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A class of separate regression type estimator under stratified random sampling

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Abstract

In this paper, a class of separate regression type estimators using the auxiliary information on strata means and strata variances is proposed under stratified random sampling. The expressions of its bias and mean square error under are obtained. Further the expression of minimum mean square error under the optimum value of the characterizing scalar is also given. An optimum allocation with the proposed class is obtained and its efficiency is compared with that of Neyman optimum allocation. Finally, a comparative study is made with that of separate ratio estimator, separate Singh (2003) product estimator, separate linear regression estimator and the usual stratified sample mean. Lastly, it is shown that the proposed allocation is always more efficient in the sense of having smaller mean square error than Neyman allocation.

Key words- Stratified Random Sampling, Separate Regression type estimator, Mean square error.

1.Introduction of the proposed estimator

Let a population of size 'N' be stratified in to 'L' non-overlapping strata, the h^{th} stratum size being N_h ($h=1,2,...,L$) and $\sum_{h=1}^{L} N_h = N$. Suppose 'y' be characteristic under study and 'x' be the auxiliary variable. We denote by Y_{hj}: The observation on the j^{th} unit of the population for the charecterstic 'y' under study ($j=1,2,...,N_h$) and X_{hj}: The observation on the j^{th} unit of the population for the auxiliary charecterstic 'x' under study ($j=1,2,...,N_h$) .

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$$\bar{Y}_h = \frac{1}{N_h}\sum_{j=1}^{N_h} Y_{hj};\ \bar{X}_h = \frac{1}{N_h}\sum_{j=1}^{N_h} X_{hj};\ S_{yh}^2 = \frac{1}{(N_h-1)}\sum_{h=1}^{N_h}(y_{hj}-\bar{Y}_h)^2;\ S_{xh}^2 = \frac{1}{(N_h-1)}\sum_{j=1}^{N_h}(x_{hj}-\bar{X}_h)^2;$$

$$\sigma_{xh}^2 = \frac{1}{N_h}\sum_{j=1}^{N_h}(x_{hj}-\bar{X}_h)^2;\ \sigma_{yh}^2 = \frac{1}{N_h}\sum_{j=1}^{N_h}(y_{hj}-\bar{Y}_h)^2;\ S_{xyh} = \frac{1}{(N_h-1)}\sum_{j=1}^{N_h}(X_{hj}-\bar{X}_h)\ (Y_{hj}-\bar{Y}_h) = \rho_h S_{xh} S_{hj};$$

where ρ_h is the population correlation coefficient between 'x' and 'y' for the h^{th} stratum $\left(h=1,2,...,L\right)$.

$R_h = \frac{\bar{Y}_h}{\bar{X}_h},\ C_{yh}^2 = \frac{S_{yh}^2}{\bar{Y}_h^2} = \frac{\mu_{02h}}{\bar{Y}_h^2},\ C_{xh}^2 = \frac{S_{xh}^2}{\bar{X}_h^2} = \frac{\mu_{20h}}{\bar{X}_h^2},\ \mu_{pqh} = \frac{1}{N_h}\sum_{j=1}^{L}(X_{hj}-\bar{X}_h)^p(Y_{hj}-\bar{Y}_h)^q :$ the $(p,q)^{th}$ product moment about mean between 'x' and 'y' for the h^{th} stratum $\left(h=1,2,...,L\right)$.

$\beta_{1h} = \frac{\mu_{30h}^2}{\mu_{20h}^2},\ \beta_{2h} = \frac{\mu_{40h}^2}{\mu_{20h}^2},\ \beta_h = \frac{S_{xyh}}{S_{xh}^2} = \rho_h\frac{S_{yh}}{S_{xh}}$ be the population regression coefficient of y on x for the h^{th} stratum $\left(h=1,2,...,L\right)$.

Let a simple random sample of size n_h be selected from the h^{th} stratum without replacement such that $\sum_{h=1}^{L} n_h = n$ and for the sake of simplicity we assume that first n_h units have been selected in sample from h^{th} stratum. We denote by:

y_{hj} : observation on the j^{th} unit of the sample for the charecterstic y under study $(j=1,2,...,n_h)$.

x_{hj} : observation on the j^{th} unit of the sample for the charecterstic x under study $(j=1,2,...,n_h)$.

Also, for the sake of simplicity we assume that N_h is so large that $1-f_h=1$.

We define $\bar{y}_h = \frac{1}{n_h}\sum_{j=1}^{n_h} y_{hj};\ \bar{x}_h = \frac{1}{n_h}\sum_{j=1}^{n_h} x_{hj};\ s_{xh}^2 = \frac{1}{n_h-1}\sum_{j=1}^{n_h}(x_{hj}-\bar{x}_h)^2;\ s_{yh}^2 = \frac{1}{n_h-1}\sum_{j=1}^{n_h}(y_{hj}-\bar{y}_h)^2;$

$$\hat{\sigma}_{xh}^2 = \frac{1}{n_h}\sum_{j=1}^{n_h}(x_{hj}-\bar{X}_h)^2;\ s_{xyh} = \frac{1}{n_h-1}\sum_{j=1}^{n_h}(x_{hj}-\bar{x}_h)(y_{hj}-\bar{y}_h);\ b_h = \frac{s_{xyh}}{s_{xh}^2};$$

Assuming the population mean $\bar{X}$ and population variance σ_x^2 of the auxiliary variable to be known that Bhushan (2010) proposed an estimator of the population mean of the study variable given by

$$\hat{\bar{Y}}_{\alpha}=\bar{y}\left[1+\frac{\alpha\left(\hat{\sigma}_x^2-\sigma_x^2\right)}{\sigma_x^2}\right]+b\left(\bar{X}-\bar{x}\right), \tag{1.1}$$

where α is the characterizing scalar to be chosen suitably and $\hat{\sigma}_x^2=\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{X})^2$ while remaining notations have the usual meaning. In this paper we propose to use the following separate regression type estimator for estimating the population mean of study variable for a stratified population

$$\hat{\bar{Y}}_{\alpha S}=\sum_{h=1}^{L}W_h\left[\bar{y}_h\left\{1+\frac{\alpha_h\left(\hat{\sigma}_{xh}^2-\sigma_{xh}^2\right)}{\sigma_{xh}^2}\right\}+b_h\left(\bar{X}_h-\bar{x}_h\right)\right] \tag{1.2}$$

where α_h are the characterizing scalars to be chosen suitably; while strata means $\bar{X}_h$ and strata variances σ_{xh}^2 of the auxiliary variable 'x' are assumed to be known. It may be pointed out that the use of population variance and population coefficient of variation has been discussed by many authors including, Searls (1964), Sen (1978), Das and Tripathi (1980), Sisodia and Dwivedi (1981), Pandey and Dubey (1988), Singh (2003), Bhushan et. al. (2010), Bhushan et. al. (2009) among others.

2. Bias and MSE of the proposed estimator

Let $\bar{y}_h-\bar{Y}=e_{0h};\ \bar{x}_h-\bar{X}=e_{1h};\ s_{xyh}-S_{xyh}=e_{2h};\ s_{xh}^2-S_{xh}^2=e_{3h};\ \hat{\sigma}_{xh}^2-\sigma_{xh}^2=e_{4h};$

$$E(e_{0h})=E(e_{1h})=E(e_{2h})=E(e_{3h})=E(e_{4h})=0; \qquad \forall h=1,2,.....L \tag{2.1}$$

Now, putting these values in (1.2) and simplifying, we have

$$\hat{\bar{Y}}_{\alpha S}=\sum_{h=1}^{L}W_h\left\{(\bar{Y}_h+e_{0h})+\alpha_h(\bar{Y}_h+e_{0h})e_{4h}/\sigma_{xh}^2+\beta_h\left(-e_{1h}-e_{1h}e_{2h}/S_{xyh}+e_{1h}e_{3h}/S_x^2+...\right)\right\} \tag{2.2}$$

Taking expectation on both sides and upto the first order of approximation, we have

$$E(\hat{\bar{Y}}_{\alpha S})=\sum_{h=1}^{L}W_h\left\{\bar{Y}_h+\alpha_h\,E(e_{0h}e_{4h})/\sigma_{xh}^2+\beta_h\left\{E(e_{1h}e_{3h})/S_x^2-E(e_{1h}e_{2h})/S_{xyh}\right\}\right\}$$

Using the results given in Sukhatme and Sukhatme (1997) and Bhushan (2007)

$$E(e_{1h}e_{3h})=\tau_{nh}\mu_{30h},\ E(e_{1h}e_{2h})=\tau_{nh}\mu_{21h},\ E(e_{0h}e_{4h})=\tau_{nh}\mu_{21h};\forall h=1,2,...,L \tag{2.3}$$

We have $E(\hat{\bar{Y}}_{\alpha S})=\bar{Y}+\sum_{h=1}^{L}W_h\tau_{nh}\left[\alpha_h\mu_{21h}/\sigma_{xh}^2+\beta_h\left\{\mu_{30h}/S_x^2-\mu_{21h}/S_{xyh}\right\}\right]$ (2.4) showing that $\hat{\bar{Y}}_{\alpha S}$ is a biased estimator of population mean $\bar{Y}$ and its bias is given by

$$B(\hat{\bar{Y}}_{\alpha S}) = E(\hat{\bar{Y}}_{\alpha S}) - \bar{Y} = \sum_{h=1}^{L} W_h \tau_{nh} \left[\alpha_h \mu_{21h} / \sigma_{xh}^2 + \beta_h \left\{ \mu_{30h} / S_x^2 - \mu_{21h} / S_{xyh} \right\} \right] \tag{2.5}$$

where $\tau_{nh} = (1 - f_{nh}) / n_h$ and $f_{nh} = n_{nh} / N_{nh}$.

Therefore mean square error of $\hat{\bar{Y}}_{\alpha S}$, using (2.2) upto first order of approximation, is given by

$$MSE(\hat{\bar{Y}}_{\alpha S}) = E(\hat{\bar{Y}}_{\alpha S} - \bar{Y})^2 = E\left\{ \sum_{h=1}^{L} W_h^{\,2} \left(e_{0h} + \alpha_h \bar{Y}_h \, e_{4h} / \sigma_{xh}^2 - \beta_h e_{1h} \right)^2 \right\}$$

$$= \sum_{h=1}^{L} W_h^{\,2} E\left\{ e_{0h}^2 + e_{4h}^2 \, \alpha_h^{\,2} \bar{Y}_h^{\,2} / \sigma_{xh}^4 + \beta_h^{\,2} e_{1h}^2 + 2 e_{0h} e_{4h} \, \alpha_h \bar{Y}_h / \sigma_{xh}^2 - 2\beta_h e_{0h} e_{1h} - 2 e_{1h} e_{4h} \, \alpha_h \bar{Y}_h \beta_h / \sigma_{xh}^2 \right\}$$

Substituting the following results given in Sukhatme and Sukhatme (1997) and Bhushan (2010)

$$E(e_{0h}^2) = \tau_{nh} S_{yh}^2; \; E(e_{1h}^2) = \tau_{nh} S_{xh}^2; \; E(e_{0h} e_{1h}) = \tau_{nh} S_{xyh}; \; E(e_{4h}^2) = \tau_{nh} \left(\mu_{40h} - \mu_{20h}^2 \right); \; E(e_{0h} e_{4h}) = \tau_{nh} \mu_{21h};$$

$$E(e_{1h} e_{4h}) = \tau_{nh} \mu_{30h}; \; \forall h = 1, 2, ..., L$$

we have,

$$MSE(\hat{\bar{Y}}_{\alpha S}) = \sum_{h=1}^{L} W_h^2 \tau_{nh} \left\{ \left(1 - \rho_h^{\,2} \right) S_{yh}^2 + \alpha_h^{\,2} \bar{Y}_h^{\,2} \left(\mu_{40h} - \mu_{20h}^2 \right) / \sigma_{xh}^4 + 2\alpha_h \bar{Y}_h \, \mu_{21h} / \sigma_{xh}^2 - 2\alpha_h \beta_h \bar{Y}_h \, \mu_{30h} / \sigma_{xh}^2 \right\} \tag{2.6}$$

(2.6) is minimum when

$$\alpha_h = \left(\beta_h \mu_{30h} - \mu_{21h} \right) \mu_{20h} / \bar{Y}_h \left(\mu_{40h} - \mu_{20h}^2 \right); \forall h = 1, 2, ..., L \tag{2.7}$$

And the minimum mean square error of $\hat{\bar{Y}}_{\alpha S}$ is given by

$$MSE\left(\hat{\bar{Y}}_{\alpha S} \right)_{\min} = \sum_{h=1}^{L} W_h^{\,2} \tau_{nh} \left\{ \left(1 - \rho_h^{\,2} \right) S_{yh}^2 - \left(\beta_h \mu_{30h} - \mu_{21h} \right)^2 / \mu_{20h}^2 \left(\beta_{2h} - 1 \right) \right\} \tag{2.8}$$

3. Optimum allocation with the proposed class

Considering the cost function $C = C_0 + \sum_{h=1}^{L} c_h n_h$, where C_0 and c_h are the cost per unit within h^{th} stratum respectively minimizing the approximate variance.

$$MSE\left(\hat{\bar{Y}}_{\alpha S}\right)_{\min} = \sum_{h=1}^{L} W_h{}^2 \left\{\left(1-\rho_h{}^2\right) S_{yh}^2 - \left(\beta_h \mu_{30h} - \mu_{21h}\right)^2 \big/ \mu_{20h}^2 \left(\beta_{2h}-1\right)\right\} \Big/ n_h \qquad (3.1)$$

By Lagrange's method of multipliers subject to the cost restriction $C - C_0 = \sum_{h=1}^{L} c_h n_h$, on the lines of Cochran (1977), n_h and the multiplier λ are found so as to minimize

$$\phi = MSE\left(\hat{\bar{Y}}_{\alpha S}\right)_{\min} + \lambda\left(\sum_{h=1}^{L} c_h n_h - C + C_0\right)$$

$$= \sum_{h=1}^{L} W_h{}^2 \left\{\left(1-\rho_h{}^2\right) S_{yh}^2 - \left(\beta_h \mu_{30h} - \mu_{21h}\right)^2 \big/ \mu_{20h}^2 \left(\beta_{2h}-1\right)\right\} \Big/ n_h + \lambda\left(\sum_{h=1}^{L} c_h n_h - C + C_0\right) \qquad (3.2)$$

Differentiating (3.2) with respect to n_h and equating to zero, we get

$$-W_h^2 \left\{\left(1-\rho_h{}^2\right) S_{yh}^2 - \left(\beta_h \mu_{30h} - \mu_{21h}\right)^2 \big/ \mu_{20h}^2 \left(\beta_{2h}-1\right)\right\} \Big/ n_h^2 + \lambda c_h = 0$$

$$n_h = \frac{1}{\sqrt{\lambda}} \frac{W_h}{\sqrt{c_h}} \left\{\left(1-\rho_h{}^2\right) S_{yh}^2 - \left(\beta_h \mu_{30h} - \mu_{21h}\right)^2 \big/ \mu_{20h}^2 \left(\beta_{2h}-1\right)\right\}^{\frac{1}{2}} ; \forall h = 1,2,...,L$$
(3.3)

Summing over all strata we have

$$n = \frac{1}{\sqrt{\lambda}} \sum_{h=1}^{L} \frac{W_h}{\sqrt{c_h}} \left\{\left(1-\rho_h{}^2\right) S_{yh}^2 - \left(\beta_h \mu_{30h} - \mu_{21h}\right)^2 \big/ \mu_{20h}^2 \left(\beta_{2h}-1\right)\right\}^{\frac{1}{2}} \qquad (3.4)$$

Taking ratio of (3.3) and (3.4) we obtain

$$n_h = n \frac{\frac{W_h}{\sqrt{c_h}} \left\{\left(1-\rho_h{}^2\right) S_{yh}^2 - \left(\beta_h \mu_{30h} - \mu_{21h}\right)^2 \big/ \mu_{20h}^2 \left(\beta_{2h}-1\right)\right\}^{\frac{1}{2}}}{\sum_{h=1}^{L} \frac{W_h}{\sqrt{c_h}} \left\{\left(1-\rho_h{}^2\right) S_{yh}^2 - \left(\beta_h \mu_{30h} - \mu_{21h}\right)^2 \big/ \mu_{20h}^2 \left(\beta_{2h}-1\right)\right\}^{\frac{1}{2}}} \quad \forall h = 1,2,...,L \qquad (3.5)$$

As a particular case for $c_h = c_1$; $\forall h = 1,2,...,L$ i.e. the given cost function $c_1 \sum_{h=1}^{L} n_h + C_0 = C = C_0 + c_1 n$

The optimum allocation (3.5) reduces to

$$n_h = n \frac{W_h \left\{\left(1-\rho_h{}^2\right) S_{yh}^2 - \left(\beta_h \mu_{30h} - \mu_{21h}\right)^2 \big/ \mu_{20h}^2 \left(\beta_{2h}-1\right)\right\}^{\frac{1}{2}}}{\sum_{h=1}^{L} W_h \left\{\left(1-\rho_h{}^2\right) S_{yh}^2 - \left(\beta_h \mu_{30h} - \mu_{21h}\right)^2 \big/ \mu_{20h}^2 \left(\beta_{2h}-1\right)\right\}^{\frac{1}{2}}} \quad \forall h = 1,2,...,L \qquad (3.6)$$

Substituting the value from (3.6) in (3.1) we have

$$MSE\left(\hat{\hat{Y}}_{\alpha S}\right)_{opt} = \frac{1}{n}\sum_{h=1}^{L}\left[W_h\left\{\left(1-\rho_h^{\,2}\right)S_{yh}^2 - \left(\beta_h\mu_{30h}-\mu_{21h}\right)^2 \big/ \mu_{20h}^2\left(\beta_{2h}-1\right)\right\}^{\frac{1}{2}}\right]^2 = M_{opt}(say) \quad (3.7)$$

4. Comparison with commonly used estimators

The variance of stratified sample mean $\bar{y}_{st}$ given by Singh (2002) and Sukhatme (1997) and Sukhatme is

$$V(\bar{y}_{st}) = \sum_{h=1}^{L}\tau_{nh}W_h^{\,2}S_{yh}^2 \quad (4.1)$$

The separate ratio estimator $\hat{\hat{Y}}_{RS} = \sum_{h=1}^{L}W_h\bar{y}_{Rh} = \sum_{h=1}^{L}W_h\bar{y}_h\,\bar{X}_h/\bar{x}_h$ has the mean square error given by

$$MSE(\hat{\hat{Y}}_{RS}) = \sum_{h=1}^{L}W_h^{\,2}\tau_{nh}(S_{yh}^2 + R_h^{\,2}S_{xh}^2 - 2\rho_{xyh}R_hS_{xh}S_{yh}) \quad (4.2)$$

The separate modified product type estimator based on Singh (2003) utilizing auxiliary information about mean and standard deviation is $\hat{\hat{Y}}_{GS} = \sum_{h=1}^{L}W_h\bar{y}_h\left(\bar{x}_h+\sigma_{xh}\right)/\left(\bar{X}_h+\sigma_{xh}\right)$ having mean square error given by

$$MSE(\hat{\hat{Y}}_{GS}) = \sum_{h=1}^{L}W_h^{\,2}\tau_{nh}(S_{yh}^2 + \eta_h^{\,2}R_h^{\,2}S_{xh}^2 + 2\rho_{xyh}\eta_hR_hS_{xh}S_{yh}) \text{ where } \eta_h = \bar{X}_h/\left(\bar{X}_h+\sigma_{xh}\right) \quad (4.3)$$

The separate modified ratio type estimator based on Singh (2003) utilizing auxiliary information about mean and coefficient of variation is $\hat{\hat{Y}}_{GR} = \sum_{h=1}^{L}W_h\bar{y}_h\left(\bar{X}_h+\sigma_{xh}\right)/\left(\bar{x}_h+\sigma_{xh}\right)$ having mean square error given by $MSE(\hat{\hat{Y}}_{GR}) = \sum_{h=1}^{L}W_h^{\,2}\tau_{nh}(S_{yh}^2 + \eta_h^{\,2}R_h^{\,2}S_{xh}^2 - 2\rho_{xyh}\eta_hR_hS_{xh}S_{yh})$ (4.4)

The separate linear regression estimator is $\bar{y}_{LRS} = \sum_{h=1}^{L}W_h\left\{\bar{y}_h + b_h(\bar{X}_h-\bar{x}_h)\right\}$ having mean square error given by $MSE(\bar{y}_{LRS}) = \sum_{h=1}^{L}W_h^2\tau_{nh}\left(1-\rho_h^2\right)S_{yh}^2$ (4.5)

Further, the minimum mean square error of the proposed class of estimators is given by

$$MSE\left(\hat{\hat{Y}}_{\alpha S}\right)_{\min} = \sum_{h=1}^{L}W_h^2\tau_{nh}\left\{\left(1-\rho_h^{\,2}\right)S_{yh}^2 - \left(\beta_h\mu_{30h}-\mu_{21h}\right)^2\big/\mu_{20h}^2\left(\beta_{2h}-1\right)\right\} = M\,(say) \quad (4.6)$$

Let us now compare the proposed estimator with the above mentioned estimators; let us first compare (4.1) and (4.6) so that we have

$$V(\bar{y}_{st}) - M = \sum_{h=1}^{L}W_h^2\tau_{nh}\left\{\rho_h^{\,2}S_{yh}^2 + \left(\beta_h\mu_{30h}-\mu_{21h}\right)^2\big/\mu_{20h}^2\left(\beta_{2h}-1\right)\right\} \geq 0 \quad (4.7)$$

Comparing (4.2) and (4.6) we have

$$MSE(\hat{\hat{Y}}_{RS}) - M = \sum_{h=1}^{L}W_h^2\tau_{nh}\left\{\left(R_hS_{xh}-\rho_hS_{yh}\right)^2 + \left(\beta_h\mu_{30h}-\mu_{21h}\right)^2\big/\mu_{20h}^2\left(\beta_{2h}-1\right)\right\} \geq 0 \quad (4.8)$$

Comparing (4.3) and (4.6) we have

$$MSE(\hat{\hat{Y}}_{GS})-M=\sum_{h=1}^{L}W_h^2\tau_{nh}\left\{\left(\eta_h R_h S_{xh}+\rho_h S_{yh}\right)^2+\left(\beta_h\mu_{30h}-\mu_{21h}\right)^2\Big/\mu_{20h}^2\left(\beta_{2h}-1\right)\right\}\geq 0 \qquad (4.9)$$

Comparing (4.4) and (4.6) we have

$$MSE(\hat{\hat{Y}}_{GR})-M=\sum_{h=1}^{L}W_h^2\tau_{nh}\left\{\left(\eta_h R_h S_{xh}-\rho_h S_{yh}\right)^2+\left(\beta_h\mu_{30h}-\mu_{21h}\right)^2\Big/\mu_{20h}^2\left(\beta_{2h}-1\right)\right\}\geq 0 \qquad (4.10)$$

Comparing (4.5) and (4.6) we have

$$MSE(\bar{y}_{LRS})-M=\sum_{h=1}^{L}W_h^2\tau_{nh}\left(\beta_h\mu_{30h}-\mu_{21h}\right)^2\Big/\mu_{20h}^2\left(\beta_{2h}-1\right)\geq 0 \qquad (4.11)$$

Thus, from (4.7) to (4.11) we conclude that the proposed class of estimators have lesser mean square error than most commonly used estimators.

5.Conclusion

A class of separate linear regression type estimator $\hat{\hat{Y}}_{\alpha S}$, utilizing the auxiliary information about strata means and strata variances, is proposed and its bias is obtained. The minimum mean square error of the proposed generalized regression type estimator $\hat{\hat{Y}}_{\alpha S}$ is given by

$$MSE\left(\hat{\hat{Y}}_{\alpha S}\right)_{\min}=\sum_{h=1}^{L}W_h^2\tau_{nh}\left\{\left(1-\rho_h^2\right)S_{yh}^2-\left(\beta_h\mu_{30h}-\mu_{21h}\right)^2\Big/\mu_{20h}^2\left(\beta_{2h}-1\right)\right\}$$

Further in the situations where the optimum value of the characterizing scalars α_h is not known, the parameter involved in α_h may be estimated by the corresponding sample value in order to get a class of estimators depending upon estimated optimum value $\hat{\alpha}_h$ which preserves the efficiency of the proposed class of estimators (cf. Bhushan et al (2009)).

The comparative study given in the preceding section shows that the proposed sampling strategy proves to be superior than stratified sample mean $\bar{y}$, separate ratio estimator $\bar{y}_R$, separate Singh product estimator $\hat{\hat{Y}}_{GS}$,separate Singh ratio estimator $\hat{\hat{Y}}_{GR}$ and separate linear regression estimator $\bar{y}_{LR}$ (cf (4.7), (4.8) , (4.9), (4.10) and (4.11) respectively). Therefore the proposed class of estimators is better in terms efficiency than the preceding estimators.

Finally, the variance of stratified sample mean $\bar{y}_{st}$ under Neyman optimum allocation

$$n_h=nW_hS_{yh}\Big/\sum_{h=1}^{L}W_hS_{yh} \quad \text{is} \quad V\left(\bar{y}_{st}\right)_{Ney}=\frac{1}{n}\left(\sum_{h=1}^{L}W_hS_{yh}\right)^2 \qquad (5.1)$$

Also from (3.6) and (3.7), we have

$$n_h = n\frac{W_h\left\{\left(1-\rho_h^2\right)S_{yh}^2-\left(\beta_h\mu_{30h}-\mu_{21h}\right)^2/\mu_{20h}^2\left(\beta_{2h}-1\right)\right\}^{\frac{1}{2}}}{\sum_{h=1}^{L}W_h\left\{\left(1-\rho_h^2\right)S_{yh}^2-\left(\beta_h\mu_{30h}-\mu_{21h}\right)^2/\mu_{20h}^2\left(\beta_{2h}-1\right)\right\}^{\frac{1}{2}}}\ \forall h=1,2,...,L$$

$$MSE\left(\hat{\bar{Y}}_{\alpha S}\right)_{opt} = \frac{1}{n}\sum_{h=1}^{L}\left[W_h\left\{\left(1-\rho_h^2\right)S_{yh}^2-\left(\beta_h\mu_{30h}-\mu_{21h}\right)^2/\mu_{20h}^2\left(\beta_{2h}-1\right)\right\}^{\frac{1}{2}}\right]^2 = M_{opt}(say) \qquad (5.2)$$

From (5.1) and (5.2), M_{opt} is always smaller than $V\left(\bar{y}_{st}\right)_{Ney}$ except for the case when $\rho_h = 0$ and $\beta_h\mu_{30h} = \mu_{21h}$ simultaneously $\forall h = 1,2,...,L$.

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