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# Transformation formulae for ordinary hypergeomatric series

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**Abstract**

*In this paper, we have established certain transformation formulae for ordinary hypergeometric series by using a known identity and some known summation formulae.*

**Keywords-** Ordinary hypergeometric series, Summation formula and transformation formula.

## 1. Introduction, Notations and Definitions

Throughout this note, we shall adopt following definitions and notations.

$$\left.\begin{aligned} (a)_n &= a(a+1)\dots(a+n-1),\ (a)_0 = 1,\\ (a)_{-n} &= \frac{(-1)^n}{(1-a)_n}\\ (a)_n &= \frac{\Gamma(a+n)}{\Gamma(a)} \end{aligned}\right\} \tag{1.1}$$

An ordinary hypergeometric series is generally defined to be a series of the type $\sum_{n=0}^{\infty} a_n$ where $\frac{a_{n+1}}{a_n}$ is a rational function of $n$.Following Gasper & Rahman (1990), we represent generalized hypergeometric series as,

$$\mathrm{r}F\mathrm{s}\left[\begin{matrix} a_1, a_2, \dots, a_r\ ; z \\ b_1, b_2, \dots, b_s \end{matrix}\right] \equiv \mathrm{r}F\mathrm{s}\left[\begin{matrix} (a_r); z \\ (b_s) \end{matrix}\right]$$

$$= \sum_{n=0}^{\infty} \frac{(a_1)_n (a_2)_n \dots\dots\dots (a_r)_n}{(b_1)_n (b_2)_n \dots\dots (b_s)_n} \frac{z^n}{(1)_n}$$

$$= \sum_{n=0}^{\infty} \frac{((a_r))_n}{((b_s))_n} \frac{z^n}{(1)_n}. \tag{1.2}$$

Since long back, mathematicians working in the field of special functions use known summation formulae to establish transformations. Bailey's transform Slater (1976) is the best source for obtaining the transformation formulae for ordinary as well as for basic hypergeometric series.

Making the use of Bailey's transform and Bailey's Lemma Denis, Singh, and Singh, (2003, 2007) have established beautiful transformations involving basic hypergeometric series. In this present paper we have made use of following identity due to Verma (1972) in order to establish transformations for generalized ordinary hypergeometric series. We shall make use of the following identity due to Verma (1972).

$$\sum_{n=0}^{\infty} A_n B_n \frac{(xw)^n}{n!} = \sum_{n=0}^{\infty} \frac{(-x)^n}{n!(\gamma+n)_n} \sum_{k=0}^{\infty} \frac{(\alpha)_{n+k}(\beta)_{n+k}}{k!(\gamma+2n+1)_k} B_{n+k} \times$$

$$\times\, x^k \sum_{j=0}^{n} \frac{(-n)_j (n+\gamma)_j}{j!(\alpha)_j(\beta)_j} A_j w^j \,. \tag{1.3}$$

George Gasper and Mizan Rahman (1990)

Putting, $B_n = 1, x = 1$ in (1.3) and summing the inner series by Gauss summation formulae,

$$\sum_{n=0}^{\infty} A_n \frac{w^n}{n!} = \Gamma\begin{bmatrix} \gamma-\alpha-\beta+1, \gamma+1 \\ 1+\gamma-\alpha, 1+\gamma-\beta \end{bmatrix} \times$$

$$\times \sum_{n=0}^{\infty} \frac{(-1)^n (\alpha)_n (\beta)_n (\gamma)_n \left(1+\frac{\gamma}{2}\right)_n}{n!(1+\gamma-\alpha)_n (1+\gamma-\beta)_n \left(\frac{\gamma}{2}\right)_n} \sum_{j=0}^{n} \frac{(-n)_j (n+\gamma)_j}{j!(\alpha)_j(\beta)_j} A_j w^j \,. \tag{1.4}$$

We shall make the use of (1.4) in next section in order to establish transformation formulae. Following summation formulae for ordinary hypergeometric series in order to establish certain interesting transformation formulae for ordinary hypergeometric series, we have

$${}_3F_2\begin{bmatrix} x, 3x+4+n, -n; \frac{3}{4} \\ \frac{3}{2}(x+1), \frac{3}{2}x+2 \end{bmatrix}$$

$$= \frac{(1)_n (2x+4)_n (x+2)_m (x+3)_{3m}}{(1+x)_n (3x+4)_n (1)_m (2x+4)_{3m}} \,. \tag{1.5}$$

Verma & Jain (1980)

(where m is the greatest integer $\leq n/3$).

$${}_3F_2\begin{bmatrix} a/3, 1+a+n, -n; \frac{3}{4} \\ \left(\frac{1}{2}+\frac{a}{2}\right), \left(1+\frac{a}{2}\right) \end{bmatrix} = \frac{(1)_n \left(1+\frac{a}{3}\right)_m}{(1+a)_n (1)_m} \,. \tag{1.6}$$

Verma and Jain (1980)

(where m is the greatest integer $\leq n/3$).

$$
{}_4F_3\left[\begin{matrix} a/3, \left(1+\frac{a}{2}\right), 1+a+n, -n; \frac{3}{4} \\ \frac{a}{2}, \left(\frac{1}{2}+\frac{a}{2}\right), \left(2+\frac{a}{2}\right) \end{matrix}\right]
= \frac{(1)_n \left(a/2\right)_n \left(1+a/3\right)_m \left(2+\frac{a}{6}\right)_m}{(1+a)_n \left(2+\frac{a}{2}\right)_n (1)_m \left(\frac{a}{6}\right)_m} . \quad (1.7)
$$

Verma and Jain (1980)

(where m is the greatest integer $\leq n/3$)

## 2. Main Results

In this paper we establish our main transformation formulae as follows

(i) Putting, $w=\frac{3}{4}, \gamma=(3x+4)$,

$$
A_j = \frac{(a)_j (\beta)_j (x)_j}{\left(\frac{3}{2}x+\frac{3}{2}\right)_j \left(\frac{3}{2}x+2\right)_j}
$$

in the equation (1.4) and using (1.5), we obtain the transformation:

$$
\frac{\Gamma(3x-\alpha+5)\Gamma(3x-\beta+5)}{\Gamma(3x-\alpha-\beta+5)\Gamma(3x+5)} {}_3F_2\left[\begin{matrix} \alpha, \quad \beta, \quad x; \quad \frac{3}{4} \\ \left(\frac{3}{2}x+\frac{3}{2}\right), \left(\frac{3}{2}x+2\right) \end{matrix}\right]
$$

$$
= \sum_{n=0}^{\infty} \frac{(\alpha)_n (\beta)_n (-1)^n \left(\frac{3}{2}x+3\right)_n (2x+4)_n (x+2)_m (x+3)_{3m}}{(3x-\alpha+5)_n (3x-\beta+5)_n \left(\frac{3}{2}x+2\right)_n (1+x)_n (1)_m (2x+4)_{3m}} . \quad (2.1)
$$

(where m is the greatest integer $\leq n/3$).

(ii) Putting, $w=\frac{3}{4}, \gamma=(1+a)$,

$$
A_j = \frac{(a)_j (\beta)_j \left(a/3\right)_j}{\left(\frac{1}{2}+\frac{a}{2}\right)_j \left(1+\frac{a}{2}\right)_j}
$$

in the equation (1.4) and using (1.6), we obtain the transformation:

$$\frac{\Gamma(2+a-\alpha)\Gamma(2+a-\beta)}{\Gamma(2+a-\alpha-\beta)\Gamma(2+a)}\ {}_3F_2\left[\begin{matrix}\alpha,\ \beta,\ a/3;\ \frac{3}{4}\\ \left(\frac{1}{2}+\frac{a}{2}\right),\left(1+\frac{a}{2}\right)\end{matrix}\right]$$

$$=\sum_{n=0}^{\infty}\frac{(-1)^n(\alpha)_n(\beta)_n\left(\frac{3}{2}+\frac{a}{2}\right)_n\left(1+\frac{a}{3}\right)_m}{(2+a-\alpha)_n(2+a-\beta)_n\left(\frac{1}{2}+\frac{a}{2}\right)_n(1)_m}. \qquad (2.2)$$

(where m is the greatest integer $\leq n/3$).

(iii) Putting, $w=\frac{3}{4},\gamma=(1+a)$,

$$A_j=\frac{(a)_j(\beta)_j\left(a/3\right)_j\left(1+\frac{a}{2}\right)_j}{\left(\frac{1}{2}+\frac{a}{2}\right)_j\left(2+\frac{a}{2}\right)_j\left(\frac{a}{2}\right)_j}$$

in the equation (1.4) and using (1.7), we obtain the transformation:

$$\frac{\Gamma(2+a-\alpha)\Gamma(2+a-\beta)}{\Gamma(2+a-\alpha-\beta)\Gamma(2+a)}\ {}_4F_3\left[\begin{matrix}\alpha,\ \beta,\ a/3,\left(1+\frac{a}{2}\right);\frac{3}{4}\\ \frac{a}{2},\left(\frac{1}{2}+\frac{a}{2}\right),\ \left(2+\frac{a}{2}\right)\end{matrix}\right]$$

$$=\sum_{n=0}^{\infty}\frac{(-1)^n(\alpha)_n(\beta)_n\left(\frac{3}{2}+\frac{a}{2}\right)_n\left(a/2\right)_n\left(1+\frac{a}{3}\right)_m\left(2+\frac{a}{6}\right)_m}{(2+a-\alpha)_n(2+a-\beta)_n\left(\frac{1}{2}+\frac{a}{2}\right)_n\left(2+\frac{a}{2}\right)_n(1)_m\left(\frac{a}{6}\right)_m}. \qquad (2.3)$$

(where m is the greatest integer $\leq n/3$).

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