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Micro-continuum investigation of heat transfer in blood flow through tapered channel

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Abstract

Steady state solutions are analysed to determine the velocity and temperature profiles of micropolar fluid model of blood flow through a vascular bed. The present model assumes the vascular bed in the form of tapered channel. Exact solutions to the governing equations are obtained by employing a constant spin boundary conditions to describe the cell rotation velocity at the boundaries. The results obtained are applicable to both arterial and venous beds and may be useful for pathological purposes.

Key words- Micro continuuam, heat transfer, tapered channel

Introduction

The straight channel flow in Newtonian and Non-Newtonian fluids have been studied by previous authors Tandan and Rai Singhani (1970), Yu and Yung (1969), Kapur and Rathy (1962). A continuing problem in hemodynamics or blood rheology have been to gain a quantitative understanding of the non-Newtonian behaviour of blood in steady flow. In order to understand the rheological properties of blood one has to make a model study, using physical models of convenient macroscopic scale, with simulated blood. The most important advantage of using these models is the possibility of simplifying the geometry and boundary conditions, so that we can concentrate at once on the most important factor involved in the problem. Within the past decade it has become evident through demographic studies that the majority of human deaths in the world occur directly as a result of diseases of the heart and cardiovascular system. One of the wide spread disease is atherosclerosis, involving hardening and thickening of blood vessel walls, resulting the accumulation in the intima of deposits of minerals, lipids, glycoproteins of collagen. At present the mechanism of atherogenesis is not well understood. An important motivation behind this is the fact that the phenomenon of atherosclerosis is known to selectively occur at arterial sides of curvature, bifurcation, tapering and branching etc. (Texon, 1960, Caro *et al* (1971). Several hemodynamics theories for the justification of this phenomenon have been suggested by Chandran *et al* (1974, 1979). In any vascular bed, the vessels bifurcate at frequent interval and the individual segments may be approximately uniform between branch points, the diameter varies quite rapidly with respect to the distance. Since the vessel diameter reduces at each bifurcation Weibel (1963).

Also the notion of tapered tube models were first suggested by Wormersley (1957) and have been discussed by Streeter (1965), and Ibrall (1964). This suggests that a vascular bed may be approximated by tapered channel rather than of uniform segments. Most of the previous studies have been treating blood as a viscous, hemogeneous incompressible fluid. But it has been concluded and experimentally verified that the suspended blood cells are responsible for non-Newtonian nature of blood rheology through such mechanism as erythrocytes aggregation and erythrocyte deformation (Copley *et al.* (1976), Chien *et al.* (1970). The number of attempts have been made to explain the anamolous behaviour of blood by proposing different theoretical models Das and Seshadari (1975), PAL and Dwivedi (1985), Dwivedi et al (1982), Tandon *et al.* (1985), Pal et al (1980), Some heat transfer problems of non-Newtonian fluid model of blood have been studied by different authors. (Damseh *et al.* (2007), Baris (2003). Motivated by the above studies, it is the objective of this paper to consider the problem of steady flow and heat transfer in a micropolar fluid model of blood flowing through rigid tapered channel. A spin boundary condition on red blood cell is used at the surface (Arimen *et al.* (1974). We have consider the constant pressure gradient throughout the analysis (Wirz *et al.* (1978). Using these assumptions analytical expressions for the flow velocity, microrotation velocity, flow rate and temperature distribution.

Formulation of the problem

While developing the theoretical model one must simplify the equations of motion sufficiently to permit calculations of the required flow variables while at realism of the model. The basic equations, proposed by Eringen (1966) for micro-polar fluid applicable to blood flow (Ariman *et al.* (1974)] and by using the assumption given Tenner (1966) may be written as

$$(\mu_s+\mu_R)\frac{\partial^2 u_x}{\partial y'^2}+2\mu_R\frac{\partial v_x}{\partial y'}-\frac{\partial p_x}{\partial x'}=0 \quad (1)$$

$$\frac{\partial p_x}{\partial y'}=0 \quad (2)$$

$$\gamma\frac{\partial^2 v_x}{\partial y'^2}-2\mu_R\frac{\partial u_x}{\partial y'}-4\mu_R v_x=0 \quad (3)$$

and energy equation

$$K\frac{\partial^2 T}{\partial y'^2}+(2\mu_s+\mu_R)\left(\frac{\partial u_x}{\partial y'}\right)^2+2\mu_R\left(\frac{1}{2}\frac{\partial u_x}{\partial y'}+v_x\right)^2+\gamma\left(\frac{\partial v_x}{\partial y'}\right)^2=0 \quad (4)$$

where μ_s, μ_R and γ are the shear viscosity, rotational viscosity and rotational gradient coefficient respectively. u_x, v_x and p_x are the axial velocity, rotational velocity and pressure respectively. x' and y' are the axial and vertical co-ordinates along the length and thickness of the channel respectively. K is the thermal conductivity and T is temperature distribution in the channel.

Boundary conditions

The boundary conditions are given by

$$\left.\begin{aligned} &u_x = 0 && \text{at } y' = \pm h' \\ &\frac{\partial v_x}{\partial y} = 0 && \text{at } y' = \pm h' \\ &T = T_c && \text{at } y' = \pm h' \end{aligned}\right| \quad (5)$$

which h' is the thickness of the channel.

By using the following non-dimensional equations

$$\left.\begin{aligned} &u_x = U_o u, \quad y' = h_o y, \quad v_x = \frac{U_o v}{h_o} \\ &x' = h_o x, \quad n_2 = \frac{\mu_s h_o^2}{\gamma}, \quad n_3 = \frac{\mu_R h_o^2}{\gamma} \\ &P_x = \frac{U_o(\mu_s + \mu_R)}{h_o} p, \quad h' = h_o h \\ &\theta = \frac{T - T_c}{T_c}, \; E_c = \frac{U_o^2}{C_p T_c}, \; P_r = \frac{C_p(2\mu_s + \mu_R)}{K} \end{aligned}\right| \quad (6)$$

The governing equations take the form after using (6)

$$\frac{\partial^2 u}{\partial y^2} + \frac{2n_3}{(n_2 + n_3)}\frac{\partial v}{\partial y} - \frac{\partial p}{\partial x} = 0 \quad (7)$$

$$\frac{\partial p}{\partial y} = 0 \quad (8)$$

$$\frac{\partial^2 v}{\partial y^2} - 2n_3\frac{\partial u}{\partial y} - 4n_3 v = 0 \quad (9)$$

$$\frac{\partial^2 \theta}{\partial y^2} + \frac{P_r E_c(2n_2 + n_3)}{(n_2 + n_3)}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{2n_3 P_r E_c}{(n_2 + n_3)}\left(\frac{1}{2}\frac{\partial u}{\partial y} + v\right)^2 + \frac{P_r E_c}{(n_2 + n_3)}\left(\frac{\partial v}{\partial y}\right)^2 = 0 \quad (10)$$

The taperness of the channel is given by the expression

$$h' = h_0 - \frac{(h_\ell - h_o)}{L}\, x'$$

or $h = 1 - mx$ (11)

$$\text{where } m = \frac{h_\ell - h_0}{L} \tag{12}$$

where $2h_\ell$ is the maximum and $2h_0$ the minimum opening the channel. L is the length of the channel.

The Boundary conditions (5) become

$$u = 0 \quad at \;\; y = \pm h, \quad \frac{\partial v}{\partial y} = 0 \quad at \; y = \pm h, \quad \theta = 0 \quad at \; y = \pm h \tag{13}$$

Solutions

The velocity profiles are obtained from the equations (7, 9) by using boundary condition (13) and are as follows

$$u = \frac{(n_2 + n_3)}{4n_2}\left(\frac{-\partial p}{\partial x}\right)\left[\frac{1}{n_2}\left(\frac{\cosh \alpha y}{\cosh \alpha h} - 1\right) - 2\left(y^2 - h^2\right)\right] \tag{14}$$

$$v = \frac{(n_2 + n_3)}{2n_2}\left(\frac{-\partial p}{\partial x}\right)\left[-\frac{1}{\alpha}\frac{\sinh \alpha y}{\cosh \alpha h} + y\right] \tag{15}$$

where
$$\alpha^2 = \frac{4n_2 n_3}{(n_2 + n_3)} \tag{16}$$

The volumetric flow rate is given by
$$Q = \frac{(n_2 + n_3)}{2n_2}\left(\frac{-\partial p}{\partial x}\right)\left[\frac{4h^3}{3} + \frac{1}{n_2}\left(\frac{1}{\alpha}\frac{\sinh \alpha h}{\cosh \alpha h} - h\right)\right] \tag{17}$$

Now since the tapered angle is very small i.e. m is very small as well as the value of h can not be zero. Hence, writing the expansions for sinh(αh) and cosh(αh) is asending powers of x and retaining the terms upto fifth powers of x, we get the simplified form for $\frac{\partial p}{\partial x}$ as

$$\frac{-\partial p}{\partial x} = \frac{\dfrac{2n_2}{(n_2 + n_3)}Q}{(1 - mx)^3\left(\dfrac{4n_2 - \alpha^2}{3n_2} + \dfrac{2\alpha^4}{15n_2}(1 - mx)^2\right)} \tag{18}$$

Integrating equation (18) we get

$$-p = \frac{6n_2^2}{m(n_2 + n_3)(4n_2 - \alpha^2)}\left[\frac{2}{5}\frac{\alpha^4}{(4n_2 - \alpha^2)}\log(1 - mx) + \frac{1}{2(1 - mx)^2} - \frac{1}{2}\left(\frac{2\alpha^4}{5(4n_2 - \alpha^2)}\right)^2\right.$$

$$\log\left(\frac{2n_2Q}{n_2+n_3}+\left(\frac{4n_2-\alpha^2}{3n_2}\right)(1-mx)^2\right)\Bigg]\Bigg]+G \tag{19}$$

The constant of integration G is evaluated with the help of following conditions

$$p=p_e \quad at\ x=0 \quad \text{and} \quad p=p_E \quad at\ x=1 \tag{20}$$

using these conditions in equation (19) we obtain:

$$p_e-P=\frac{6n_2^2Q}{(n_2+n_3)(4n_2-\alpha^2)m}\left[\frac{2\alpha^4}{5(4n_2-\alpha^2)}\log(1-mx)+\frac{1}{2}\left(\frac{1}{(1-mx)^2}-1\right)-\frac{1}{2}\left(\frac{2\alpha^4}{5(4n_2-\alpha^2)}\right)^2\right.$$

$$\left.\log\left(\frac{\frac{2n_2Q}{(n_2+n_3)}+\left(\frac{4n_2-\alpha^2}{3n_2}\right)(1-mx)^2}{\frac{2n_2Q}{(n_2+n_3)}+\frac{(4n_2-\alpha^2)}{3n_2}}\right)\right]\Bigg] \tag{21}$$

$$P_e-P_\epsilon=\frac{3n_2^2Q}{(n_2+n_3)(4n_2-\alpha^2).m}\left[\left(-1+\frac{1}{(1-m)^2}\right)-\left(\frac{2\alpha^4}{5(4n_2-\alpha^2)}\right)^2\log\left(\frac{\frac{2Qn_2}{(n_2+n_3)}+\left(\frac{4n_2-\alpha^2}{3n_2}\right)(1-m^2)}{\frac{2Qn_2}{(n_2+n_3)}+\frac{(4n_2-\alpha^2)}{3n_2}}\right)\right]\Bigg] \tag{22}$$

The equation (21) gives pressure difference at any point in the channel and equation (22) gives the flow rate Q for different values of the pressure difference (P_e-P_E) between the entry and exit cross sections, and for different values of m. In the absence of tapeness it is obvious from the equation (21), a constant pressure gradient acts throughout the straight channel.

Heat Transfer

Putting the value of $\frac{\partial u}{\partial y}$, $\frac{\partial v}{\partial y}$ and ν in equation (10) and integrating twice and using Boundary conditions (13), we get

$$\theta=\frac{-P_rE_c(n_2+n_3)}{(2n_2)^2}\left(\frac{-\partial p}{\partial x}\right)^2\left[\frac{(2n_2+n_3)\alpha^2}{4n_2^2}\left(\frac{1}{8\alpha^2}(\cosh 2\alpha y-\cosh 2\alpha h)-\frac{1}{4}(y^2-h^2)\right)+\right.$$

$$\frac{1}{3}(2n_2+n_3)(y^4-h^4)-\frac{2\alpha}{n_2}(2n_2+n_3)\left(\frac{1}{\alpha^2}(y\sinh\alpha y-h\sinh\alpha h)-\frac{2}{\alpha^3}(\cosh\alpha y-\cosh\alpha h)\right)$$

$$+\frac{1}{2}\left(y^2-h^2\right)-\frac{2}{\alpha^2}\left(\frac{\cosh \alpha y}{\cosh \alpha h}-1\right)\Bigg] \tag{23}$$

Results and discussion

We have taken the constant pressure gradient throughout the numerical analysis, because constant pressure gradient is one of the possible cases (as regards the conservation of mass, momentum and energy) under which the problem can be solved and can be verified by the experimental observations (Wirz and Smolderen (1978).

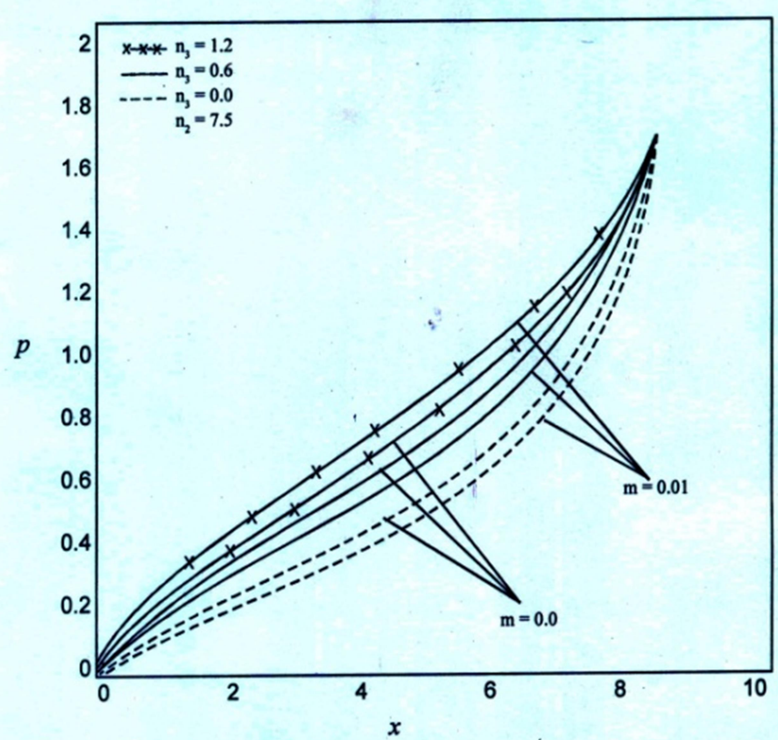

Fig. 8- Plot of pressure different verous axial distance for different value of m and n_3

Fig-8 depicts the variation of pressure difference with axial distance for different value of coupling constant, and tapered angle and fixed value of viscosity coefficient (n_2 = 7.5). It is obvious from the figure that the pressure difference at any point of the channel is more in case of micropolar fluid as compared to corresponding viscous fluid (i.e. n_3 = 0). Also pressure difference at any point increases with the increase of coupling constant and axial distance but decrease with tapered angle.

From above analysis it is concluded that axial velocity, microrotational velocity, volume flow rate, pressure difference and temperature are highly influence by assumption of taperness. We have also observed from the figures that the coupling constant has more influence on above characteristics than usual viscosity coefficient.

Fig-1 The variation of axial velocity profile with coupling constant (n_3), usual velocity coefficient (n_2) and tapeness (m) for fixed position at x = .60 of the channel have been shown in fig-1. It is found that for a micropolar fluid model of blood, axial velocity increases with increase in n_3 for fixed tapered angle and decreases with m for fixed value of coupling constant. It is also clear from the figure that axial velocity decreases with increasing the usual viscosity coefficient.

Fig-2 depicts the variation of micro-rotational velocity with m, n_2 and n_3 for fixed axial position x = .60. We conclude that an increase in n_3 and leads to an increase in the micro-rotational velocity and the micro-rotational velocity increases, with decreasing values of m and n_2. It is seen that the qualitative behaviour of micro-rotational velocity is same as axial velocity.

Fig-3&4 show the variation of axial velocity and micro-rotational velocity with axial distance for different values of n_3, n_2 and m for fixed radial position. y = 0.75. It is clear from the figure that the axial velocity and micro-rotational velocity increases with axial distance and coupling constant, but both velocities decrease with increasing value of usual viscosity coefficient and tapered angle.

Fig-5 shows the variation of volume flow rate with axial distance for different values of coupling constant, viscosity coefficient and tapered angle. It is found that an increase in coupling constant leads to increase in the volume flow rate. But increase in viscosity coefficient tapered angle leads to decrease in volume flow rate.

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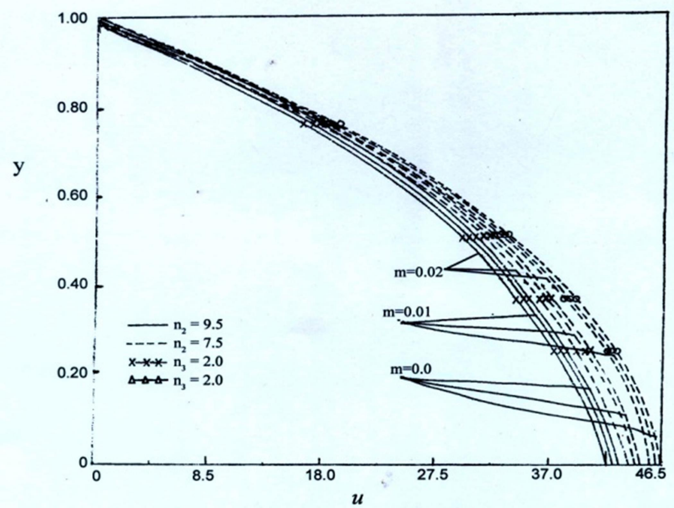

Fig. 1- Effect of m, n_2 and n_3 on velocity profile at the axcal posting x = 0.60

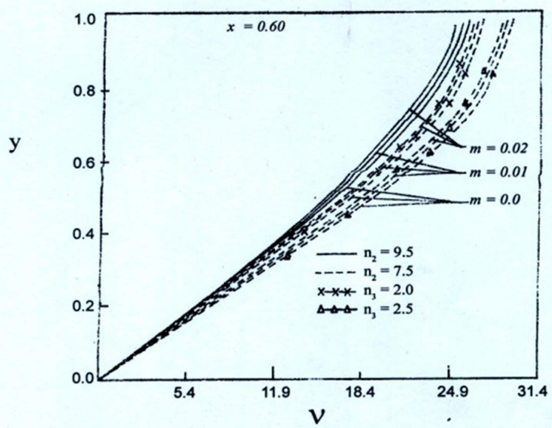

Fig. 2- Effect of m, n_2 and n_3 on micro-rotational velocity profile

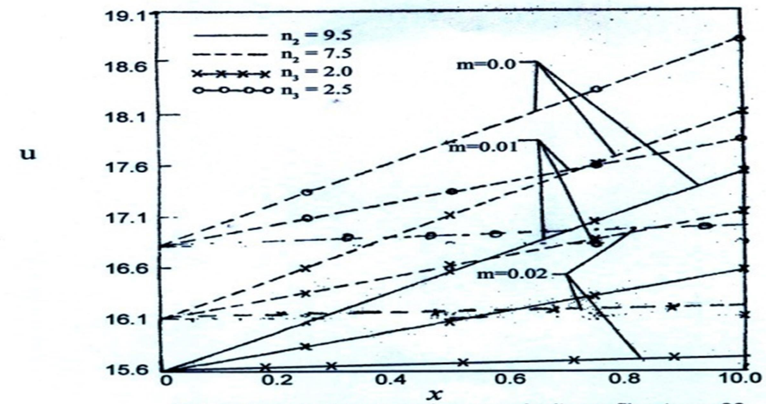

Fig. 3- Effect of m, n_2 and n_3 on velocity profile at y = .60

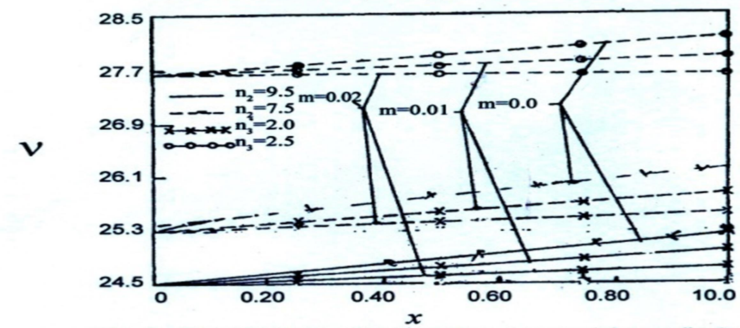

Fig. 4- Plot of micro-rotational velocity versus axial co-ordinate at y = 0.60

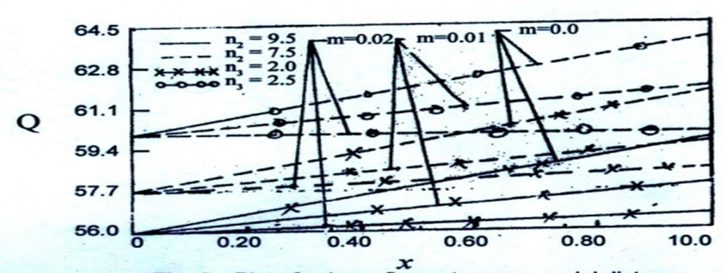

Fig. 5- Plot of volume flow rate versus axial distance

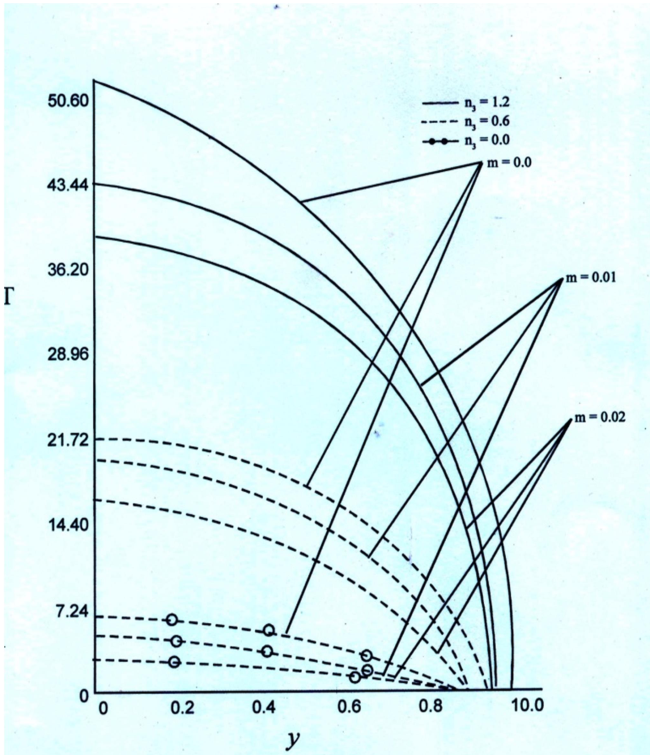

Fig. 6- Effect of m, and n_3 on temperature at x = 0.60

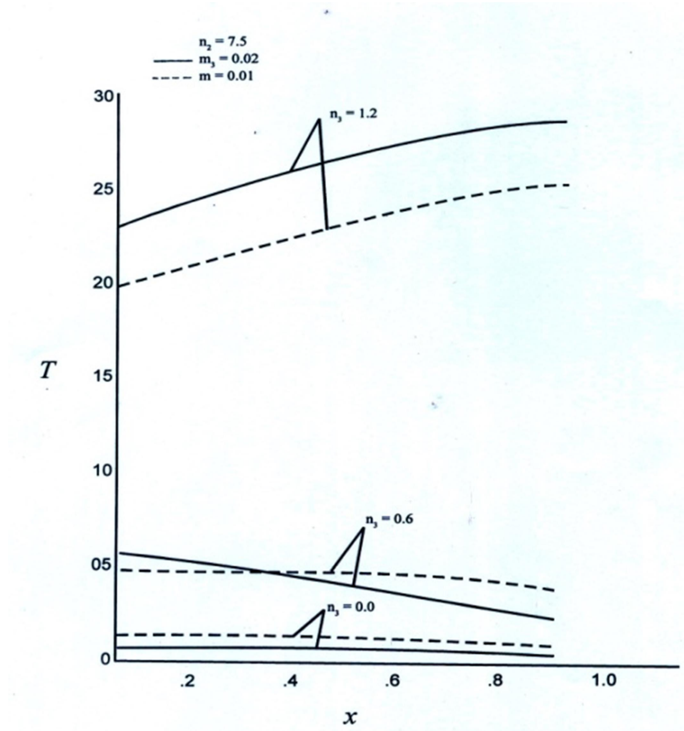

Fig. 7- Plot of temperature with axial distance for different values of m and n_3

Fig-6 depicts the variation of Temperature distribution for different value of coupling constant, viscosity coefficient and tapered angle for fixed value of x = 0.60. We conclude that temperature decreases with increasing the tapered angle and viscosity coefficient. But increases with increasing coupling constant for fixed values of other parameters.

Fig-7 shows the variation of temperature with axial distance for different value of other parameter. It is clear from the figure that the temperature first increases with taperness but later on decreases with increasing taperness for fixed value of n_3 = 0.6. But for n_3 = 1.2 the temperature increases throughout with axial distance for increasing value of taperness. Temperature also increases with coupling constant. From figure we can conclude that the temperature is larger in micropolar fluid than the corresponding viscous fluid.