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Quasi-conhormonic curvature tensor on a Lorentzian Para-Sasakian manifold

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Abstract

In this paper we consider quasi-Conhormonically flat, quasi-Conhormonically conservative and ϕ – quasi-conhormonically flat LP-Sasakin manifold. It also been proved that Lorentzian para-Sasakian (LP-Sasakian) manifold satisfying the relation $R(X,Y).\tilde{H} = 0$, where $\tilde{H}$ is quasi-Conhormonically curvature tensor, then it is a manifold of constant curvature.

Keywords- LP-sasakian manifold, Einstein manifold, η – Einstein manifold, quasi-conhormonic curvature tensor, ϕ – quasi-conhormonically flat manifold.

Introduction

In 1989 Matsumoto introduced the notion of LP-Sasakian manifold. Mihai and Rosca (1992) defined the some notation independently and there after many authors Ventesha and Bagewadi (2008), Aleshian and Asghari (2010) & (2011), De and Modal (2010), Özgür (2003), Prasad and Yadav (2018) and many others.

Curvature tensors are useful tools for understand the global differential geometric properties of the manifolds with some special structures. Also, they give us some information about the global curvature properties of the manifold, as they are associated with some special transformations. A type of curvature tensor was defined by Lal, Srivastava and Prasad (2013) with the name quasi-conhormonic curvature tensor (QCC). A QCC tensor $\tilde{H}$ of type (1,3) on Riemannian manifold ($n > 3$) defined as:

$$\begin{aligned}\tilde{H}(X,Y)Z = & a R(X,Y)Z + b [Ric(Y,Z)X - Ric(X,Z)Y] \\ & + c [g(Y,Z)QX - g(X,Z)QY]\end{aligned}$$

$$- \frac{r}{n} \left[\frac{2a}{n-2} + b + c \right] [g(Y, Z)X - g(X, Z)Y], \quad (1.1)$$

for all $X, Y, Z \in M$, where R is the Riemannian curvature tensor Ric is the Ricci tensor of the type $(0, 2)$, Q is the Ricci operator of the type $(1, 1)$ defined by $Ric(X, Y) = g(QX, Y)$, r is the scalar curvature tensor and a, b, c are constants. If $a = 1$ and $b = c = -\frac{1}{n-2}$ then (1.1) takes the form

$$\begin{aligned} \tilde{H}(X, Y)Z &= R(X, Y)Z - \frac{1}{n-2} [Ric(Y, Z)X - Ric(X, Z)Y + \\ &g(Y, Z)QX - g(X, Z)QY] = H(X, Y)Z, \end{aligned}$$

where H is the Conharmonic curvature tensor Mishra (1984). Hence the Conharmonic curvature tensor H is a particular case of the tensor $\tilde{H}$.

For this reason $\tilde{H}$ is called Quasi-Conharmonic curvature tensor (QCC).

Equation (1.1) can be written as of type $(0, 4)$ as follows

$$\begin{aligned} {}'\tilde{H}(X, Y, Z, W) &= a {}'R(X, Y, Z, W) + b [Ric(Y, Z)g(X, W) - Ric(X, Z)g(Y, W)] \\ &+ c [g(Y, Z)Ric(X, W) - g(X, Z)Ric(Y, W)] \\ &- \frac{r}{n} \left[\frac{2a}{n-2} + b + c \right] [g(Y, Z)g(X, W) - g(X, Z)g(Y, W)] \quad (1.2) \end{aligned}$$

where

$$\left. \begin{aligned} {}'\tilde{H}(X, Y, Z, W) &= g(\tilde{H}(X, Y)Z, W) \\ {}'R(X, Y, Z, W) &= g(R(X, Y)Z, W). \end{aligned} \right\} \quad (1.3)$$

QCC tensor $\tilde{H}$ satisfies the following algebraic properties:

$${}'\tilde{H}(X, Y, Z, W) + {}'\tilde{H}(Y, X, Z, W) = 0$$

$${}'\tilde{H}(X, Y, Z, W) + {}'\tilde{H}(X, Y, W, Z) = 0,$$

$${}'\tilde{H}(X, Y, Z, W) - {}'\tilde{H}(Z, W, X, Y) \neq 0,$$

and

$${}'\tilde{H}(X, Y, Z, W) + {}'\tilde{H}(Y, Z, X, W) + {}'\tilde{H}(Z, X, Y, W) = 0.$$

Lal et al (2013) proved that quasi-conharmonically flat manifold is of zero scalar curvature provided that $a(3n - 4) + 2(n - 1)(n - 2)(b + c) \neq 0$.

The present paper is organized as follows:

In section 2, we give the definitions and some preliminary results that will be needed thereafter. In section 3, we discuss quasi-conharmonically flat LP-Sasakian manifold and it is shown that manifold is η - Einstein LP-Sasakian manifold with $(div\tilde{H})(X,Y)Z = 0$. In section 5, we consider ϕ - quasi quasi-conharmonically flat LP-Sasakian manifold and prove that manifold is η - Einstein with non-zero scalar curvature manifold, provided $a = b = c) \neq 0$. ξ - quasi-conharmonically flat LP-Sasakian manifold is η - Einstein shown in section 6.

2. Preliminaries

An n-dimensional differentiable manifold M^n is a Lorentzian para-Sasakian (LP-Sasakian) manifold, if it admits a (1,1) tensor field ϕ , ξ is a vector of type (0,1), 1-form η is a tensor of the type (1,0) and a Lorentzian metric g which satisfy Matsumoto (1989)

$$\phi^2 X = X + \eta(X)\xi, \quad \text{or } \phi^2 I = I + \eta \otimes \xi, \quad (2.1)$$

$$\eta(\xi) = -1, \quad (2.2)$$

$$g(\phi X, \phi Y) = g(X, Y) + \eta(X)\eta(Y), \quad (2.3)$$

$$g(X, \xi) = \eta(X), \quad (2.4)$$

$$(D_X \phi)(Y) = g(X, Y)\xi + \eta(Y)X + 2\eta(X)\eta(Y), \quad (2.5)$$

and

$$D_X \xi = \phi X, \quad (2.6)$$

for arbitrary vector fields X and Y on C where D denotes covariant differentiation with respect to g , I denotes the identity map and symbol $\otimes$ is the tensor product.

In an LP-Sasakian manifold M^n with structure (ϕ, ξ, η, g) , it is easily seen that

$$(a) \phi\xi = 0 \quad (b) \eta(\phi X) = 0 \text{ or } \eta\phi = 0 \quad (c) \text{rank } \phi = (n - 1) \quad (2.7)$$

$$\text{Let us put } F(X, Y) = g(\phi X, Y) \quad (2.8)$$

Thus the tensor field F is (0,2) type symmetric tensor field. That is,

$$F(X, Y) = F(Y, X) \quad (2.9)$$

Since the 1-form η is closed in an LP-Sasakian manifold, we have

$$(D_X \eta)(Y) = (D_Y \eta)(X) \quad (2.10)$$

$$\text{where we put } F(X, Y) = (D_X \eta)(Y) \quad (2.11)$$

Let M^n be an n -dimensional LP-Sasakian manifold with structure (ϕ, ξ, η, g) . Then we have Matsumoto and Mihai (1988) and Mihai, Shaikh and De (1999):

$$'R(X, Y, Z, W) = g(Y, Z)g(X, W) - g(X, Z)g(Y, W)$$

$$\text{or } R(X, Y)Z = g(Y, Z)X - g(X, Z)Y$$

$$\text{or } R(X, Y, Z, \xi) = g(R(X, Y)Z, \xi) = \eta(R(X, Y)Z) = g(Y, Z)\eta(X) - g(X, Z)\eta(Y), \quad (2.12)$$

$$R(X, Y)\xi = \eta(Y)X - \eta(X)Y, \quad (2.13)$$

$$-R(X, \xi)Y = R(\xi, X)Y = g(X, Y)\xi - \eta(Y)X, \quad (2.14)$$

$$-R(X, \xi)\xi = R(\xi, X)\xi = X + \eta(X)\xi, \quad (2.15)$$

$$Ric(X, \xi) = (n - 1)\eta(X), \quad (2.16)$$

$$Q\xi = (n - 1)\xi, \quad (2.17)$$

and

$$Ric(\phi X, \phi Y) = Ric(X, Y) + (n - 1)\eta(X)\eta(Y), \quad (2.18)$$

An LP-Sasakian manifold M^n is said to be Einstein manifold if its Ricci tensor Ric is of the form

$$Ric(X, Y) = kg(X, Y), \quad (2.19)$$

where k is constant.

An LP-Sasakian manifold M^n is said to be η -Einstein manifold if its Ricci tensor Ric is of the form

$$Ric(X, Y) = \alpha'g(X, Y) + \beta'\eta(X)\eta(Y), \quad (2.20)$$

for any vector fields X and Y , where α' and β' are function on M^n .

3. LP-Sasakian manifold satisfying $\tilde{H}(X, Y)Z = 0$.

In this section we assume that $\tilde{H}(X, Y)Z = 0$.

Then from (1.2), we get

$$\begin{aligned} & a'R(X, Y, Z, W) + b[Ric(Y, Z)g(X, W) - Ric(X, Z)g(Y, W)] \\ & + c[g(Y, Z)Ric(X, W) - g(X, Z)Ric(Y, W)] \\ & - \frac{r}{n} \left[\frac{2a}{n-2} + b + c \right] [g(Y, Z)g(X, W) - g(X, Z)g(Y, W)] = 0 \end{aligned} \quad (3.1)$$

Putting ξ for X and W in (3.1), we get

$$\begin{aligned} & a'R(\xi, Y, Z, \xi) + b[Ric(Y, Z)g(\xi, \xi) - Ric(\xi, Z)g(Y, \xi)] \\ & + c[g(Y, Z)Ric(\xi, \xi) - g(\xi, Z)Ric(Y, \xi)] \\ & - \frac{r}{n} \left[\frac{2a}{n-2} + b + c \right] [g(Y, Z)g(\xi, \xi) - g(\xi, Z)g(Y, \xi)] = 0 \end{aligned} \quad (3.2)$$

In view of (2.4), (2.12), (2.16) and (3.2), we get

$$\begin{aligned} Ric(Y, Z) = & \left[-\frac{a}{b} + (n-1)\frac{c}{b} + \frac{r}{nb} \left(\frac{2a}{n-2} + b + c \right) \right] g(Y, Z) + \\ & \left[-\frac{a}{b} - (n-1) + (n-1)\frac{c}{b} + \frac{r}{nb} \left(\frac{2a}{n-2} + b + c \right) \right] \eta(Y)\eta(Z), \end{aligned} \quad (3.3)$$

Further contraction of (3.3) gives

$$r = \frac{(n-1)[-a+b+(n-1)c]}{\frac{2a}{n-2}-c+\frac{1}{n}\left(\frac{2a}{n-2}+b+c\right)} = \text{constant} \quad (3.4)$$

From (3.3) and (3.4), we get the following theorem:

Theorem (3.1). Let M^n be an n -dimensional, ($n > 3$), quasi-conharmonically flat LP-Sasakian manifold. Then M^n is an η -Einstein manifold with the constant scalar curvature, provided a , b and c are not equal to zero.

Contracting (1.1) with respect to X , we get

$$(C_1^1 \tilde{H})(Y, Z) = [a + (n-1)b - c] Ric(Y, Z) + r \left[c - \frac{1}{n} \left(\frac{2a}{n-2} + b + c \right) \right] g(Y, Z), \quad (3.5)$$

where contraction of $\tilde{H}(X, Y)Z$ with respect to X is denoted by $(C_1^1 \tilde{H})(Y, Z)$.

Let us assume that in an LP-Sasakian manifold,

$$(C_1^1 \tilde{H})(Y, Z) = 0. \quad (3.6)$$

In view of (3.5) and (3.6), we get

$$Ric(X, Y) = kg(X, Y), \quad (3.7)$$

$$\text{where } k = \frac{r \left[\frac{1}{n} \left(\frac{2a}{n-2} + b + c \right) - c \right]}{a + (n-1)b - c}.$$

Hence we have the following theorem:

Theorem (3.2). If in an LP-Sasakian manifold relation $(C_1^1 \tilde{H})(Y, Z) = 0$ holds, then the manifold is Einstein manifold, provided $a + (n-1)b - c \neq 0$.

4. An Einstein LP-Sasakian manifold satisfying $(div \tilde{H})(X, Y)Z = 0$.

Definition (4.1). A manifold (M^n, g) ($n > 3$) is said to quasi-conharmonically conservative if $div \tilde{H} = 0$ (Hicks, 1969).

In this section, we assume that

$$\operatorname{div} \tilde{H} = 0, \quad (4.1)$$

where div denotes divergence.

Now, differentiating (1.1) covariantly with respect to U , we get

$$\begin{aligned} (D_U \tilde{H})(X, Y)Z &= a(D_U R)(X, Y)Z + b[(D_U \operatorname{Ric})(Y, Z)X - (D_U \operatorname{Ric})(X, Z)Y] + \\ &\quad c[g(Y, Z)(D_U Q)(X) - g(X, Z)(D_U Q)Y] - \\ &\quad \frac{D_U r}{n} \left[\frac{2a}{n-2} + b + c \right] [g(Y, Z)QX - g(X, Z)Y], \end{aligned} \quad (4.2)$$

which gives on contraction

$$\begin{aligned} (\operatorname{div} \tilde{H})(X, Y)Z &= (\operatorname{div} R)(X, Y)Z + b[(D_X \operatorname{Ric})(Y, Z) - (D_Y \operatorname{Ric})(X, Z)] + \\ &\quad c[g(Y, Z)(\operatorname{div} Q)(X) - g(X, Z)(\operatorname{div} Q)Y] - \\ &\quad \frac{1}{n} \left[\frac{2a}{n-2} + b + c \right] [g(Y, Z)dr(X) - g(X, Z)dr(Y)] \end{aligned} \quad (4.3)$$

We have Eisenhart (1926)

$$(\operatorname{div} R)(X, Y)Z = (D_X \operatorname{Ric})(Y, Z) - (D_Y \operatorname{Ric})(X, Z). \quad (4.4)$$

If an LP-Sasakian manifold is an Einstein manifold, then (2.19) and (4.4), we get

$$(\operatorname{div} R)(X, Y)Z = 0. \quad (4.5)$$

Hence (4.3) and (4.5), we get

$$(\operatorname{div} \tilde{H})(X, Y)Z = -\frac{n(n^2-4)c+4a+2(n-2)b}{2n(n-2)} \left[\frac{2a}{n-2} + b + c \right] [g(Y, Z)dr(X) - g(X, Z)dr(Y)] \quad (4.6)$$

From (4.1) and (4.6), we get

$$g(Y, Z)dr(X) - g(X, Z)dr(Y) = 0, \text{ provided } n(n^2 - 4)c + 4a + 2(n - 2)b \neq 0.$$

This shows that r is constant if r is constant then from (4.6), we get

$$(\operatorname{div} R)(X, Y)Z = 0.$$

Hence we can state the following theorem:

Theorem (4.1). An Einstein LP-Sasakian manifold $(M^n, g)(n > 3)$ is quasi-conharmonically conservative if and only if the scalar curvature tensor is constant, provided $n(n^2 - 4)c + 4a + 2(n - 2)b \neq 0$.

5. ϕ –quasi conharmonically flat LP-Sasakian manifold.

Definition (5.1). A differentiable manifold $(M^n, g)(n > 3)$, satisfying the condition the manifold Cobrerizo, Fernandez, Fernandez and Zhen (1999)

$$\phi^2 \tilde{V}(\phi X, \phi Y)\phi Z = 0, \quad (5.1a)$$

is called ϕ –quasi conharmonically flat in a K-contact manifold . condition (5.1a) considered by the author Özgür (2003) in LP-Sasakian manifold.

Here our aim is to find the characterization of LP-Sasakian manifold satisfying the condition

$$\phi^2 \tilde{H}(\phi X, \phi Y)\phi Z = , \quad (5.1b)$$

in case of quasi-conhormonic curvature tensor.

In this section we assume that LP-Sasakain manifold is ϕ –quasi conharmonically flat. Then for any vector fields X, Y, Z and W .

Hence in view of (1.1) and (5.1), we get

$$\begin{aligned} & a'R(\phi X, \phi Y, \phi Z, \phi W) = \\ & -b[Ric(\phi Y, \phi Z)g(\phi X, \phi W) - Ric(\phi X, \phi Z)g(\phi Y, \phi W)] \\ & -c[g(Y\phi, \phi Z)Ric(\phi X, \phi W) - g(\phi X, \phi Z)Ric(\phi Y, \phi W)] \\ & + \frac{r}{n} \left[\frac{2a}{n-2} + b + c \right] [g(\phi Y, \phi Z)g(\phi X, \phi W) - g(\phi X, \phi Z)g(\phi Y, \phi W)] \end{aligned} \quad (5.2)$$

Let $\{e_1, e_2, \dots, e_{n-1}, \xi\}$ be a local orthogonormal basis of vector fields in M^n . Using the fact that $\{\phi e_1, \phi e_2, \dots, \phi e_{n-1}, \xi\}$ is also a local orthonormal basis. Putting e_i for X and W in (5.2) and sum up with respect to i , then we have

$$\begin{aligned}
 a \sum_{i=1}^{n-1} 'R(\phi e_i, \phi Y, \phi Z, \phi e_i) \\
 = -b \sum_{i=1}^{n-1} [Ric(\phi Y, \phi Z)g(\phi e_i, \phi e_i) - Ric(\phi e_i, \phi Z)g(\phi Y, \phi e_i)] \\
 -c \sum_{i=1}^{n-1} [g(Y\phi, \phi Z)Ric(\phi e_i, \phi e_i) - g(\phi e_i, \phi Z)Ric(\phi Y, \phi e_i)] \\
 + \frac{r}{n} \left[\frac{2a}{n-2} + b + c \right] \sum_{i=1}^{n-1} [g(\phi Y, \phi Z)g(\phi e_i, \phi e_i) \\
 - g(\phi e_i, \phi Z)g(\phi Y, \phi e_i)] \quad (5.3)
 \end{aligned}$$

On an LP-Sasakian manifold, we have Özgür (2003)

$$\left. \begin{aligned}
 \sum_{i=1}^{n-1} 'R(\phi e_i, \phi Y, \phi Z, \phi e_i) &= Ric(\phi Y, \phi Z) + g(\phi Y, \phi Z) \\
 \sum_{i=1}^{n-1} Ric(\phi e_i, \phi e_i) &= r + n - 1 \\
 \sum_{i=1}^{n-1} g(\phi e_i, \phi Z)Ric(\phi Y, \phi e_i) &= Ric(\phi Y, \phi Z) \\
 \sum_{i=1}^{n-1} g(\phi e_i, \phi e_i) &= n + 1 \\
 \sum_{i=1}^{n-1} g(\phi e_i, \phi Z)g(\phi Y, \phi e_i) &= g(\phi Y, \phi Z)
 \end{aligned} \right\} \quad (5.4)$$

In consequences of (5.3) and (5.4), we get

$$(a + nb - c)Ric(\phi Y, \phi Z) = \left[-a - (n-1)c + r \left(\frac{2a}{n-2} + b + c \right) \right] g(\phi Y, \phi Z) \quad (5.5)$$

From (2.3) and (5.4), we get

$$Ric(Y, Z) = \left[\frac{-a - (n-1)c + r \left(\frac{2a}{n-2} + b + c \right)}{(a + nb - c)} \right] g(Y, Z) + \left[\frac{-na - nb + r \left(\frac{2a}{n-2} + b \right)}{(a + nb - c)} \right] \eta(Y)\eta(Z). \quad (5.6)$$

Equation (5.6) can be written as

$$Ric(Y, Z) = A g(Y, Z) + B\eta(Y)\eta(Z), \quad a + nb - c \neq 0,$$

where $A = \frac{-a-(n-1)c+r\left(\frac{2a}{n-2}+b+c\right)}{(a+nb-c)}, \quad a + nb - c \neq 0,$

and

$$B = \frac{-na-nb+r\left(\frac{2a}{n-2}+b\right)}{(a+nb-c)}, \quad a + nb - c \neq 0.$$

Hence we have the following theorem.

Theorem (5.1). Let M^n be an n -dimensional ($n > 3$), ϕ –quasi-conharmonically flat LP-Sasakian manifold. Then M^n is an η – Einstein manifold with non-zero scalar curvature, provided $a + nb - c \neq 0$.

6. ξ –quasi-conharmonically flat LP-Sasakian manifold.

ξ –Conformally flat K-contact manifolds have been studied by Zhen, Cabrerzo and Fernandez (1997) by condition

$$V(X, Y)\xi = 0, \tag{6.1}$$

where V is the conformal curvature tensor Mishra (1984).

Analogous to the definition of ξ – Conformally flat K-contact manifold, Mondal and De (2009) defined ξ –projectively flat Sasakian manifold.

Definition (6.1). A LP-Sasakian manifold M^n is called ξ –quasi-conharmonically flat if the condition

$$\tilde{H}(X, Y)\xi = 0, \tag{6.2}$$

holds on M^n .

Putting ξ for Z in (1.1) and using (6.2), we get

$$\begin{aligned} a R(X, Y)\xi + b [Ric(Y, \xi)X - Ric(X, \xi)Y] + c [g(Y, \xi)QX - g(X, \xi)QY] \\ - \frac{r}{n} \left[\frac{2a}{n-2} + b + c \right] [g(Y, \xi)QX - g(X, \xi)Y], \end{aligned} \tag{6.3}$$

Again putting ξ for X in (6.3) and using (2.4), (2.13), (2.16), (2.17), we get

$$cQY = \left[-a + (n-1)b + \frac{r}{n} \left(\frac{2a}{n-2} + b + c \right) \right] Y + \left[-a - (n-1)b - (n-1)c + \frac{r}{n} \left(\frac{2a}{n-2} + b + c \right) \right] \eta(Y)\xi \quad (6.4)$$

Associated g in equation (6.4), we get

$$Ric(Y, Z) = A_1 g(Y, Z) + B_1 \eta(Y)\eta(Z), \quad (6.5)$$

where $A_1 = \frac{1}{c} \left[-a + (n-1)b + \frac{r}{n} \left(\frac{2a}{n-2} + b + c \right) \right],$

and

$$B_1 = \frac{1}{c} \left[-a - (n-1)b - (n-1)c + \frac{r}{n} \left(\frac{2a}{n-2} + b + c \right) \right].$$

In view of (6.5), we have the following theorem.

Theorem (6.1). Let M^n be an n -dimensional ($n > 3$), ξ –quasi-conhormonically flat LP-Sasakian manifold. Then M^n is an η –Einstein manifold, provided A_1 and B_1 is not equal to zero.

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