,Y)Z &= a R(X,Y)Z + b[g(Y,Z)QX - g(X,Z)QY] \\ &\quad - \frac{r}{n} \left[\frac{a}{n-1} + b \right] [g(Y,Z)X - g(X,Z)Y], \end{aligned} \quad (1.3)$$

and

$$\tilde{V}(X,Y)Z = a R(X,Y)Z + \frac{r}{n} \left[\frac{a}{n-1} + 2b \right] [g(Y,Z)X - g(X,Z)Y]. \quad (1.4)$$

Following the same line Prasad, Narain and Maurya in 2008 defined Pseudo W_4 –curvature tensor $\widetilde{W}_4$ on a Riemannian manifold by the expression:

$$\begin{aligned}\widetilde{W}_4(X, Y)Z &= a R(X, Y)Z + b[g(X, Y)QZ - g(X, Z)QY] \\ &\quad - \frac{r}{n} \left[\frac{a}{n-1} + b \right] [g(X, Y)QZ - g(X, Z)QY],\end{aligned}\tag{1.5}$$

Narain, Prakash and Prasad (2009), Prasad, Gupta and Pandey (2010), Prakash (2011), Kumar (2012), Mishra, Prasad and Upadhyay (2019), Prakash and Yadav (2018) and Ahmad, Haseb and Jun (2019) extended this notation to LP-Sasakian manifold, P-Sasakian manifold, LP-Sasakian manifold with coefficient α and Lorentzian β - Kenmotsu manifold.

If $a = 1$ and $b = -\frac{1}{n-1}$ then equations (1.3), (1.4) and (1.5) reduces to

$$\begin{aligned}\widetilde{W}_2(X, Y)Z &= R(X, Y)Z - \frac{1}{n-1} [g(Y, Z)QX - g(X, Z)QY] \\ &= W_2 - \text{Curvature tensor (Pokhariyal and Mishra 1970)} \\ \widetilde{V}(X, Y)Z &= R(X, Y)Z - \frac{r}{(n-1)(n-2)} [g(Y, Z)X - g(X, Z)Y] \\ &= V \text{ Conircular curvature tensor (Mishra 1984)} \\ \widetilde{W}_4(X, Y)Z &= R(X, Y)Z - \frac{1}{n-1} [g(X, Y)QZ - g(X, Z)QY] \\ &= W_4 - \text{Curvature tensor (Pokhariyal 1971)}\end{aligned}$$

Thus we see that projective curvature tensor P , W_2 –curvature tensor, concircular curvature tensor V , and W_4 –curvature tensor are the particular cases of the tensor $\tilde{P}$, $\widetilde{W}_2$, $\tilde{V}$ and $\widetilde{W}_4$. For this reason $\tilde{P}$, $\widetilde{W}_2$, $\tilde{V}$ and $\widetilde{W}_4$ justified their nomenclature. In continuation of the above study, the object of this paper is to define another type of a new curvature tensor on a Riemannian manifold which we called it “Pseudo W_8 –curvature tensor $\widetilde{W}_8$ ” and obtained its various geometrical properties.

2. PSEUDO W_8 –CURVATURE TENSOR $\widetilde{W}_8$

Let (M^n, g) be an n –dimensional Riemannian manifold and $\chi(M^n)$ the Lie algebra of vector fields in M^n . Throughout in the paper we assume that X, Y, Z and $W \in \chi(M^n)$, unless specifically stated otherwise. In an n –dimensional Riemannian manifold (M^n, g) , Pseudo W_8 –curvature tensor $\widetilde{W}_8$ of the type (1, 3) is given by the expression:

$$\widetilde{W}_8(X, Y)Z = a R(X, Y)Z + b[Ric(X, Y)Z - Ric(Y, Z)X]$$

$$- \frac{r}{n} \left[\frac{a}{n-1} - b \right] [g(X, Y)Z - g(Y, Z)X] \quad (2.1)$$

where a and b are constant such that $a, b \neq 0$, other symbols have their usual meanings.

If $a = 1$ and $b = \frac{1}{n-1}$ then equations (2.1) takes the form $\widetilde{W}_8(X, Y)Z = R(X, Y)Z + \frac{1}{n-1} [Ric(X, Y)Z - Ric(Y, Z)X] = W_8$ -curvature tensor (Pokhariyal 1982). Thus we see that W_8 -curvature tensor is a particular case of the tensor $\widetilde{W}_8(X, Y)Z$. For this reason $\widetilde{W}_8$ is called pseudo W_8 - curvature tensor.

Let us denote that

$$' \widetilde{W}_8(X, Y, Z, W) = g(\widetilde{W}_8(X, Y)Z, W) \text{ and } 'R(X, Y, Z, W) = g(R(X, Y)Z, W). \quad (2.1^*)$$

Hence in view of (2.1) and (2.1*), $\widetilde{W}_8(X, Y)Z$ can be written as follows:

$$\begin{aligned} ' \widetilde{W}_8(X, Y, Z, W) &= a'R(X, Y, Z, W) + \\ &b[Ric(X, Y)g(Z, W) - Ric(Y, Z)g(X, W)] \\ &- \frac{r}{n} \left[\frac{a}{n-1} - b \right] [g(X, Y)g(Z, W) - g(Y, Z)g(X, W)]. \end{aligned} \quad (2.2)$$

Theorem(2.1): Pseudo W_8 - curvature tensor $\widetilde{W}_8$ satisfies the following algebraic properties:

$$' \widetilde{W}_8(X, Y, Z, W) + ' \widetilde{W}_8(Y, X, Z, W) \neq 0, \quad (2.3)$$

$$' \widetilde{W}_8(X, Y, Z, W) + ' \widetilde{W}_8(X, Y, W, Z) \neq 0, \quad (2.4)$$

$$' \widetilde{W}_8(X, Y, Z, W) - ' \widetilde{W}_8(Z, W, X, Y) \neq 0, \quad (2.5)$$

$$' \widetilde{W}_8(X, Y, Z, W) + ' \widetilde{W}_8(Y, Z, X, W) + ' \widetilde{W}_8(Z, X, Y, W) = 0, \quad (2.6)$$

$$' \widetilde{W}_8(X, Y, E_i, E_i) = (n-1)[b Ric(X, Y) - \frac{r}{n} \{a - (n-1)\}g(X, Y)], \quad (2.7)$$

$$' \widetilde{W}_8(E_i, E_i, Z, W) = b[r g(Y, W) - Ric(Z, W)] - \frac{r}{n} [a - b(n-1)]g(Z, W), \quad (2.8)$$

$$' \widetilde{W}_8(E_i, Y, Z, E_i) = [a - (n-1)b] \left[\widetilde{Ric}(Y, Z) + \frac{r}{n} g(Y, Z) \right] = \widetilde{W}_8(Y, Z). \quad (2.9)$$

Proof : In view of (2.2) , we can prove (2.3), (2.4), (2.5), (2.6), (2.7),(2.8) and (2.9), where $\{E_i, 1 \leq i \leq n\}$ be an orthonormal basis of the tangent space at each point of the manifold.

Definition (2.1): Let (M^n, g) be a Riemannian manifold with Levi-Civita connection D . A quadratic killing tensor is defined as a second order symmetric tensor $\tilde{A}$ satisfying the condition Wald (1984) and Walkar and Penorer (1970) as follows:

$$(D_X \tilde{A})(Y, Z) + (D_Y \tilde{A})(Z, X) + (D_Z \tilde{A})(X, Y) = 0. \quad (2.10)$$

Definition (2.2): Let (M^n, g) be a Riemannian manifold with Levi-Civita connection D . A quadratic conformal killing tensor is defined as a second order symmetric tensor $\tilde{B}$ satisfying the condition Wald (1984) and Walkar and Penorer (1970) as follows:

$$\begin{aligned} (D_X \tilde{B})(Y, Z) + (D_Y \tilde{B})(Z, X) + (D_Z \tilde{B})(X, Y) = \\ K(X)g(Y, Z) + K(Y)g(Z, X) + K(Z)g(X, Y). \end{aligned} \quad (2.11)$$

Now we have the following theorem:

Theorem (2.2): If the Ricci tensor of M^n admitting Pseudo W_8 –curvature tensor $\tilde{W}_8$ is quadratic conformal killing tensor $\tilde{W}_8(Y, Z)$ of the type (0,2), then it is also quadratic conformal killing tensor.

Proof: From (2.9), we get

$$\tilde{W}_8(Y, Z) = [a - (n - 1)b] [Ric(Y, Z) + \frac{r}{n} g(Y, Z)] \quad (2.12)$$

Differentiating covariantly (2.12) with respect to, we get

$$(D_X \tilde{W}_8)(Y, Z) = [a - (n - 1)b] \left[D_X Ric(Y, Z) + \frac{(D_X r)}{n} g(Y, Z) \right]. \quad (2.13)$$

From (2.13), we get

$$\begin{aligned} (D_X \tilde{W}_8)(Y, Z) + (D_Y \tilde{W}_8)(Z, X) + (D_Z \tilde{W}_8)(X, Y) = \\ [a - (n - 1)b] \{ D_X Ric(Y, Z) + D_Y Ric(Z, X) + D_Z Ric(X, Y) \} \\ + \frac{1}{n} \{ (D_X r) g(Y, Z) + (D_Y r) g(Z, X) + (D_Z r) g(X, Y) \}. \end{aligned} \quad (2.14)$$

If we assume that the Ricci tensor is a quadratic conformal killing tensor, then from (2.11) and (2.14), we get

$$\begin{aligned} (D_X \tilde{W}_8)(Y, Z) + (D_Y \tilde{W}_8)(Z, X) + (D_Z \tilde{W}_8)(X, Y) = [a - (n - 1)b] \\ \left[\left\{ K(X) + \frac{1}{n} (D_X r) \right\} g(Y, Z) + \left\{ K(Y) + \frac{1}{n} (D_Y r) \right\} g(Z, X) + \right. \end{aligned}$$

$$\left\{ K(Z) + \frac{1}{n} (D_Z r) \right\} g(X, Y) \Big]. \quad (2.15)$$

By taking $[a - (n - 1)b] \left\{ K(X) + \frac{1}{n} (D_X r) \right\} = \alpha(X)$, then (2.15) can be written $(D_X \widetilde{W}_8)(Y, Z) + (D_Y \widetilde{W}_8)(Z, X) + (D_Z \widetilde{W}_8)(X, Y) = \alpha(X)g(Y, Z) + \alpha(Y)g(Z, X) + \alpha(Z)g(X, Y)$.

This complete the proof.

Theorem (2.3): If the Ricci tensor of M^n admitting a pseudo W_8 -curvature tensor $\widetilde{W}_8$ be quadratic killing tensor. A necessary and sufficient condition for $\widetilde{W}_8(X, Y)$ to be a quadratic killing tensor is that the scalar curvature tensor be constant, provided $a - (n - 1)b \neq 0$.

Proof. Let us consider that the Ricci tensor of the manifold M^n is quadratic tensor. Therefore, equation (2.10) and (2.14), we get

$$(D_X \widetilde{W}_8)(Y, Z) + (D_Y \widetilde{W}_8)(Z, X) + (D_Z \widetilde{W}_8)(X, Y) = [a - (n - 1)b] \frac{1}{n} \{ (D_X r) g(Y, Z) + (D_Y r) g(Z, X) + (D_Z r) g(X, Y) \}. \quad (2.16)$$

If $\widetilde{W}_8(X, Y)$ be a quadratic killing tensor, then from (2.16) we get

$$[a - (n - 1)b] \{ (D_X r) g(Y, Z) + (D_Y r) g(Z, X) + (D_Z r) g(X, Y) \} = 0. \quad (2.17)$$

Walkar's Lemma (1950) states that if $\tilde{a}(X, Y)$ and $\tilde{b}(X)$ are numbers such that

$$\tilde{a}(X, Y) = \tilde{a}(Y, X) \text{ and } \tilde{a}(X, Y) \tilde{b}(Z) + \tilde{a}(Y, Z) \tilde{b}(X) - \tilde{a}(Z, X) \tilde{b}(Y) = 0, \quad (2.18)$$

for all X, Y, Z then either all $\tilde{a}(X, Y)$ are zero or all $\tilde{b}(X)$ zero. Hence by Walkar's Lemma, we get from (2.17) and (2.18) that either $g(X, Y) = 0$ or $(D_X r) = 0$. But $g(X, Y) \neq 0$ and hence we get $D_X r = 0 \Rightarrow r$ is constant, then from (2.16), we get $\widetilde{W}_8(X, Y)$ is a quadratic killing tensor. Hence the theorem.

3. PSEUDO W_8 -FLAT MANIFOLD

In this section, we assume that $\widetilde{W}_8(X, Y)Z = 0$. Then from (2.1), we get

$$a R(X, Y)Z + b [Ric(X, Y)Z - Ric(Y, Z)Y] - \frac{r}{n} \left[\frac{a}{n-1} - b \right] [g(X, Y)Z - g(Y, Z)X] = 0. \quad (3.1)$$

From (3.1), we get

$$a R(X, Y, Z, W) + b [Ric(X, Y)g(Z, W) - Ric(Y, Z)g(X, W)]$$

$$-\frac{r}{n} \left[\frac{a}{n-1} - b \right] [g(X, Y)g(Z, W) - g(Y, Z)g(X, W)] = 0. \quad (3.2)$$

Putting E_i for X and W in equation (3.2), we get

$$[a - (n-1)b] [Ric(Y, Z) + \frac{r}{n} g(Y, Z)] = 0. \quad (3.3)$$

If $a - (n-1)b \neq 0$, then from (3.3) we get

$$Ric(Y, Z) + \frac{r}{n} g(Y, Z) = 0. \quad (3.4)$$

Hence in view of (3.1) and (3.4), we get

$$a \left[R(X, Y)Z - \frac{r}{n(n-1)} \{ g(X, Y)Z - g(Y, Z)X \} \right] = 0.$$

Hence we can state the following theorem:

Theorem (3.1): A pseudo W_8 – flat manifold is not a constant curvature, provided $a - (n-1)b \neq 0$ and, $b \neq 0$.

4. Pseudo W_8 – Conservative

In this section, we assume that $div \widetilde{W}_8 = 0$ Hicks (1969) where div denotes divergence. (4.1)

Differentiating (2.1) covariantly with respect to, we get

$$\begin{aligned} (D_U \widetilde{W}_8)(X, Y)Z &= a (D_U R)(X, Y)Z \\ &+ b [(D_U Ric)(X, Y)Z - (D_U Ric)(Y, Z)X] \\ &- \frac{D_U r}{n} \left[\frac{a}{n-1} + b \right] [g(X, Y)Z - g(Y, Z)X], \end{aligned}$$

which gives on contraction

$$\begin{aligned} (div \widetilde{W}_8)(X, Y)Z &= a (div R)(X, Y)Z \\ &+ b [(D_Z Ric)(X, Y) - (D_X Ric)(Y, Z)] \\ &- \frac{1}{n} \left[\frac{a}{n-1} + b \right] [g(X, Y)dr(Z) - g(Y, Z)dr(X)]. \end{aligned} \quad (4.2)$$

But we know that

$$(div R)(X, Y)Z = (D_X Ric)(Y, Z) - (D_Y Ric)(X, Z). \quad (4.3)$$

Hence from (4.1), (4.2) and (4.3), we get

$$a [(D_X Ric)(Y, Z) - (D_Y Ric)(X, Z)] + b [(D_Z Ric)(X, Y) -$$

$$(D_X Ric)(Y, Z) - \frac{1}{n} \left[\frac{a}{n-1} + b \right] [g(X, Y)dr(Z) - g(Y, Z)dr(X)] = 0. \quad (4.4)$$

From (4.4), we get $dr(X) = 0$, provided $a(n+1) + 3b(n-1) \neq 0$.

Hence we can state the following theorem:

Theorem (4.1)- If in a Riemannian manifold (M^n, g) ($n > 2$), the pseudo W_8 – curvature tensor $\widetilde{W}_8$ is such that $a(n+1) + 3b(n-1) \neq 0$, then the manifold is pseudo W_8 conservative if the scalar curvature is constant.

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