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On certain new transformations for basic hypergeometric series

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Abstract

Using the transformation formula given by Bhargava, Somashekara and Fathima and also making use of the Ramanujan's summation ${}_1\Phi_1$, *we have established certain new transformation formulae for basic hypergeometric series.*

Keywords: Transformation/ Summation formula.

Introduction

Recently, Bhargava, Somashekara and Fathima (2005) established the following transformation formula

If $|q|<1$ and $\left|\frac{b}{a}\right|<|z|<1$, then

$$\sum_{n=-\infty}^{\infty}\frac{(a)_n}{(b)_n}z^n=\frac{(q,az;q)_n}{(b,z;q)_n}\sum_{n=0}^{\infty}\frac{(b/q;q)_n(z;q)_n}{(q;q)_n(az;q)_n}q^n$$

$$+\frac{(b-a)}{(az-q)}\frac{(q,q/az;q)_\infty}{(q/a,b/az;q)_\infty}\sum_{n=0}^{\infty}\frac{(b/az,q/a;q)_n}{\left(q,q^2/az;q\right)_n}q^n, \tag{1.1}$$

where $(a;q)_\infty=\prod_{n=0}^{\infty}\left(1-aq^n\right)$ and $(a;q)_n=\prod_{n=0}^{\infty}\frac{(a;q)_\infty}{\left(aq^n;q\right)_\infty}$, n : any integer.

In order to prove (1.1), they used the famous Hein's transformation, viz.;

$$\sum_{n=0}^{\infty}\frac{(a,b;q)_n}{(q,c;q)_n}z^n=\frac{(b,az;q)_n}{(c,z;q)_n}\sum_{n=0}^{\infty}\frac{(c/b,z;q)_n}{(q,az;q)_n}b^n. \tag{1.2}$$

In this paper making the use of the Ramanujan's summation formula for ${}_1\Psi_1$ (Gasper and Rahman,1991)

$$ {}_1\Psi_1\left[\begin{matrix}\alpha;q;z\\ \beta\end{matrix}\right]=\sum_{n=-\infty}^{\infty}\frac{(\alpha;q)_n}{(\beta;q)_n}z^n=\frac{(\alpha z,q/\alpha z,q,\beta/\alpha;q)_\infty}{(z,\beta/\alpha z,\beta,q/\alpha;q)_\infty} \tag{1.3}$$

and certain transformations for ${}_2\Phi_1$-series, we have established some new transformations for basic hypergeometric series.

Following transformations are needed in our analysis due to Gasper and Rahman, 1991

$$ {}_2\Phi_1\left[\begin{matrix}a,b;q;z\\ c\end{matrix}\right]=\frac{(abz/c;q)_\infty}{(z;q)_\infty}\,{}_2\Phi_1\left[\begin{matrix}c/a,c/b;q;abz/c\\ c\end{matrix}\right], $$

(1.4)

$$ {}_2\Phi_1\left[\begin{matrix}a,b;q;z\\ c\end{matrix}\right]=\frac{(c/b,bz;q)_\infty}{(c,z;q)_\infty}\,{}_2\Phi_1\left[\begin{matrix}abz/c,b;q;c/b\\ bz\end{matrix}\right]. \tag{1.5}$$

and

$$ {}_2\Phi_1\left[\begin{matrix}a,b;q;z\\ c\end{matrix}\right]=\frac{(bz;q)_\infty}{(z;q)_\infty}\sum_{n=0}^{\infty}(-)^n q^{n(n-1)/2}\frac{(b,c/a;q)_n}{(q,c,bz;q)_n}(az)^n. \tag{1.6}$$

We shall also make use of the following Rogers-Fine identity,

$$ \sum_{n=0}^{\infty}\frac{(\alpha;q)_n z^n}{(\beta;q)_n}=\sum_{n=0}^{\infty}\frac{(\alpha;q)_n(\alpha zq/\beta;q)_n\beta^n z^n q^{n^2-n}\left(1-\alpha zq^{2n}\right)}{(\beta;q)_n(z;q)_{n+1}}. \tag{1.7}$$

2. Main Results

In this section we obtain establish our main transformation formulae by using the relation

We know $$\sum_{n=-\infty}^{\infty}\frac{(\alpha;q)_n z^n}{(\beta;q)_n}=\sum_{n=0}^{\infty}\frac{(\alpha;q)_n z^n}{(\beta;q)_n}+\frac{(\beta-q)}{z(\alpha-q)}\sum_{n=0}^{\infty}\frac{\left(q^2/\beta;q\right)_n}{\left(q^2/\alpha;q\right)_n}\left(\frac{\beta}{\alpha z}\right)^n \tag{2.1}$$

(i) Applying (1.7) on the first part and (1.2) on the second part of right hand side of (2.1) with the help of (1.3) we get;

$$\sum_{n=0}^{\infty} \frac{(\alpha;q)_n (\alpha zq/\beta;q)_n \beta^n z^n q^{n^2-n}\left(1-\alpha zq^{2n}\right)}{(\beta;q)_n (zq;q)_n}$$

$$\frac{(\alpha z, q/\alpha z, q, \beta/\alpha;q)_\infty}{(zq,\beta/\alpha z,\beta,q/\alpha;q)_\infty} - \frac{(\beta-q)(1-z)\left(q,q^2/\alpha z;q\right)_\infty}{z(\alpha-q)\left(q^2/\alpha,\beta/\alpha z;q\right)_\infty}\times$$

$$\times {}_2\Phi_1\left[\begin{matrix} q/\alpha,\beta/\alpha z;q;q \\ q^2/\alpha z \end{matrix}\right]. \tag{2.2}$$

(ii) Taking z/α for z in (2.2) and the $\alpha \to \infty$ we get;

$$\sum_{n=0}^{\infty} \frac{(-)^n (q/\beta;q)_n \left(1-zq^{2n}\right)\beta^n z^n q^{\frac{3}{2}n(n-1)}}{(\beta;q)_n}$$

$$= \frac{(z,q/z,q;q)_\infty}{(\beta/z,\beta;q)_\infty} - \frac{(\beta-q)\left(q,q^2/z;q\right)_\infty}{z(\beta/z;q)_\infty}\times {}_1\Phi_1\left[\begin{matrix} \beta/z;q;q \\ q^2/z \end{matrix}\right]$$

$$= \frac{(z,q/z,q;q)_\infty}{(\beta,\beta/z;q)_\infty} - \frac{(\beta-q)}{z}\frac{(q,q/z;q)_\infty}{(1-q/z)(\beta/z;q)_\infty}\times$$

$$\times \sum_{n=0}^{\infty} \frac{(\beta/z;q)_n q^n}{\left(q^2/z;q\right)_n (q;q)_n},$$

$$= \frac{(q;q)_\infty (q/z;q)_\infty}{(\beta/z;q)_\infty}\times$$

$$\times\left\{\frac{(z;q)_\infty}{(\beta;q)_\infty} - \frac{(\beta-q)}{(z-q)}\sum_{n=0}^{\infty}\frac{(\beta/z;q)_n q^n}{\left(q,q^2/z;q\right)_n}\right\}. \tag{2.3}$$

(iii) As $\beta \to zq$ in (2.3) we get,

$$\sum_{n=0}^{\infty}\frac{(-)^n(1/z;q)_n\left(1-zq^{2n}\right)z^{2n}q^{n(3n-1)/2}}{(zq;q)_n}$$

$$=(1-z)(q/z;q)_\infty\left\{1+\frac{q}{(z-q)}\sum_{n=0}^{\infty}\frac{q^n}{\left(q^2/z;q\right)_n}\right\}. \tag{2.4}$$

(iv) Applying (1.2) on both part of the right hand side of (2.1) and summing its left hand side by making use of (1.3) we get,

$${}_2\Phi_1\left[\begin{matrix}\beta/q,z;q;q\\ \alpha z\end{matrix}\right]=\frac{(q/\alpha z,\beta/\alpha;q)_\infty}{(\beta/\alpha z,q/\alpha;q)_\infty}-\frac{(\beta-q)}{z(\alpha-q)}\frac{\left(q^2/\alpha z,\beta,z;q\right)_\infty}{\left(\alpha z,q^2/\alpha,\beta/\alpha z;q\right)_\infty}\times$$

$$\times{}_2\Phi_1\left[\begin{matrix}q/\alpha,\beta/\alpha z;q;q\\ q^2/\alpha z\end{matrix}\right], \tag{2.5}$$

(v) Applying (1.7) on the second part of the right hand side of (2.1) and summing its left hand side by making use of (1.3) we get,

$$\sum_{n=0}^{\infty}\frac{(\alpha;q)_n}{(\beta;q)_n}z^n=\frac{(\alpha z,q/\alpha z,q,\beta/\alpha;q)_\infty}{(z,\beta/\alpha z,\beta,q/\alpha;q)_\infty}$$

$$-\frac{(\beta-q)}{z(\alpha-q)}\sum_{n=0}^{\infty}\frac{\left(q/z,q^2/\beta;q\right)_n\left(\frac{\beta q^2}{\alpha^2 z}\right)\left(1-q^{2n+2}/\alpha z\right)q^{n(n-1)}}{\left(q^2/\alpha;q\right)_n(\beta/\alpha z;q)_{n+1}}. \tag{2.6}$$

(a) For z = q , (2.6) yields

$$\sum_{n=0}^{\infty}\frac{(\alpha;q)_n}{(\beta;q)_n}q^n=\frac{q}{(\alpha q-\beta)}\left[(1-\beta/q)-\frac{(\alpha;q)_\infty}{(\beta;q)_\infty}\right]. \tag{2.6.A}$$

(vi) Applying (1.7) on both part of the right hand side of (2.1) and summing its left hand side by using (1.3) we get,

$$\sum_{n=0}^{\infty}\frac{(\alpha;q)_n(\alpha zq/\beta;q)_n\left(1-\alpha zq^{2n}\right)\beta^n z^n q^{n(n-1)}}{(\beta;q)_n(z;q)_{n+1}}$$
$$=\frac{(\alpha z,q/\alpha z,q,\beta/\alpha;q)_\infty}{(z,\beta/\alpha z,\beta,q/\alpha;q)_\infty}-\frac{(\beta-q)}{z(\alpha-q)}\times$$
$$\times\sum_{n=0}^{\infty}\frac{\left(q/z,q^2/\beta;q\right)_n\left(1-q^{2n+2}/\alpha z\right)\left(\beta q^2/\alpha^2 z\right)q^{n(n-1)}}{\left(q^2/\alpha;q\right)_n(\beta/\alpha z;q)_{n+1}}. \tag{2.7}$$

(vii) Applying (1.4) on both part of the right hand side of (1.2.1) and summing its left hand side by using (1.3) we find,

$${}_2\Phi_1\left[\begin{matrix}\beta/\alpha,\beta/q;q;\alpha zq/\beta\\ \beta\end{matrix}\right]$$
$$=\frac{(\alpha z,q/\alpha z,q,\beta/\alpha;q)_\infty}{(\alpha zq/\beta,\beta/\alpha z,\beta,q/\alpha;q)_\infty}-\frac{(\beta-q)(z,q/z;q)_\infty}{z(\alpha-q)(\beta/\alpha z,\alpha zq/\beta;q)_\infty}\times$$
$$\times{}_2\Phi_1\left[\begin{matrix}\beta/\alpha,q/\alpha;q;q/z\\ q^2/\alpha\end{matrix}\right]. \tag{2.8}$$

(a) Taking $\alpha=-q^{\frac{2}{3}},\beta=-q^{\frac{5}{3}},z=q^{\frac{1}{3}}$ in (2.8) and then replacing q by q^3 we get,

$${}_2\Phi_1\left[\begin{matrix}q^3,-q^2;q^3;q\\ -q^5\end{matrix}\right]+\frac{\left(1+q^2\right)}{(1+q)}{}_2\Phi_1\left[\begin{matrix}q^3,-q;q^3;q^2\\ -q^4\end{matrix}\right]$$
$$=\frac{2\left(1+q^2\right)}{q^{2/3}}\frac{\eta^3(6\tau)}{\eta(2\tau)}.$$

(2.8.A)

(viii) Applying (1.7) on the first part and (1.4) on the second part of the right hand side of (2.1) and summing its left hand side by (1.3) we obtain,

$$\sum_{n=0}^{\infty}\frac{(\alpha;q)_n(\alpha zq/\beta;q)_n\beta^n z^n\left(1-\alpha zq^{2n}\right)q^{n(n-1)}}{(\beta;q)_n(z;q)_{n+1}}$$

$$=\frac{(\alpha z,q/\alpha z,q,\beta/\alpha;q)_\infty}{(z,\beta/\alpha z,\beta,q/\alpha;q)_\infty}-\frac{(\beta-q)(q/z;q)_\infty}{z(\alpha-q)(\beta/\alpha z;q)_\infty}\times$$

$$\times{}_2\Phi_1\left[\begin{matrix}\beta/\alpha,q/\alpha;q;q/z\\ q^2/\alpha\end{matrix}\right]. \tag{2.9}$$

(ix) Applying (1.4) on the first part and (1.7) on the second part of the right hand side of (2.1) and (1.3) on its left hand side we get,

$${}_2\Phi_1\left[\begin{matrix}\beta/\alpha,\beta/q;q;\alpha zq/\beta\\ \beta\end{matrix}\right]=\frac{(\alpha z,q/\alpha z,q,\beta/\alpha;q)_\infty}{(\alpha zq/\beta,\beta/\alpha z,\beta,q/\alpha;q)_\infty}-$$

$$-\frac{(\beta-q)(z;q)_\infty}{z(\alpha-q)(\alpha zq/\beta;q)_\infty}\times$$

$$\times\sum_{n=0}^{\infty}\frac{\left(q^2/\beta;q\right)_n(q/z;q)_n\left(1-q^{2n+2}/\alpha z\right)\left(\beta q^2/\alpha^2 z\right)^n q^{n(n-1)}}{\left(q^2/\alpha;q\right)_n(\beta/\alpha z;q)_{n+1}}. \tag{2.10}$$

(x) Applying (1.5) on both part of the right hand side of (2.1) and (1.3) on its left hand side we get;

$$\sum_{n=0}^{\infty}\frac{(\alpha zq/\beta;q)_n}{(zq;q)_n}\left(\frac{\beta}{q}\right)^n$$

$$=\frac{(\alpha z,q/\alpha z,q,\beta/\alpha;q)_\infty}{(zq,\beta/\alpha z,\beta/q,q/\alpha;q)_\infty}-\frac{q(1-z)}{(\alpha z-\beta)}\times$$

$$\times\sum_{n=0}^{\infty}\frac{(q/z;q)_n(q/\alpha)^n}{(\beta q/\alpha z;q)_n}. \tag{2.11}$$

(a) Taking z = q in (2.11) we get,

$$\sum_{n=0}^{\infty}\frac{\left(\alpha q^{2}/\beta;q\right)_{n}(\beta/q)^{n}}{(q;q)_{n+1}}=\frac{q}{(\alpha q-\beta)}\left\{1+\frac{(\alpha q;q)_{\infty}}{(\beta/\alpha;q)_{\infty}}\right\}. \tag{2.11.A}$$

(xi) Applying (1.7) on the first part and (1.5) on the second part of the right hand side and (1.3) on its left hand side we get,

$$\sum_{n=0}^{\infty}\frac{(\alpha;q)_{n}(\alpha zq/\beta;q)_{n}\beta^{n}z^{n}\left(1-\alpha zq^{2n}\right)q^{n(n-1)}}{(\beta;q)_{n}(z;q)_{n+1}}$$

$$=\frac{(\alpha z,q/\alpha z,q,\beta/\alpha;q)_{\infty}}{(z,\beta/\alpha z,\beta,q/\alpha;q)_{\infty}}-\frac{(\beta-q)}{(\alpha z-\beta)}\,{}_{2}\Phi_{1}\left[\begin{matrix}q/z,q;q;q/\alpha\\ \beta q/\alpha z\end{matrix}\right] \tag{2.12}$$

(a) Again taking z = q in (2.12) we get,

$$\sum_{n=0}^{\infty}\frac{(\alpha;q)_{n}\left(\alpha q^{2}/\beta;q\right)_{n}\beta^{n}q^{n^{2}}\left(1-\alpha q^{2n+1}\right)}{(\beta;q)_{n}(q;q)_{n+1}}=\frac{1}{(\alpha q-\beta)}\left\{\frac{q(\alpha q;q)_{\infty}}{(\beta/q;q)_{\infty}}-(\beta-q)\right\} \tag{2.12.A}$$

(xii) Applying (1.7) on the first part and (1.6) on the second part of the right hand side of (2.1) and (1.3) on its left hand side we get,

$$\sum_{n=0}^{\infty}\frac{(\alpha;q)_{n}(\alpha zq/\beta;q)_{n}\beta^{n}z^{n}\left(1-\alpha zq^{2n}\right)q^{n(n-1)}}{(\beta;q)_{n}(z;q)_{n+1}}=\frac{(\alpha z,q/\alpha z,q,\beta/\alpha;q)_{\infty}}{(z,\beta/\alpha z,\beta,q/\alpha;q)_{\infty}}-\frac{\alpha(\beta-q)}{(\alpha-q)(\alpha z-\beta)}\times$$

$$\times\sum_{n=0}^{\infty}\frac{(-)^{n}q^{n(n-1)}(\beta/\alpha;q)_{n}\left(\frac{q^{2}}{\alpha z}\right)^{n}}{(\beta q/\alpha z;q)_{n}\left(q^{2}/\alpha;q\right)_{n}}. \tag{2.13}$$

(xiii) Applying (1.6) on the first part and (1.7) on the second part of the right hand side of (2.1) and (1.3) on its left hand side, we get;

$$\sum_{n=0}^{\infty}(-)^{n}q^{n(n-1)/2}\frac{(\beta/\alpha;q)_{n}(\alpha z)^{n}}{(\beta,zq;q)_{n}}=\frac{(\alpha z,q/\alpha z,\beta/\alpha,q;q)_{\infty}}{(zq,\beta/\alpha z,\beta,q/\alpha;q)_{\infty}}-$$

$$-\frac{(\beta-q)(1-z)}{z(\alpha-z)}\sum_{n=0}^{\infty}\frac{\left(q^2/\beta,q/z;q\right)_n\left(\beta q^2/\alpha^2 z\right)^n q^{n(n-1)}}{\left(q^2/\alpha;q\right)_n(\beta/\alpha z;q)_{n+1}}. \tag{2.14}$$

(xiv) Applying (1.6) on both part of the right hand side of (2.1) and (1.3) on its left hand side we get,

$$\sum_{n=0}^{\infty}(-)^n q^{n(n-1)/2}\frac{(\beta/\alpha;q)_n}{(\beta;q)_n}\frac{(\alpha z)^n}{(z;q)_{n+1}}$$

$$=\frac{(\alpha z,q/\alpha z,\beta/\alpha,q;q)_\infty}{(z,\beta/\alpha z,\beta,q/\alpha;q)_\infty}-\frac{(\beta-q)}{(\alpha-q)(\alpha z-q)}\times$$

$$\times\sum_{n=0}^{\infty}\frac{(-)^n q^{n(n-1)/2}(\beta/\alpha;q)_n\left(q^2/\alpha z\right)^n}{\left(q^2/\alpha,\beta q/\alpha z;q\right)_n}. \tag{2.15}$$

(a) Taking z = q in (2.15) we get,

$$\sum_{n=0}^{\infty}\frac{(-)^n q^{n(n-1)/2}(\beta/\alpha;q)_n(\alpha q)^n}{(\beta;q)_n(q;q)_{n+1}}+\frac{(\beta-q)}{(\alpha q-\beta)}\times\times\sum_{n=0}^{\infty}\frac{(-)^n q^{n(n-1)/2}(q/\alpha)^n}{(q/\alpha;q)_{n+1}}$$

$$=\frac{(\alpha q,1/\alpha,q,\beta/\alpha;q)_\infty}{(q,\beta/\alpha q,\beta,q/\alpha;q)_\infty}=\frac{(\alpha q;q)_\infty}{(\beta;q)_\infty}\frac{(1-1/\alpha)}{(1-\beta/\alpha q)}$$

$$=\frac{q(\alpha-1)}{(\alpha q-\beta)}\frac{(\alpha q;q)_\infty}{(\beta;q)_\infty}=\frac{q}{(\beta-\alpha q)}\frac{(\alpha;q)_\infty}{(\beta;q)_\infty} \tag{2.15.A}$$

(b) Again taking $\alpha=-q^{1/2},\beta=-q$ and choosing $q\to q^2$ we get

$$\sum_{n=0}^{\infty}\frac{(-)^n q^{n(n-1)/2}\left(q^{1/2};q\right)_n\left(-q^{3/2}\right)^n}{(-q;q)_n(q;q)_{n+1}}+$$

$$+\frac{2q^{3/2}}{\left(-q^{3/2}+q\right)}\sum_{n=0}^{\infty}\frac{(-)^n q^{n(n-1)/2}\left(-q^{1/2}\right)^n}{\left(q^{1/2};q\right)_{n+1}}$$

$$=\frac{q}{\left(-q+q^{3/2}\right)}\frac{\left(q^{1/2};q\right)_\infty}{(-q;q)_\infty}$$

$$\sum_{n=0}^{\infty}\frac{q^{n(n-1)}\left(q;q^2\right)_n}{\left(-q^2;q^2\right)_n\left(q^2;q^2\right)_{n+1}}+\frac{2q}{(1-q)}\sum_{n=0}^{\infty}\frac{q^{n(n-1)}(q)^n}{\left(-q;q^2\right)_{n+1}}$$

$$=-\frac{1}{(1-q)}\frac{\left(-q;q^2\right)_\infty}{\left(-q^2;q^2\right)_\infty}=-\frac{q^{1/8}}{(1-q)}\frac{\eta^3(2\tau)}{\eta^2(4\tau)\eta(\tau)}. \tag{2.15.B}$$

3. Special Cases

(i) Taking z = q in (2.2) we get,

$$\sum_{n=0}^{\infty}\frac{(\alpha;q)_n\left(\alpha q^2/\beta;q\right)_n\left(1-\alpha q^{2n+1}\right)\beta^n q^{n^2}}{(\beta;q)_n\left(q^2;q\right)_n}$$

$$=\frac{(\alpha q,1/\alpha,q,\beta/\alpha;q)_\infty}{\left(q^2,\beta/\alpha q,\beta,q/\alpha;q\right)_\infty}-\frac{(\beta-q)(1-q)}{q(\alpha-q)}\frac{(q/\alpha;q)_\infty(\beta/\alpha;q)_\infty}{\left(q^2/\alpha,\beta/\alpha q;q\right)_\infty}$$

$$=\frac{(1-q)(1-1/\alpha)}{(1-\beta/\alpha q)}\frac{(\alpha q;q)_\infty}{(\beta;q)_\infty}-\frac{(\beta-q)(1-q)}{\alpha q(\alpha q-\beta)}$$

$$=\frac{(1-q)(\alpha q-q)}{(\alpha q-\beta)}\frac{(\alpha q;q)_\infty}{(\beta;q)_\infty}-\frac{(\beta-q)(1-q)}{(\alpha q-\beta)}$$

The summation formula thus obtained is

$$\sum_{n=0}^{\infty}\frac{(\alpha;q)_n(\alpha q/\beta;q)_{n+1}\left(1-\alpha q^{2n+1}\right)\beta^n q^{n^2}}{(\beta;q)_n(q;q)_{n+1}}$$

$$=\frac{q(1-\alpha)}{\beta}\frac{(\alpha q;q)_\infty}{(\beta;q)_\infty}+\frac{(\beta-q)}{\beta}.=\frac{q}{\beta}\left\{\frac{(\alpha;q)_\infty}{(\beta;q)_\infty}-\left(1-\frac{\beta}{q}\right)\right\}. \tag{3.1}$$

(ii) Taking $\beta=q$ in (2.2) we get

$$\sum_{n=0}^{\infty}\frac{(\alpha;q)_n(\alpha z;q)_{n+1}\left(1-\alpha zq^{2n+1}\right)z^n q^{n^2}}{(zq;q)_n(q;q)_n}=\frac{(\alpha z;q)_\infty}{(zq;q)_\infty}. \tag{3.2}$$

(iii) Taking z = 1 in (2.2) we get,

$$\sum_{n=0}^{\infty}\frac{(\alpha;q)_n(\alpha q/\beta;q)_n\left(1-\alpha q^{2n}\right)\beta^n q^{n(n-1)}}{(\beta;q)_n(q;q)_n}=\frac{(\alpha;q)_\infty}{(\beta;q)_\infty}. \tag{3.3}$$

(iv) Putting $\beta=\alpha$ in (2.2) we get,

$$\sum_{n=0}^{\infty}\left(1-\alpha zq^{2n}\right)(\alpha z)^n q^{n(n-1)}$$

$$=-\frac{(1-z)}{z}\frac{\left(q,q^2/\alpha z;q\right)_\infty}{\left(q^2/\alpha,1/z;q\right)_\infty}{}_2\Phi_1\begin{bmatrix}q/\alpha,1/z;q;q\\q^2/\alpha z\end{bmatrix}$$

$$=\left(1-\frac{1}{z}\right)\frac{\left(q,q^2/\alpha z;q\right)_\infty}{\left(1/z,q^2/\alpha;q\right)_\infty}\frac{\left(q/z,q^2/\alpha;q\right)_\infty}{\left(q^2/\alpha z,q;q\right)_\infty}$$

$$=\left(1-\frac{1}{z}\right)\frac{1}{\left(1-\frac{1}{z}\right)}=1 \tag{3.4}$$

(v) Setting $\alpha=-q^{1/2},\beta=-q^2,z=q^{1/2}$ in (2.2) and then replacing q by q^2 we get,

$$\frac{(1-q)\left(q;q^2\right)_\infty q^{1/12}}{\left(1+q^2\right)}\sum_{n=0}^{\infty}\frac{\left(-q;q^2\right)_\infty q^{2n}}{\left(-q^2;q^2\right)_n}=\frac{q^{1/8}\eta^4(2\tau)}{\eta^2(4\tau)\eta(\tau)}+2\eta(2\tau), \tag{3.5}$$

where $\eta(\tau)=q^{1/24}(q;q)_\infty;\left(q=e^{2\pi i\tau}\right)$.

(vi) Setting $\alpha=-q^{2/3},\beta=-q^{5/3},z=q^{1/3}$ and then replacing q by q^3 in (2.2) we get,

$$\sum_{n=0}^{\infty}\frac{\left(-q^2;q^3\right)_n\left(q;q^3\right)_n\left(1+q^{6n+3}\right)(-)^n q^{3n(n+1)}}{\left(q,-q^2;q^3\right)_{n+1}}$$

$$+\frac{q^{-1/4}\eta(6\tau)}{\left(-q;q^3\right)_\infty\left(q^2;q^3\right)_\infty}\sum_{n=0}^{\infty}\frac{\left(q^2,-q;q^3\right)_n q^{3n}}{\left(q^6;q^6\right)_n}=\frac{2\eta^3(6\tau)}{q^{2/3}\eta(2\tau)(1-q)^2\left(1+q^2\right)^2} \quad (3.6)$$

(vii) Taking z = q in (2.5) we get the following summation formula;

$$\sum_{n=0}^{\infty}\frac{(\beta/q;q)_n q^n}{(\alpha q;q)_n}=\frac{(\alpha-1)q}{(\alpha q-\beta)}-\frac{(\beta-q)}{(\alpha q-\beta)}\frac{(\beta;q)_\infty}{(\alpha q;q)_\infty}$$

$$=\frac{(\alpha-1)q}{(\alpha q-\beta)}+\frac{q}{(\alpha q-\beta)}\frac{(\beta/q;q)_\infty}{(\alpha q;q)_\infty}=\frac{(1-\alpha)q}{(\alpha q-\beta)}\left\{\frac{(\beta/q;q)_\infty}{(\alpha;q)_\infty}-1\right\} \quad (3.7)$$

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