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A study of quasi-concircular curvature tensor on a Riemannian manifold

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Abstract

In this paper we studied a quasi-concircular curvature tensor in a Riemannian manifold which generalizes the concircular curvature tensor. We start by deducing some fundamental geometric properties of quasi-concircualr curvature tensor $(QCCT)$. After that, we study quasi-concircualr symmetric manifolds.

Keywords and phrases- Concircular curvature tensor, quasi-concircular curvature tensor, pseudo quasi-concircualar symmetric manifolds.

1.Introduction

Curvature tensor plays a crucial role in the development of differential geometry and physics. According to Chern (1990) "A fundamental notion is the curvature, in its different forms". Therefore, finding the curvature tensors are very interesting topics for the workers. Therefore, in this paper we studied a $(QCCT)$ which generalizes the some known curvature tensors.

Investing conformally flat Riemannian manifolds of dimension of class one, Sen and Chaki (1967) found that the curvature tensor R of type (0,4) satisfies

$$R_{hijk,l} = 2a_l R_{hij} + a_h R_{lijk} + a_i R_{hljk} + a_j R_{hilk} + a_k R_{hijl},$$

where "comma" denotes the covariant derivative with respect to metric and R_{hij} are the components of the curvature tensor R of the type (0,4). Here after Chaki (1987) and Chaki and De (1989) examine the Riemannian manifold with the above condition. The first author called such a manifolds pseudo symmetric, since locally symmetric manifold satisfies the above condition with $a_l = 0$.

The above expression in index free notation can be written as

$$(D_X R)(Y,Z,U,V) = 2A(X)R(Y,Z,U,V) + A(Y)R(X,Z,U,V) +$$
$$A(Z)R(Y,X,U,V) + A(U)R(Y,Z,X,V) + A(V)R(Y,Z,U,X),$$

where A is a non-zero 1-form, ρ is a vector field defined by $g(X,\rho) = A(X)$ for all X, D denotes the operator of covariant differentiation with respect to metric g and $R(Y,Z,U,V) = (\mathcal{R}(Y,Z)U,V)$, where $\mathcal{R}$ is the curvature tensor of the type (1,3). The 1-form A is called the associated 1-form of the manifold. If $A = 0$, then the manifold becomes locally symmetric manifold in the sence of Cartan. An n-dimensional pseudo symmetric manifold is denoted by $(PS)_n$. Pseudo symmetric manifolds have been studied by several authors such as Mantica and Molinari (2012), Mantica and Suh (2013), Zengin and Tasci (2014), De and Majhi (2018) and many others.

A transformation of an n-dimensional Riemannian manifold M, which transforms every geodesic circle of M into a geodesic circle is called a concircular transformation Yano (1940). A concircular transformation is always a conformal transformation Yano (1940). Here geodesic circle means a curve in M whose first curvature is constant and second curvature is identically zero. Thus the geometry of concircular transformation that is the concircular geometry in the that the change of metric is more general than that induced by a circle preserving diffeomorphism Yano and Bochner (1953). An interesting invariant of a concircular transformation is the concircular curvature tensor L with respect to the Levi-Civita connection. It is defined by Yano (1940)

$$L(X,Y)Z = \mathcal{R}(X,Y)Z - \frac{r}{n(n-1)}[g(Y,Z)X - g(X,Z)Y], \tag{1.1}$$

whereX, Y, Z are differentiable vector fields, $\mathcal{R}$ and r are curvature tensor and the scalar curvature tensor with respect to Levi-Civita connection respectively. A Riemannian manifold with vanishing concircular curvature tensor is of constant curvature. Thus, the concircular curvature tensor is the measure of the failure of a Riemannian manifold to be of constant curvature.

In 2007, Prasad and Maurya, defined quasi-concircualr curvature tensor $(QCCT)$ by the expression

$$\tilde{L}(X,Y)Z = a\mathcal{R}(X,Y)Z + \frac{r}{n}\left(\frac{a}{n-1} + 2b\right)[g(Y,Z)X - g(X,Z)Y], \tag{1.2}$$

where$a$ and b are constant such that $a, b \neq 0$. In particular, if$= 1$, $b = -\frac{1}{n-1}$, then the equation (1.2) reduces in concircualr curvature tensor. Hence$L(X,Y)Z$ is a particular case of $\tilde{L}(X,Y)Z$. This justifies their nomenclature.

From equation (1.2), we can write an expression of the type (0,4) as follows:

$$'\tilde{L}(X,Y,Z,W) = a.R(X,Y,Z,W) + \frac{r}{n}\left(\frac{a}{n-1} + 2b\right)[g(Y,Z)g(X,W) - g(X,Z)g(Y,W)], \tag{1.3}$$

where$g(\tilde{L}(X,Y)Z,W) = {'\tilde{L}}(X,Y,Z,W)$. Several authors studied $(QCCT)$ in different way such as (Narain et.al 2009), (Kumar et. al, 2009), (Ahmad et.al, 2019) and many others.

The $(QCCT)$ satisfying the following algebraic properties:

$$\left.\begin{aligned} &'\tilde{L}(X,Y,Z,W) + {'\tilde{L}}(Y,X,Z,W) = 0,\\ &'\tilde{L}(X,Y,Z,W) + {'\tilde{L}}(X,Y,W,Z) = 0,\\ &'\tilde{L}(X,Y,Z,W) - {'\tilde{L}}(Z,W,X,Y) = 0,\\ &'\tilde{L}(X,Y,Z,W) + {'\tilde{L}}(Y,Z,X,W) + {'\tilde{L}}(Z,X,Y,W) = 0. \end{aligned}\right\} \tag{1.4}$$

Let $\{e_i\}$ be an orthonormal basis of the tangent space at each point of the manifold where $1 \leq i \leq n$.

Now, from (1.3), we have

$$\sum_{i=1}^{n} {'\tilde{L}}(X,Y,e_i,e_i) = \sum_{i=1}^{n} {'\tilde{L}}(e_i,e_i,Z,W) = 0,$$

$$\sum_{i=1}^{n} {'\tilde{L}}(e_i,Y,Z,e_i) = aRic(Y,Z) + \frac{r}{n}[a + 2(n-1)b]g(Y,Z) = L_1(Y,Z). \tag{1.5}$$

A non-flat Riemannian manifold $(M^n, g), (n > 2)$, is said to be a pseudo quasi-concircular symmetric manifold if the quasi-concircular curvature tensor $(QCCT)$ $'\tilde{L}$ of type (0,4) satisfies the condition:

$$\left(D_X{}'\tilde{L}\right)(Y,Z,U,V) = 2A(X){}'\tilde{L}(Y,Z,U,V) + A(U){}'\tilde{L}(X,Z,U,V) + A(Z){}'\tilde{L}(Y,X,U,V) + A(U){}'\tilde{L}(Y,Z,X,V) + A(V){}'\tilde{L}(Y,Z,U,X), \tag{1.6}$$

where$A$ is a non-zero 1-form, ρ is a vector field defined by $g(X,\rho) = A(X)$.

An n-dimensional pseudo quasi-concircular symmetric manifold is denoted by $(PQ'\tilde{L}S)$, where P stands for pseudo, $'\tilde{L}$ stands for quasi-concircular and S stands for symmetric. Moreover, if $a = 1$and $b = -\frac{1}{n-1}$, then $(Q'\tilde{L}S)$ manifolds includes (QLS) introduced by (De and Tarafdar, 1992).

2. Some Properties of $(QCCT), n > 3$

Let $(QCCT)$ be flat (1.3) and (2.1), we get

$$'\tilde{L}(X,Y,U,V) = 0. \tag{2.1}$$

Hence, in view of (1.3) and (2.1), we get

$$a.R(X,Y,U,V) = -\frac{r}{n}\left(\frac{a}{n-1} + 2b\right)[g(Y,U)g(X,V) - g(X,U)g(Y,V)\,]. \tag{2.2}$$

Taking a frame field and contracting Y and U in (2.2), we have

$$a.Ric(X,V) = -\frac{r}{n}\left(\frac{a}{n-1} + 2b\right)(n-1)g(X,V). \tag{2.3}$$

Again, contracting X and V in (2.3), we get

$$r[a + (n-1)b] = 0. \tag{2.4}$$

Hence, from (2.4), we can state the following theorem:

Theorem (2.1): If the $(QCCT)$ is flat, then then the scalar curvature vanishes, provided $a + (n-1)b \neq 0$.

Let $(QCCT), (n > 2)$ be symmetric. That is,

$$\left(D_X\tilde{L}\right)(Y,Z)U = 0. \tag{2.5}$$

Now, differentiating (1.2) covariantly, we get

$$\left(D_X\tilde{L}\right)(Y,Z)U = a(D_X\mathcal{R})(Y,Z)U + \frac{D_X r}{n}\left(\frac{a}{n-1} + 2b\right)[g(Z,U)Y - g(Y,U)Z]. \tag{2.6}$$

According to our assumption, we have from (2.5) and (2.6),

$$a(D_X\mathcal{R})(Y,Z)U + \frac{D_X r}{n}\left(\frac{a}{n-1} + 2b\right)[g(Z,U)Y - g(Y,U)Z] = 0. \tag{2.7}$$

Contracting (2.7) with respect to Y, we get

$$a(D_X Ric)(Z,U) = -\frac{D_X r}{n}\ \ [a + 2(n-1)b]g(Z,U). \tag{2.8}$$

Again, contracting Z and U in (2.8), we get

$$dr(X) = 0, \quad a + (n-1)b \neq 0. \tag{2.9}$$

Conversely, if$dr(X) = 0$. Then, from (2.8), we get

$$(D_X Ric)(Z,U) = 0, \text{ provided } a \neq 0. \tag{2.10}$$

Hence, in view of (2.9) and (2.10), we get

Theorem (2.2): If the $(QCCT), (n > 2)$ is symmetric, then then the manifold is Ricci symmetric if and only if $dr(X) = 0$, provided $a + (n-1)b \neq 0$.

Now, contracting (2.6) with respect to X, we get

$$\left(div\tilde{L}\right)(Y,Z)U = a(div\mathcal{R})(Y,Z)U + \frac{1}{n}\left(\frac{a}{n-1} + 2b\right)[g(Z,U)dr(Y) - g(Y,U)dr(Z)]. \tag{2.11}$$

But we know that

$$(divL)(Y,Z)U = (div\mathcal{R})(Y,Z)U - \frac{1}{n(n-1)}[g(Z,U)dr(Y) - g(Y,U)dr(Z)]. \tag{2.12}$$

Above equation (2.12) can be put as

$$(div\mathcal{R})(Y,Z)U = (divL)(Y,Z)U + \frac{1}{n(n-1)}[g(Z,U)dr(Y) - g(Y,U)dr(Z)]. \tag{2.13}$$

From (2.11) and (2.13), we get

$$\left(div\tilde{L}\right)(Y,Z)U = a(divL)(Y,Z)U + 2[a + 2(n-1)b][g(Z,U)dr(Y) - g(Y,U)dr(Z)]. \tag{2.14}$$

Hence in view of (2.14), we have the following theorem:

Theorem (2.3): For the $(QCCT), (n > 2)$, $div\tilde{L}$ is equal to $divL$ if and only if $dr(X) = 0$,provided $a + (n-1)b \neq 0$.

3. Bianchi's 2nd identity for $(PQ'\tilde{L}S), (n > 2)$

In this section, we prove that in a $(PQ'\tilde{L}S), (n > 2)$, the $(QCCT)\tilde{L}$ satisfies Bianchi's 2nd identity. That is

$$\left(D_X{}'\tilde{L}\right)(Y,Z,U,V) + \left(D_Y{}'\tilde{L}\right)(Z,X,U,V) + \left(D_Z{}'\tilde{L}\right)(X,Y,U,V) = 0. \tag{3.1}$$

In view of (1.3), (1.4) and (1.6), we get

$$\left(D_X{}'\tilde{L}\right)(Y,Z,U,V) + \left(D_Y{}'\tilde{L}\right)(Z,X,U,V) + \left(D_Z{}'\tilde{L}\right)(X,Y,U,V) = A(U)[{}'\tilde{L}(Y,Z,X,V) + {}'\tilde{L}(Z,X,Y,V) + {}'\tilde{L}(X,Y,Z,V)] + A(V)[{}'\tilde{L}(Y,Z,U,X) + {}'\tilde{L}(Z,X,U,Y) + {}'\tilde{L}(X,Y,U,Z)] \tag{3.2}$$

Using Bianch's 1st identity in (3.2), we get

$$\left(D_X{}'\tilde{L}\right)(Y,Z,U,V) + \left(D_Y{}'\tilde{L}\right)(Z,X,U,V) + \left(D_Z{}'\tilde{L}\right)(X,Y,U,V) = A(V)[g(Z,U)g(Y,X) - g(Y,U)g(Z,X) + g(X,U)g(Z,Y) - g(Z,U)g(X,Y) + g(Y,U)g(X,Z) - g(X,U)g(Y,Z)].\frac{1}{n}\left(\frac{a}{n-1} + 2b\right).$$

$$\Rightarrow \left(D_X{}'\tilde{L}\right)(Y,Z,U,V) + \left(D_Y{}'\tilde{L}\right)(Z,X,U,V) + \left(D_Z{}'\tilde{L}\right)(X,Y,U,V) = 0.$$

Thus, we can state the following theorem:

Theorem (3.1): The quasi-concircular curvature tensor $'\tilde{L}$ in $(PQ'\tilde{L}S), (n > 2)$ satisfies the Bianchi's 2nd identity.

Now, from (1.5), we get

$$L_1(Z,U) = aRic(Z,U) + \frac{r}{n}[a + 2(n-1)b]g(Z,U). \tag{3.3}$$

Contacting (3.3), we get

$$l_1 = 2r[a + (n-1)b]. \tag{3.4}$$

In $(PQ'\tilde{L}S), (n > 2)$, we have

$$(D_X{}'\tilde{L})(Y,Z,U,V) = 2A(X)'\tilde{L}(Y,Z,U,V) + A(X)'\tilde{L}(X,Z,U,V) + A(Z)'\tilde{L}(Y,X,U,V) + A(U)'\tilde{L}(Y,Z,X,V) + A(V)'\tilde{L}(Y,Z,U,X). \tag{3.5}$$

Contracting Y and V in (3.5), we get

$$(D_X L_1)(Z,U) = 2A(X)L_1(Z,U) + {}'\tilde{L}(X,Z,U,\rho) + A(Z)L_1(X,U) + A(U)L_1(Z,X) + {}'\tilde{L}(\rho,Z,U,X). \tag{3.6}$$

Again contracting Z and U in (3.6), we get

$$D_X l_1 = 2A(X)l_1 + 4L_1(X,\rho). \tag{3.7}$$

From (3.4), we get

$$D_X l_1 = 2[a + (n-1)b](D_X r). \tag{3.8}$$

Hence, in view of (3.3), (3.4), (3.7) and (3.8), we have

$$[a + (n-1)b]dr(X) = A(X).\,2r\left[\frac{a(n+1) + (n-1)(n+2)b}{n}\right] + 2Ric(X,\rho). \tag{3.9}$$

Thus, we can state the following theorem:

Theorem (3.1): In $(PQ'\tilde{L}S), (n > 2)$, we have

$$[a + (n-1)b]dr(X) = A(X).\,2r\left[\frac{a(n+1) + (n-1)(n+2)b}{n}\right] + 2Ric(X,\rho).$$

In particular, if$a = 1$ and $b = -\frac{1}{n-1}$, then from (3.9), we get

$$Ric(X,\rho) = \frac{r}{n}g(X,\rho). \tag{3.10}$$

Thus, we get, in a (PLS), $\frac{r}{n}$ is an eigen values of the Ricci tensor Ric and ρ is an eigen vector corresponding to this eigen value. Thus we find the De and Tarafdar (1997) result.

4. Einstein $(PQ'\tilde{L}S), (n > 2)$

In this section, we consider Einstein$(PQ'\tilde{L}S), (n > 2)$. Since for every Einstein manifold the scalar curvature r is constant, hence for Einstein $(PQ'\tilde{L}S), (n > 2)$, we have $dr(X) = 0$. Hence from (3.9), we get

$$A(X).\,r\left[\frac{a(n+1) + (n-1)(n+2)b}{n}\right] + Ric(X,\rho) = 0. \tag{4.1}$$

For an Einstein manifold (M^n, g), we have

$$Ric(X,Y) = \frac{r}{n}g(X,Y). \tag{4.2}$$

Hence in view of (4.1) and (4.2), we get

$$A(X).r\left[a+(n-1)b+\frac{a+2(n-1)b+1}{n}\right]=0.$$

If $a \neq 0,\ b \neq 0$and$A(X) \neq 0$, then $r = 0$.

Therefore, we can state the following theorem:

Theorem (4.1): An Einstein $(PQ'\tilde{L}S), (n > 2)$ is of zero scalar curvature provided $a \neq 0,\ b \neq 0$.

If possible, let $(PQ'\tilde{L}S), (n > 2)$ be a space of constant curvature. Then we have

$$\mathcal{R}(X,Y)Z = k[g(Y,Z) - g(X,Z)Y]\ , \tag{4.3}$$

where$k$ is constant. Contracting X in (4.3), we get

$$Ric(Y,Z) = k(n-1)g(Y,Z). \tag{4.4}$$

Again, contracting Y and Z in (4.4), we get

$$r = kn(n-1). \tag{4.5}$$

Using (4.5) in (4.3), we get

$$\mathcal{R}(X,Y)Z = \frac{r}{n(n-1)}[g(Y,Z) - g(X,Z)Y]. \tag{4.6}$$

Since every space of constant curvature is an Einstein manifold, then from Theorem (4.1) we have $r = 0$. Hence from (4.6) it follows that $\mathcal{R}(X,Y)Z = 0$, which is admissible by definitions. This leads to following Corollary of the above Theorem under condition $a, b \neq 0$ as:

Corollary (4.2): A $(PQ'\tilde{L}S), (n > 2)$ can not be of constant curvature.

Again, from (1.2), we get

$$\tilde{L}(X,Y)\rho\ =\ a\mathcal{R}(X,Y)\rho + \frac{r}{n}\left(\frac{a}{n-1} + 2b\right)[g(Y,\rho)X - g(X,\rho)Y\ ]. \tag{4.7}$$

First we assume that $r = 0$in $(PQ'\tilde{L}S), (n > 2)$. Then from (4.7), we get

$$\tilde{L}(X,Y)\rho\ =\ a\mathcal{R}(X,Y)\rho. \tag{4.8}$$

Next, we assume that in $(PQ'\tilde{L}S), (n > 2)$, the relation (4.6) holds. Then from (4.7), we get

$$r[a + 2(n-1)b][A(Y)X - A(X)Y = 0]. \tag{4.9}$$

Hence, equation (4.9) gives

$r = 0$, provided $a + 2(n-1)b \neq 0,\ A(X) \neq 0$.

This leads to the following theorem:

Theorem (4.3): A $(PQ'\tilde{L}S), (n > 2)$, is of zero scalar curvature if and only if $\tilde{L}(X,Y)\rho\ =\ a\mathcal{R}(X,Y)\rho$ provided provided $a + 2(n-1)b \neq 0$.

5. $(PQ'\tilde{L}S), (n > 2)$with $divL = 0$

For $(PQ'\tilde{L}S), (n > 2)$, we have

$$(D_X{}'\tilde{L})(Y,Z,U,V) = 2A(X)'\tilde{L}(Y,Z,U,V) + A(Y)'\tilde{L}(X,Z,U,V) + A(Z)'\tilde{L}(Y,X,U,V)$$

$$+ A(U)'\tilde{L}(Y,Z,X,V) + A(V)'\tilde{L}(Y,Z,U,X). \quad (5.1)$$

Equation (5.1) can be written as

$$g\left((D_X\tilde{L})(Y,Z)U,V\right) = 2A(X)g\left(\tilde{L}(Y,Z)U,V\right) + A(Y)g\left(\tilde{L}(X,Z)U,V\right) +$$

$$A(Z)g\left(\tilde{L}(Y,X)U,V\right) + A(U)g\left(\tilde{L}(Y,Z)X,V\right) + g(V,\rho)g\left(\tilde{L}(Y,Z)U,X\right). \quad (5.2)$$

where$A$ is non-zero 1-form, ρ is a vector field defined by $g(X,\rho) = A(X)$.

Put $X = V = e_i$ in (5.2), we get

$$\left(div\tilde{L}\right)(Y,Z)U = \sum_{i=1}^{n} g\left((D_{e_i}\tilde{L})(Y,Z),e_i\right) = \sum_{i=1}^{n}\{2\,A(e_i)g\left(\tilde{L}(Y,Z)U,e_i\right) +$$

$$A(Y)g\left(\tilde{L}(e_i,Z)U,e_i\right) + A(Z)g\left(\tilde{L}(Y,e_i)U,e_i\right) +$$

$$A(U)g\left(\tilde{L}(Y,Z)e_i,e_i\right) + g(e_i,\rho)g\left(\tilde{L}(Y,Z)U,e_i\right)\},$$

$$\Rightarrow (divL)(Y,Z)U = 3A\left(\tilde{L}(Y,Z)U\right) + A(Y)L_1(Z,U) - A(Z)L_1(Y,U) \quad (5.3)$$

Hence, we assume that

$$\left(div\tilde{L}\right)(Y,Z)U = 0. \quad (5.4)$$

Hence from (5.3) and (5.4), we get

$$3A\left(\tilde{L}(Y,Z)U\right) + A(Y)L_1(Z,U) - A(Z)L_1(Y,U) = 0. \quad (5.5)$$

Contracting with respect to $Z$and U, we get

$$L_1(Y,\rho) + r.A(Y)[a + (n-1)b] = 0. \quad (5.6)$$

Hence, from (1.5) and (5.6), we get

$$Ric(X,\rho) = -\frac{r[a(n+1)+(n-1)(n+1)b]}{na}g(X,\rho). \quad (5.7)$$

Hence from (5.7), we have

$$Ric(X,\rho) = \lambda g(X,\rho),$$

where$\lambda = -\frac{r[a(n+1)+(n-1)(n+1)b]}{na}$ is a scalar. Hence in view of the above results we are a position to state the following theorem:

Theorem (5.1): For $(PQ'\tilde{L}S)$, $(n > 2)$with $div\tilde{L} = 0$, λ is an eigen values of the Ricci tensor Ric corresponding to the eigen vector ρ.

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