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A study of Characterization for Euler Graph

Sudhir Prakash Srivastava
IET, Dr R.M L Avadh University, Faizabad, India
Email-sudhir_ietfzd@yahoo.com

Abstract
The concepts of Euler trails mainly deal with the nature of connectivity in graphs. These concepts have applications to the area of puzzle and games. In this paper we discuss the relation between a local property, namely, degree of a vertex and global properties like the existence of Euler graph.

Key Words - Vertices, Edges, Graph, Trail, Walk, Paths, Circuit

Introductions
Graph theory was born in 1736 with Euler's famous paper in which he solved the Konigsberg problem. Euler posed a more general problem that problem in which type of graph G is possible to find a closed walk running through every edge of G exactly once? A closed trail containing all points and lines is called an Euler trail. A graph having an Euler trail is called an Euler graph. Obviously in Euler graph, for every pair of points u and v there exist at least two edge disjoint u-v trails and consequently there are at least two edge –disjoint a u-v paths.

The graph given in Fig. 1and 2 is an Euler graph

Properties of Euler Graph
First we prove a simple properties that is needed to study of Euler graph
Properties1: If G is a graph in which the degree of every vertex is at least two then G contains a cycle

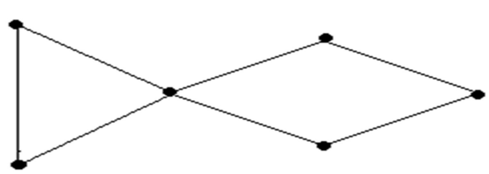

Fig. 1

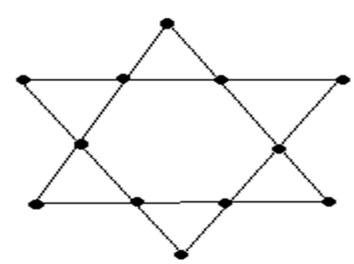

Fig. 2

Proof. Construct a sequence v, v_1, v_2 ....of vertices as follows. Choose any vertex v. Let v_1 be any vertex adjacent to vertex v. Let v_2 be any vertex adjacent to v_1 other than v. At any stage, if vertex

v_i, $i \geq 2$ is already chosen, then choose v_{i+1} to be any vertex adjacent to v_i other than v_{i-1} Since degree of each vertex is at least 2, the existence of v_{i+1} is always guaranteed.

Since G has only a finite number of vertices, at some stage we have to choose a vertex which has been chosen before.
Let v_k be the first such vertex and let $v_k = v_i$ where $i < k$. The $v_i v_{i+1 ....} v_k$ is a cycle.
The following properties answer the problem. In what type of graph G is it possible to find a closed trail running through every edge of G?

Properties 2

The following statements are equivalent for a connected graph G.
(a) If G is Euler graph
(b)Every point of G has even degree.
(c)The set of edges of G can be portioned into cycles.

Proof. (a) ⇨(b) : Let T be an Euler trail in G, with origin u. Every time a vertex v occurs in T in a place other than the origin and end, two of the edges incident with v are accounted for.
Since an Euler trail contains every edges of G, d (v) is even for every $v \neq u$. For u, one of the edges incident with u is accounted for by the origin of T, another by terminus of T and others are accounted for in pairs.
Hence d (u) is also even.

(b) ⇨ (c) : Since G is connected and nontrivial every vertex of G has degree at least 2. Hence G contains a cycle Z. The removal of the lines of Z results in a spanning sub graph G_1 in which again every vertex has even degree. If G_1 has no edges, then all the lines of G form one cycle and hence (c) holds.
Otherwise, G_1 has a cycle Z_1 , Removal of the lines of Z_1 from G_1 results in spanning sub graph G_2 in which every vertex has even degree. Containing the above process, when a graph G_n with no edge is obtained, we obtain a partition of the edges of G into n cycles.

(c) ⇨ (a) : If the partition has only one cycle, then G is obviously Euler, since it is connected. Otherwise let Z_1 ,Z_2,… Z_n be the cycles forming a partition of the lines of G. Since G is connected there exists a cycle $Z_i \neq Z_1$ having a common point v_1 with Z_1. Without loss of generality let it be Z_2. The walk beginning at v_1 and consisting of the cycle Z_1 and Z_2 in succession is closed trail containing the edges of these two cycles. Continuing these process, we can construct the edges of these two cycles. Continuing this process, we can construct a closed trail containing all the edges of G. Hence G is euleiran.

Note. The above properties and its proof hold for pseudo graphs also. Even otherwise, a pseudo graph G^* becomes a graph G when we introduce two points of degree 2 on each loop and a point of degree 2 on every other edge. Every vertex of G is of even degree iff every vertex of G^* is of even degree. Also G has closed trail running through every edge if every vertex of G^* is of even degree.

The proof of the above properties gives a method for finding a Eulerian trail when such a trail exists.

1.3<u>Konigsberg Bridge Problem</u>

The "graph" of the Konigsberg bridges has vertices of odd degree. Hence it cannot have a closed trail running through every edge. Hence one cannot walk through each of the Konigsberg bridges exactly once and come back to the starting place.

Lemma 1. Let G be connected graph with exactly 2n ($n \geq 1$) odd vertices. Then the edge set of G can be portioned into n open trails.

Proof. Let the odd vertices of G be labeled v_1, v_2 ,…v_n in any arbitrary order. Add n edges to G between the vertex pairs (v_1,w_1),(v_2,w_2) …..(v_n,w_n) to form a new graph G No two of these n edges are incident with the same vertex. Further every vertex of G is of even degree and hence G has an Eulerian trial T. If then edges that we added to G are now removed from T, it will spilt into n open trails . These are open trails in G and form a partition of the edges of G.

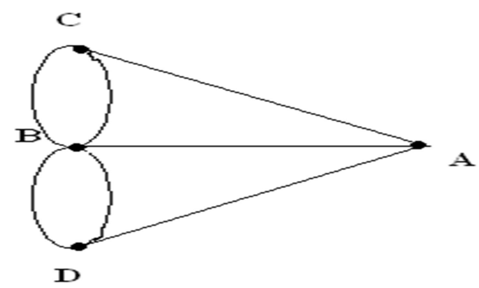

Fig 3

Proof. This is only a particular case of Lemma1.
Obviously the open trail mentioned in Lemma 1 begins at one of the odd vertices and ends at the other.
Lemma 1 answers the question: Which diagrams can be drawn without lifting one's pen form the paper not covering any line segment more than once?

1.4 Arbitrarily Traversable Graph

A graph is said to be arbitrarily traversable from a vertex v if the following procedure always results in a Eulerian trail.
Start at v by traversing any incident edge. On arriving at a vertex u depart through any incident edge not yet traversed and continue until all the lines are traversed. If a graph is arbitrarily traceable from a vertex 3 then it is obviously Eulerian. For example the graph Fig 4 is arbitrarily traceable from vertex 3. but start with vertex 1, and trace 1, 2, 3. Now at the vertex 3 we have three choices going to 1,4, or 5. Now we took the first choice (ie, going to vertex1) , we are left with a circuit 1,2,3 1 not an Euler line, so this graph is not arbitrarily traceable form vertex 1. Graph shown is fig 5 is arbitrarily traceable graph from all vertex and graph in fig6 is not arbitrarily traceable from any vertex. One can draw the observation by seeing the graph in fig4 & fig 5 that a graph can arbitrarily traceable from a vertex v if every circuit in G contains v.
The graph in Fig 1 is arbitrarily traversable form v. From no other point it is arbitrarily traversable.

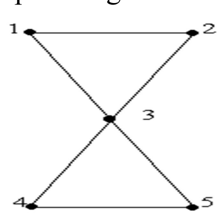

Fig.4

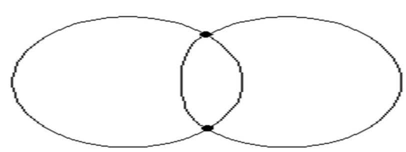

Fig. 5

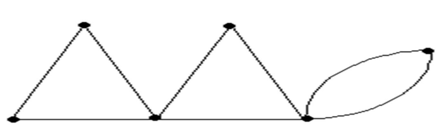

Fig. 6

The following theorem due to Ore (1951) tell just when a given graph is arbitrarily traversable

Theorem 1.1 An Euler form a chosen point graph G is arbitrary traversable from a vertex v in G if every cycle in G contains v.

1.5 Fleury's Algorithm

There is a good algorithm, due to Fleury, to construct an Eulerain trail in an Eulerian graph. Algorithm approach is to construct a trail that grows to desired Euler line, the Algorithm says.

Step 1. Choose an arbitrary vertex V_0 and set $W_0 = v_o$

Step 2. Suppose that trail W has been chosen. Then choose an edge from in such a way that
(i) e_{1+1} is incident with v_i
(ii) Unless there is no alternative, e_{i+1} is not a bridge of $G - (e_1, e_{2,\ldots}\ e_i)$

Step 3. Stop when step 2 can no longer be implemented.

Obliviously, Fleuryr's algorithm construct a trail in G. It can be proved that if G is Euler, then any trail in G constructed by Fleruy's algorithm is an Euler trial in G.

Conclusion

In this paper we discussed about a very important graph called Euler graph with special properties. Many physical problem can be represent by graph and solved by observing the relevant properties of the corresponding graph. If any system like as Euler graph then using of above properties we can find the optimize many thing . Chinese Postman problem is an example. There are many algorithm aviable to constructing an Euler path in Euler graph,one of them is Fleuryr's algorithm.This type of apporch is to construct a trail that grows to the desired Euler path.

References

1. Axlex, S.(1998). Modern Graph Theory, Springer, Chapter 1: 14-16.
2. Berge, C. (1962). The Theory of graph and its application, J.W. and S, New York.
3. Euler, L.(1736). Solution Problematics and Geometriam Situs Pertinantis, 8(1736): 128 -140.
4. English translation (1953). *Sci. Am.* July 66-70.
5. Harry, F.(1965). A Text Book of Graph Theory, Springer, Chapter 9, 1-10
6. Harary, F.(1967). Boolean operation on Graph Maths, Scand, Vol. 20; 41-51.
7. Narsingh, D.(1986). Graph Theory, PHI, 96 chapter 4:23-26.
8. Ore, O. (1960). Graph & Application, Chapter 2 and Chapter:1-8.
9. Ore, O. (1961) Note Euler & Hamiltonian circuit, *An. Math. Morthy* Vol. 67: 55.
10. Sooryanarayana, B. (2005) Graph Theory and It's Application , S. Chand, Chapter 5 :56-75.
11. Tutte, W. T.(1966) Connection of Graph University,Toronoto.

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