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Quarter symmetric connection in a conformal K – contact Riemannian Manifold

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Abstract

In 1975, Golab, S introduced the notion of quarter-symmetric connection in a Riemannian manifold with affine connection. Further it was developed by Biswas, S.C., De,U.C. and other geometers. In this paper we have studied quarter symmetric connection in a conformal K – contact Riemannian manifold.

Keywords- Quarter symmetric, conformal K – contact, Riemannian Manifold

1. Introduction

Let us consider an n – dimensional differentiable manifold M^n of differentiability class C^r indowed with a tensor field F of the type (1,1), a 1 – form u and a vector field U, satisfying.

$$\bar{\bar{X}} = -X + u(X)U , \qquad (1.1)\ (a)$$

where

$$\bar{X} = F(X). \qquad (1.1)(b)$$

$$\text{rank } F = n-1 \quad , u = 0 \, , u(\bar{X}) = 0 \ , \ u(U) = 1 \qquad (1.2)$$

then M^n is said to be an almost contact manifold with contact structure (F, u, U).

Let there be defined in an almost contact manifold M^n, a metric tensor g satisfying

$$g(\bar{X}, \bar{Y}) = g(X, Y) - u(X)u(Y), \qquad (1.3)\ (a)$$

where

$$u(X) = g(X, U), \qquad 1.3)\ (b)$$

then the manifold is called almost contact metric manifold.

If we put

$$`F(X, Y) = g(\bar{X}, Y). \qquad (1.4)$$

Then using (1.1)(a), (1.2), (1.3) and (1.4), we have

$$`F(\bar{X}, \bar{Y}) = -g(X, \bar{Y}) = g(\bar{X}, Y) = `F(X, Y), \qquad (1.5)\ (a)$$

$$`F(X, Y) + `F(Y, X) = 0. \qquad (1.5)(b)$$

If in an almost contact metric manifold

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$$2`F(X,Y) = (D_X u) - (D_Y u)(X), \tag{1.6}$$

where D is Riemannian connection, then M^n is called an almost Sasakian manifold.
If in an almost Sasakian manifold (Miyazawa and Yamaguchi, 1966)

$$(D_X u)(Y) + (D_Y u)(X) = 2\rho g(X,Y), \tag{1.7}$$

Then the manifold is called conformal K – contact Riemannian manifold.
A connection on B is said to be quarter- symmetric connection if torsion tensor satisfies

$$S(X,Y) = \pi(Y)\bar{X} - \pi(X)\bar{Y}$$

where π is 1 form and $FX = \bar{X}$, F is a tensor field of type (1,1).
A quarter symmetric connections B is given by (Golab, 1975)

$$B_X Y = D_X Y - u(X)\bar{Y}, \tag{1.8}$$

where D is a Riemannian connection.
In an almost contact manifold

$$(B_X u)(Y) = (D_X u)(Y). \tag{1.9}$$

Also in almost contact metric manifold with quarter- symmetric connection B, we have

$$(B_X g)(Y,Z) = 0 \quad (1.10)\ (a)$$

$$(B_X `F)(Y,Z) = (D_X `F)(Y,Z) \quad (1.10)\ (b)$$

$$(B_X `F)(Y,Z) = g((B_X F)(Y),Z) \quad (1.10)\ (c)$$

An almost contact metric manifold with connection B is an almost Sasakian manifold if

$$2`F(X,Y) = (B_X u)(Y) - (B_Y u)(X). \tag{1.11}$$

2. Some theorem on conformal K- contact Riemannian manifold admitting quarter symmetric connection B

Theorem (2.1) In conformal K- contact Riemannian manifold with quarter -symmetric connection B, we have

$$(B_X u)(Y) + (B_Y u)(X) = 2\rho g(X,Y) \tag{2.1}$$

Proof. From (1.8) and (1.10), we have

$$(B_X u)(Y) + (B_Y u)(X) = (D_X u)(Y) + (D_Y u)(X)$$
$$= 2\rho g(X,Y),$$

which proves the theorem.
In view of above theorem we have the following theorem.

Corollary(2.1) : In a conformal K-contact Riemannian manifold with the quarter-symmetric connection B, we have

$$`F(X,Y) = (B_X u)(Y) - \rho g(X,Y) \tag{2.2}$$

Proof. Using (1.12) in (2.1), we get (2.2).

Theorem (2.2): In a conformal K-contact Riemannian manifold with the quarter-symmetric connection B, we have

$$u((B_X F)(Y)) = \rho `F(X,Y) - g(\bar{X},\bar{Y}) \text{ and} \quad (2.3)\ (a)$$

$$(B_X `F)(\bar{Y},\bar{Z}) + (B_X `F)(Y,Z) = u(Z)\rho `F(X,Y) + u(Y)\rho `F(X,Z)$$
$$+u(Y)\, g(\bar{X},\bar{Z}) + u(Z) g(\bar{X},\bar{Y}). \quad (2.3)(b)$$

Proof. Using (2.2) and (1.4), we have

$$u((B_X F)(Y)) = u(B_X \bar{Y}) = -(B_X u)(\bar{Y})$$
$$= \rho`F(X,Y) - g(\bar{X},\bar{Y}),$$

which proves (2.3) (a) .

Further we have from (1.5)(a)

$$`F(\bar{Y},\bar{Z}) = `F(Y,Z)$$
$$(B_X`F)(\bar{Y},\bar{Z}) + `F(B_X\bar{Y},\bar{Z}) + `F(\bar{Y},B_X\bar{Z}) = (B_X`F)(Y,Z) + `F(B_XY,Z) + `F(Y,B_XZ) . \tag{2.4}$$

Now

$$`F(B_X\bar{Y},\bar{Z}) = g(\overline{B_X\bar{Y}},Z)$$
$$= g(B_X\bar{Y},Z) - u(B_X\bar{Y})u(Z)$$
$$= g((B_XF)(Y),Z) + g(\overline{B_XY},Z) - u((B_XF)(Y))u(Z)$$
$$`F(B_X\bar{Y},\bar{Z}) = (B_X`F)(Y,Z) + `F(B_XY,Z) - \rho`F(X,Y)u(Z) - g(\bar{X},\bar{Y})u(Z). \tag{2.5}$$

Similarly

$$`F(\bar{Y},B_X\bar{Z}) = -(B_X`F)(Z,Y) + `F(Y,B_XZ) - \rho`F(X,Z)u(Y) - g(\bar{X},\bar{Z})u(Y). \tag{2.6}$$

Using (2.5), (2.6) in (2.4), and using $((B_X`F)(Y,Z)) = -(B_X`F)(Z,Y)$,we have

$$(B_X`F)(\bar{Y},\bar{Z}) + (B_X`F)(Y,Z) = \rho`F(X,Y)u(Z) + \rho`F(X,Z)u(Y) + g(\bar{X},\bar{Y})u(Z) + g(\bar{X},\bar{Z})u(Y) \tag{2.7}$$

which proves (2.3) (b).

Theorem (2.3): In a conformal K-contact Riemannian manifold with quarter -symmetric connection B, we have

$$(B_Z`F)(X,Y) = -`R(X,Y,Z,U) + (B_{S(X,Y)}u)(Z) - Y\rho g(X,Z) + X\rho g(Y,Z) \tag{2.8}$$

where

$$R(X,Y,Z) = B_XB_YZ - B_YB_XZ - B_{[X,Y]}Z$$
$$`R(X,Y,Z,U) = u(R(X,Y,Z)) = g(R(X,Y,Z),U)$$

and $S(X,Y) = u(Y)\bar{X} - u(X)\bar{Y}$ is the torsion tensor of the connection B.

Proof. From (2.2) we have

$$`F(Y,Z) = (B_Yu)(Z) - \rho g(Y,Z)$$
$$(B_X`F)(Y,Z) + `F(B_XY,Z) + `F(Y,B_XZ)$$
$$= B_X((B_Yu)(Z)) - B_X(\rho g(Y,Z))$$
$$= (B_XB_Yu)(Z) + (B_Yu)(B_XZ) - X\rho g(Y,Z) - \rho(B_Xg)(Y,Z) - \rho g(B_XY,Z) - \rho g(Y,B_XZ)$$
$$= (B_XB_Yu)(Z) + (B_Yu)(B_XZ) - X\rho g(Y,Z) - \rho g(B_XY,Z) - \rho g(Y,B_XZ)$$

Using (2.8), we have

$$(B_X`F)(Y,Z) + (B_{B_XY}u)(Z) - \rho g(B_XY,Z) + (B_Yu)(B_XZ) - \rho g(Y,B_XZ)$$
$$= (B_XB_Yu)(Z) + (B_Yu)(B_XZ) - X\rho g(Y,Z) - \rho g(B_XY,Z) - \rho g(Y,B_XZ)$$

9) $(B_X`F)(Y,Z) = (B_XB_Yu)(Z) - (B_{B_XY}u)(Z) - X\rho g(Y,Z)$ (2.9)

Similarly, using

$$`F(X,Z) = (B_Xu)(Z) - \rho g(X,Z)$$

$$-`F(Z,X) = (B_X u)(Z) - \rho g(X,Z)$$
$$-(B_Y `F)(Z,X) - `F(B_Y Z,X) - `F(Z,B_Y X)$$
$$= (B_Y B_X u)(Z) + (B_X u)(B_Y Z) - Y\rho g(X,Z) - \rho g(B_X Y,Z)$$
$$-\rho g(X,B_Y Z)$$
$$-(B_Y `F)(Z,X) + (B_X u)(B_Y Z) - \rho g(X,B_Y Z) + (B_{B_Y X} u)(Z) - \rho g(B_Y X,Z)$$
$$= (B_Y B_X u)(Z) + (B_X u)(B_Y Z) - Y\rho g(X,Z)$$
$$-\rho g(B_Y X,Z) - \rho g(X,B_Y Z)$$
$$-(B_Y `F)(Z,X) = (B_Y B_X u)(Z) - (B_{B_Y X} u)(Z) - Y\rho g(X,Z) \tag{2.10}$$

Subtracting (2.10) from (2.9), we get

$$(B_X `F)(Y,Z) + (B_Y `F)(Z,X)$$
$$= (B_X B_Y u)(Z) - (B_Y B_X u)(Z) - \{(B_{S(X,Y)-[X,Y]} u)(Z)\} + Y\rho g(X,Z) - X\rho g(Y,Z)$$

$$U(R(X,Y,Z)) - (B_{S(X,Y)} U)(Z) + Y\rho g(X,Z) - X\rho g(Y,Z)$$
$$-(B_Z `F)(X,Y) = `R(X,Y,Z,U) - (B_{S(X,Y)} u)(Z) + Y\rho g(X,Z)$$
$$-X\rho g(Y,Z)$$
$$(B_Z `F)(X,Y) = -`R(X,Y,Z,U) + (B_{S(X,Y)} U)(Z) - Y\rho g(X,Z)$$
$$+X\rho g(Y,Z)$$

which proves (2.3).

Further let $(B_{S(X,Y)} u)(Z) = 0$, then

$$(B_Z `F)(X,Y) = -`R(X,Y,Z,U) - Y\rho g(X,Z) + X\rho g(Y,Z)$$
$$`R(X,Y,Z,U) = -(B_Z `F)(X,Y) - Y\rho(X,Z) + X\rho g(Y,Z) \tag{2.11}$$

Barring X and Y , we have

$$(B_Z `F)(\bar{X},\bar{Y}) = -`F(\bar{X},\bar{Y},Z,U) - \bar{Y}\rho g(\bar{X},Z) + \bar{Y}\rho g(\bar{Y},Z)$$
$$`R(\bar{X},\bar{Y},Z,U) = -(B_Z `F)(\bar{X},\bar{Y}) - \bar{Y}\rho g(\bar{X},Z) + \bar{X}\rho g(\bar{Y},Z) \tag{2.12}$$

Adding (2.11) and (2.12),we get

$$`R(\bar{X},\bar{Y},Z,U) + `R(X,Y,Z,U)$$
$$= -((B_Z `F)(\bar{X},\bar{Y}) + (B_Z `F)(X,Y)) - \bar{Y}\rho g(\bar{X},Z) + \bar{X}\rho g(\bar{Y},Z)$$
$$- Y\rho g(X,Z) + X\rho g(Y,Z)$$
$$-\rho `F(Z,X)u(Y) - \rho `F(Z,Y)u(X) - g(\bar{Z},\bar{X})u(Y) - g(\bar{Z},\bar{Y})u(X)$$
$$-\bar{Y}\rho g `F(X,Z) + \bar{X}\rho `F(Y,Z) - Y\rho g(X,Z) + X\rho g(Y,Z)$$
$$= X\rho g(Y,Z) + \bar{X}\rho `F(Y,Z) - \rho `F(Z,X)u(Y) - \rho `F(Z,Y)u(X)$$
$$-g(\bar{Z},\bar{X})u(Y) - g(\bar{Z},\bar{Y})u(X) - \bar{Y}\rho `F(X,Z) - Y\rho g(X,Z).$$

Hence, we can state following theorem.

Theorem (2.4) In a conformal K-contact Riemannian manifold with quarter -symmetric connection B, we have

$$`R(\bar{X},\bar{Y},Z,U) + `R(X,Y,Z,U) = X\rho g(Y,Z) - Y\rho g(X,Z) + \bar{X}\rho `F(Y,Z)$$
$$-\bar{Y}\rho `F(X,Z) - \rho(`F(Z,X)u(Y) + `F(Z,Y)u(X))$$
$$-(g(\bar{Z},\bar{X})u(Y) + g(\bar{Z},\bar{Y})u(X))$$

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