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The Optimization and sensitivity of deteriorating inventory system with partial backlogging

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Abstract

In this study, a single item deterministic inventory model is considered, in which items are subject to constant deterioration and shortages are allowed. The unsatisfied demand is backlogged partially, which is a function of waiting time. The optimum order quantity is obtained by minimizing the total cost. The results are illustrated through numerical example. The convexity of the cost function is shown numerically. Sensitivity analysis is carried out to understand the effect of critical parameters on decision variables and total cost of the inventory system.

Key-words- Deterministic Inventory Models, constant demand, deterioration, waiting time partial backlogging, optimality, infinite replenishment.

Introduction

The effect of deterioration is very important in many inventory systems. Deterioration is defined as decay or damage such that item/commodity cannot be used for its original purpose. Food items, drugs, pharmaceuticals and radioactive substances are examples of items in which deterioration take place during the normal storage period of the units and therefore, this loss must be taken into account for analyzing the system. Hence deterioration cannot be avoided in business senerios. It is sometimes assumed by recent authors that the storages are either wholly backlogged or completely lost. The acceptance of the backlogged demand depends on the waiting time. Hence the waiting time for the next replenishment plays a vital role in this acceptance. Chang and Dye (1999) developed a model for deteriorating items with time varying demand and shortages in which the backlogging rate is assumed to be inversely proportional to the waiting time for the next replenishment. The literature survey by Raafat (1991), Shah and Shah (2000) and Goyal and Giri (2001) cite upto date review on deteriorating inventory models. Abad (1996,2001) derived a pricing and ordering policy for a variable rate of deterioration and partial backlogging. Dye (2002) considered an inventory model with stock dependent demand and partial backlogging. Dye *et al*. (2007) took into account the backorder cost and lost sale. Ouyang *et al*. (2005) gave an inventory model for deteriorating items with exponentially

declining demand and partial backlogging. Shah and Shukla (2009) gave a deteriorating inventory model for waiting time partial backlogging.
Ghare and Schrader (1963) developed an inventory model with a constant rate of deterioration. An order level inventory model for items deteriorating at a constant rate was discussed by Shah and Jaiswal (1977). Aggarwal (1977) in calculating the average inventory holding cost. Bahari-Kashani (1989) discussed a heuristic model with time-proportional demand. An economic order quantity model for deteriorating items with shortages and linear trend in demand was studied by Goswami and Chaudhari (1991). Deb and Chaudhuri (1986) studied a model with a finite rate of production and a time proportional deterioration rate, allowing backlogging. Goswami and Chaudhuri (1992) assumed that the demand rate, production rate and deterioration rate were all time dependent.Ouyang *et al*. (1999) considered the continuous inventory system with partial backorders. This paper attempts to apply the results of Shah and Shukla (2009) in the case where the backlogging rate is negative exponential function of waiting time. In the present paper, a deterministic inventory model in which items are subject to constant deterioration and shortages are allowed. The unsatisfied demand is partially backlogged. The partial backlogging was assumed to be exponential function of waiting time till the next replenishment. The optimal order quantity is derived by minimizing the total cost.

Notations and Assumptions

Notations

O = the ordering cost (OC) per order, P = the purchase cost (PC) per unit, h = the inventory holding cost (IHC) per unit per time unit, C_b = the backordered cost (BC) per unit short per time unit, C_L = the cost of lost sales (LS) per unit, t_1 = the time at which the inventory level reaches zero. $t_1 \geq 0$, t_2 = the length of period during which shortages are allowed, T = $t_1 + t_2$ = the length of cycle time, a = constant demand θ = deterioration rate, α = backlogging parameter, I_M = the maximum level during [0, T], I_B = the maximum backordered units during stock out period, Q (= I_M+I_B) the order quantity during a cycle of length of T, I(t) = the inventory level at time t, $I_1(t)$ = the level of positive inventory at times t. $0 \leq t \leq t_1$ $I_2(t)$= the level of negative inventory at time t. $t_1 \leq t \leq t_1 + t_2$, $G(t_1, t_2)$ = the total cost per time unit.
$\Delta(t) = e^{-\alpha(t_1+t_2-t)}$ the backlogging rate;α>0 denotes the backlogging parameter. $t_1 \leq t \leq T$.

Assumptions

Demand rate is known and constant, Ordering cost is fixed, Inventory holding cost is constant, Backorder cost is constant, Replenishment is instantaneous, The planning horizon is infinite, Backlogging rate is variable and is dependent on the length of the time for the next replenishment Deterioration rate is constant.

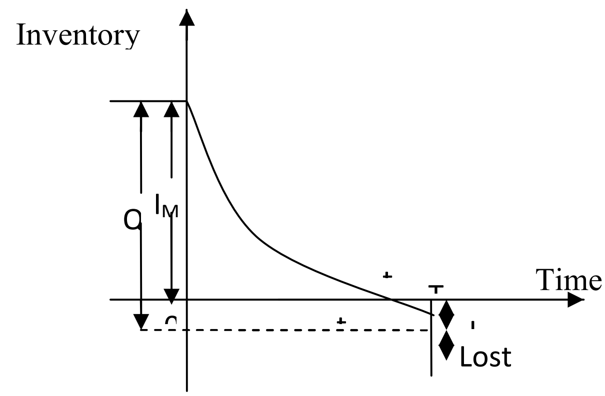
Graphical and mathematical model

Under above assumption, the on – hand inventory level at any instant of time is exhibited in figure 1.

During the period $[0, t_1]$, the inventory depletes due to the cumulative effects of demand and deterioration. Hence the inventory level at any instant of time during $[0, t_1]$ is described by the differential equation:

$$\frac{dI_1(t)}{dt} = -a - \theta I_1(t);\ 0 \le t \le t_1 \tag{1}$$

Fig.1



From time t_1 onwards shortages occur and inventory level reaches to zero. During the interval $[t_1, t_1+t_2]$, the inventory level depends on demand and a fraction of the demand is backlogged. The state of inventory during $[t_1, t_1+t_2]$, can be represented by the differential equation,

$$\frac{dI_2(t)}{dt} = -ae^{-\alpha(t_1+t_2-t)}\ ; \quad t_1 \le t \le t_1 + t_2 \tag{2}$$

The solution of the differential equation (1) with boundary condition $I_1(t_1) = 0$ at t $= t_1$,

$$I_1(t) = \frac{a(e^{\theta(t_1-t)} - 1)}{\theta};0 \le t \le t_1 \tag{3}$$

Using boundary condition $I_2(t_1) = 0$, the solution of differential equation (2) is,

$$I_2(t) = \frac{a}{\alpha}[e^{-\alpha t_2} - e^{-\alpha(t_1+t_2-t)}] \tag{4}$$

The maximum positive inventory is I_M= $-I_1(0) = \frac{a}{\theta}\left(e^{\theta t_1} - 1\right)$ (5)

The maximum backordered units are I_B= $-I(t_1+t_2) = \frac{-a}{\alpha}\left[e^{-\alpha t_2} - 1\right]$ (6)

Hence, the order size during [0, T] is Q = $\frac{a}{\theta}\left(e^{\theta t_1} - 1\right) - \frac{a}{\alpha}\left(e^{-\alpha t_2} - 1\right)$ (7)

The total cost per cycle consists of following cost components

1. Ordering cost per cycle (OC) = O
2. Inventory holding cost per cycle IHC = $h\int_0^{t_1} I_1(t)\,dt = \frac{ha}{\theta^2}\left(e^{\theta t_1} - 1 - \theta t_1\right)$.

3. Backordered cost per cycle BC = $C_b \int_{t_1}^{t_1+t_2} -I_2(t)dt \quad = \frac{C_b}{\alpha^2} a\left[1 - e^{-\alpha t_2} - \alpha t_2 e^{-\alpha t_2}\right]$

4. Cost due to lost sales per cycle (LS) LS = $C_L \int_{t_1}^{t_1+t_2} a\left(1 - e^{-\alpha(t_1+t_2-t)}\right)dt \quad = \frac{aC_L[\alpha t_2 + e^{-\alpha t_2} - 1]}{\alpha}$

5. Purchase cost (PC) = P [$\frac{a}{\theta}\left(e^{\theta t_1} - 1\right) - \frac{a}{\alpha}\left(e^{-\alpha t_2} - 1\right)$]

Therefore, the total cost under above condition per time unit is:

$$G(t_1, t_2) = \frac{1}{(t_1 + t_2)}\left[OC + IHC + BC + LS + PC\right] \tag{8}$$

The necessary conditions for the total cost per time unit, to be minimize are

$$\frac{\partial G}{\partial t_1} = 0 \quad \text{and} \quad \frac{\partial G}{\partial t_2} = 0 \tag{9}$$

Provided that

$$\left(\frac{\partial^2 G}{\partial t_1^2}\right)\left(\frac{\partial^2 G}{\partial t_2^2}\right) - \left(\frac{\partial^2 G}{\partial t_1 \partial t_2}\right) > 0 \tag{10}$$

The equations (9) and (10) are highly non-linear. So, by using suitable computer aid, these can be solved for known values of parameters.

Numerical example

Consider an inventory system with following parametric values in proper units.

$[O, P, h, C_b, C_L, a, \alpha]$ = [250, 8, 0.05, 12, 15, 25, 2]

Table 1. Variation in deterioration rate θ

Theta(θ)	t_1	t_2	Q	G	Profit
0.0500	4.2800	0.2500	124.2297	307.6580	30.7658
0.1000	3.3700	0.3300	106.2241	330.6158	33.0616
0.1500	2.8200	0.4200	94.8593	349.6183	34.9618

It is observed in this table that increase in deterioration rate increases shortages, total cost and profit per time unit of an inventory system and decreases positive inventory time period and procurement quantity.

Table variation in backlogging parameter ‘α’

α	t1	t2	Q	G	Profit
5.0000	4.3200	0.1700	123.4141	308.7177	30.8718
10.0000	4.3400	0.1100	122.8399	309.3427	30.9343
15.0000	4.3600	0.0600	122.7826	309.5393	30.9539

It is observed in table that increase in backlogging parameter decreases positive inventory time period, total cost, profit per time unit of an inventory system.

Table variation in demand 'a'

a	t1	t2	Q	G	Profit
50	4.32	0.1700	246.8282	561.756	56.1756
100	4.34	0.1100	493.1492	1066.345	106.6345
150	4.36	0.060	738.5367	1571.704	157.1704

In this table, increase in demand increases procurement quantity, profit and total cost per time unit of an inventory system.

Conclusion

Model indicate optimal replenishment schedule is derived under the assumption of waiting time partial backlogging when units in an inventory are subject to constant deterioration. The optimization of cost function is done on the basis of calculus method. The sensitivity of cost function is shown by varying the values of parameters in above tables. This model may exhibit prevailing realistic market.

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