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Fuzzy relational matrix model applied to optimize the production in BHEL

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Abstract

BHEL a public sector undertaking is the largest engineering and manufacturing enterprise in India in the energy related / infrastructure sector. In the present paper using the data from BHEL, we analyze the same via a fuzzy relational matrix and obtain the best relationship between employee and employer in BHEL. Hence using these suggestions got from the study, BHEL and other industries can maximize their production. For we have applied the technique of FRM introduced by W.B.V. Kandasamy with slight modifications Bawa(2009), Taber(1994) and Kandasamy et al.(2007) .

Introduction

Let us give some basic information about BHEL. The full form of BHEL is Bharat Heavy Electricals Limited. It was set up at Bhopal under the name Heavy Electrical (India) in collaboration with AEI, U.K. Currently, three more plants have been set up at Hyderabad, Haridwar and Trichy. BHEL manufactures over 180 products under 30 major product groups and caters sectors like power generation and transmission, industry, transportation, telecommunication, renewable energy, etc. BHEL has 14 manufacturing divisions, 4 power sector regional centers, more than 100 project sites, 8 service centers and 18 regional offices on 31 Dec 2008. BHEL order book stands of Rs. 1,13,500 cr. Order book grew 9% on sequential basis and 46% on yearly basis. BHEL expects in order book swelling to Rs. 1,20,000 cr by FY 09 end. Following are year wise Networth:

Y_1: March, 2005 Rs. 6026.89 cr
Y_2: March, 2006 Rs. 7301.38 cr
Y_3: March, 2007 Rs. 8788.26 cr
Y_4: March, 2008 Rs. 10,774.2 cr
Let Y be a set consisting Y_1 to Y_4, that is, $Y = \{Y_1, Y_2, Y_3, Y_4\}$

The company BHEL provides following attributes related to the Employee.

X_1 – Salaries and wages
X_2 – Bonus
X_3 – Provident fund
X_4 – Medical
X_5 – Leave traveling allowances
X_6 – Employee other welfares
X_7 – Staff training expense
X_8 – Loan facilities.

Let us call these as set X, that is X = set of attributes related to the Employee from the above it is clear that when the company provides attributes given in X, employees with a pleasing mood work hard with increase in production and the result is clear is the set Y.

The relational map's shown in the following figure:

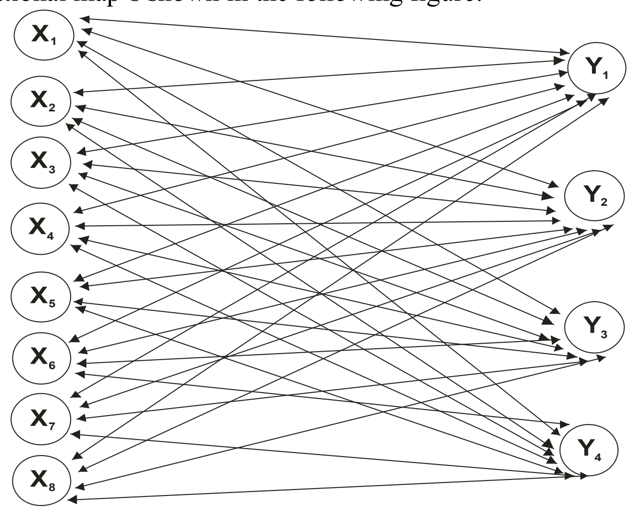


Fig. (1)

From the figure fig (1) let us construct the initial 4×8 relational matrix from the relational map using weights. The weights of each edge is given any positive number, this number is adjusted by on past experience and by dialogues with employees and employer. Let X_1 and Y_1denote the two nodes of the relational map. The nodes X_j and Y_i are taken as the attributes of the employee and employer respectively. The directed edge from X_i to Y_j and Y_j to X_i denote the causality relations. Every edge in the relational map is weighted with a positive real number.

	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8
Y_1	70	68	15	5	3	4	10	5
Y_2	75	80	12	7	2	5	12	5
Y_3	77	85	14	5	3.5	3	15	7
Y_4	80	83	18	6	4.5	3	14	8

Let us convert the relational matrix into average matrix which is given by $[a_{ij}]$; it will simplify our calculation.

$$\left[a_{ij}\right]_{4\times 8} = \begin{bmatrix} 35 & 34 & 7.5 & 2.5 & 1.5 & 2 & 5 & 2.5 \\ 37.5 & 40 & 6.0 & 3.5 & 1.0 & 2.5 & 6 & 2.5 \\ 38.5 & 42.5 & 7.0 & 2.5 & 1.75 & 1.5 & 7.5 & 3.5 \\ 40 & 41.5 & 9.0 & 3 & 2.25 & 1.5 & 7 & 4 \end{bmatrix}$$

To find a best form of relationship between employee and employer, let us use mean, mean deviation and the parameter $\lambda \in [0, 1]$.

FORMATION OF FRM (I.E FUZZY RELATIONAL MATRIX)

Let, μ_j, M_j be the mean and mean deviation of j^{th} column for some $\square \in [0,1]$

The matrix $[a_{ij}]$ will be converted to $[b_{ij}]$

As

$$b_{ij} \in \left[0, \frac{a_{ij} - \mu_j + \alpha * M_j}{\left(\mu_j + \alpha * M_j\right) - \left(\mu_j - \alpha * M_j\right)}, 1 \right] \quad \ldots\ldots(A)$$

where i = 1, 2, 3, 4, and j runs from 1, 2, 3, 4, 5, 6, 7, 8 b_{ij} is the $(i,j)^{th}$ element of the converted matrix.

$b_{ij}=0$ if $a_{ij} < \mu_j - \alpha^* M_j$

$$b_{ij} = \frac{a_{ij} - \mu_j + \alpha * M_j}{\left(\mu_j + \alpha * M_j\right) - \left(\mu_j - \alpha * M_j\right)} \text{ if } \mu_j - \alpha^* M_j \le a_{ij} < \mu_j + \alpha * M_j$$

$b_{ij} = 1$ if $a_{ij} \ge \mu_j + \alpha^* M_j$

$\alpha = 0.1$; then

Our average matrix is given by

$$\begin{bmatrix} 35 & 34 & 7.5 & 2.5 & 1.5 & 2 & 5 & 2.5 \\ 37.5 & 40 & 6.0 & 3.5 & 1.0 & 2.5 & 5 & 2.5 \\ 38.5 & 42.5 & 7.0 & 2.5 & 1.75 & 1.5 & 7.5 & 3.5 \\ 40 & 41.5 & 9.0 & 3 & 2.25 & 1.5 & 7 & 4 \end{bmatrix}$$

Let us calculate average of each column and mean deviation for α=0.1:

μ_j	37.7	39.500	7.400	2.900	1.62	1.870	6.370	3.120
M_j	1.5	7.5	.88	.38	.38	.38	.87	.62
$\alpha * M$	.015	.075	.088	.038	.038	.038	.087	.062
$\mu - \alpha * M$	36.685	39.425	7.312	2.862	1.582	1.832	6.283	3.058
$\mu + \alpha * M$	37.715	39.575	7.488	2.938	1.658	1.908	6.457	3.182

$$FRM = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ .78 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$

The corresponding row sum matrix is given by:

R_1 = The first row sum = 2,

R_2 = The second row sum = 3.78,

R_3 = Third row sum = 5,

R_4 = fourth row sum = 7,

Min $[R_1, R_2, R_3, R_4]$ = 2,

Max $[R_1, R_2, R_3, R_4]$ = 7,

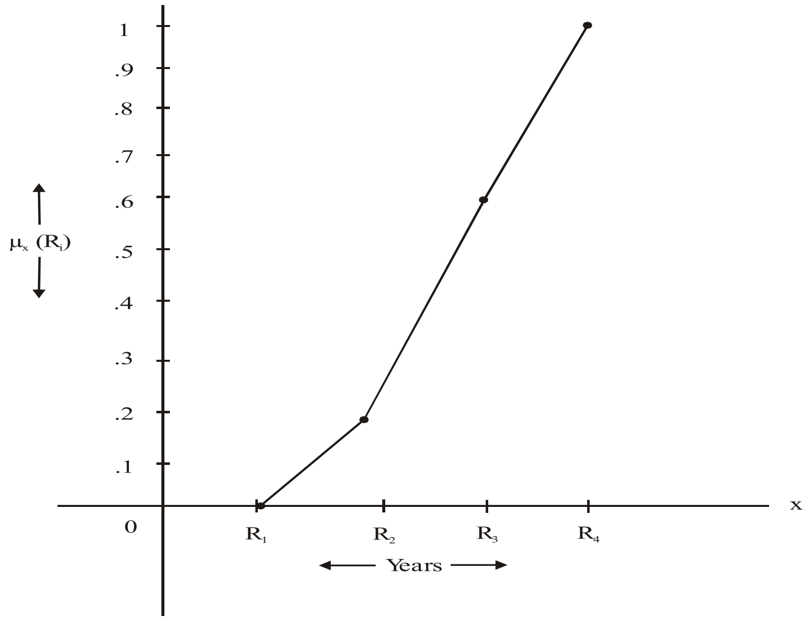
where R_i are nothing but the corresponding years of Y_i.

Let us now define the new fuzzy membership function for graphical illustration which will convert the row sum to take sum values in the interval [0,1]. Let us define

$$\mu_x\left(R_i\right) = \frac{R_i - Row\ sum\ of\ Min.\ value}{Row\ sum\ of\ Max.\ values - Row\ sum\ of\ Min.\ value} \quad \text{….(B)}$$

$\mu_x(R_1)=0$, $\mu_x(R_2)=.176$, $\mu_x(R_3)=.6$, $\mu_x(R_4)=1$

Graphical illustration for $\alpha = .1$



Graph-1

Here the fourth row sum R_4 is getting the highest membership grade i.e. 1 and next highest membership grade is .6 for third row sum R_3.

For □ = .3, we have

$$\begin{bmatrix} 35 & 34 & 7.5 & 2.5 & 1.5 & 2 & 5 & 2.5 \\ 37.5 & 40 & 6.0 & 3.5 & 1.0 & 2.5 & 5 & 2.5 \\ 38.5 & 42.5 & 7.0 & 2.5 & 1.75 & 1.5 & 7.5 & 3.5 \\ 40 & 41.5 & 9.0 & 3 & 2.25 & 1.5 & 7 & 4 \end{bmatrix}$$

μ_j	37.700	39.500	7.400	2.900	1.620	1.870	6.370	3.120
M_j	1.5	2.5	.88	.38	.38	.38	.87	.62
□*M_j	.450	.750	.264	.114	.114	.114	.261	.186
μ_j-□□*M_j	37.250	38.750	7.136	2.786	1.506	1.756	6.109	2.934
μ_j+□□*M_j	38.150	40.250	7.664	3.014	1.734	1.984	6.631	3.306

According to (A), FRM is given by

$$\begin{bmatrix} 0 & 0 & .68 & 0 & 0 & 1 & 0 & 0 \\ .27 & .83 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & .93 & 1 & 0 & 1 & 1 \end{bmatrix}$$

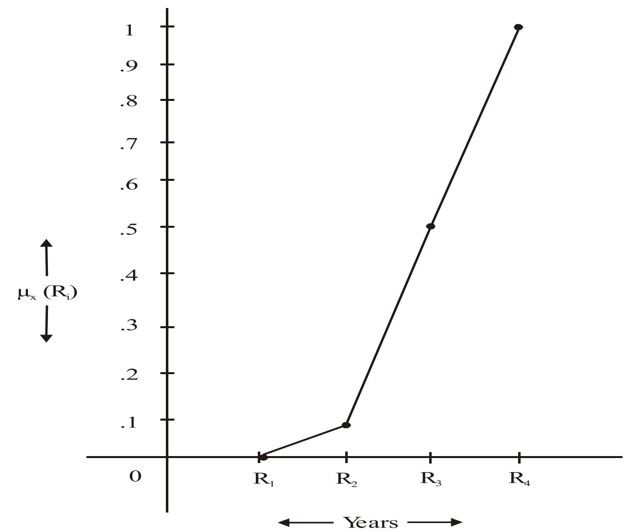
Corresponding row sum matrix is given by

$R_1 = 2.60$
$R_2 = 3.10$
$R_3 = 5.00$
$R_4 = 6.93$
Min of $[R_1, R_2, R_3, R_4] = 2.60$;
Max of $[R_1, R_2, R_3, R_4] = 6.93$

Now from fuzzy membership function formula given by (B), we have

$$\mu_x(R_1) = 0,\ \mu_x(R_2) = .11,\ \mu_x(R_3) = 0.55,\ \mu_x(R_4) = 1$$

Graphical illustration for □ = .3



Graph-2

Here we see that membership grade for R_4 is 1 and next highest membership grade is .55, got by R_3.

For □ = .5, we have

$$\begin{bmatrix} 35 & 34 & 7.5 & 2.5 & 1.5 & 2 & 5 & 2.5 \\ 37.5 & 40 & 6.0 & 3.5 & 1.0 & 2.5 & 6 & 2.5 \\ 38.5 & 42.5 & 7.0 & 2.5 & 1.75 & 1.5 & 7.5 & 3.5 \\ 40 & 41.5 & 9.0 & 3 & 2.25 & 1.5 & 7 & 4 \end{bmatrix}$$

μ_j	37.700	39.500	7.400	2.900	1.620	1.870	6.370	3.120
M_j	1.5	2.5	.88	.38	.38	.38	.87	.62
□*M_j	.750	1.250	.420	.190	.190	.190	.435	.310
μ_j-□□*M_j	36.950	38.250	6.980	2.710	1.430	1.680	5.935	2.810
μ_j+□□*M_j	38.450	40.750	7.820	3.090	1.810	2.060	6.805	3.430

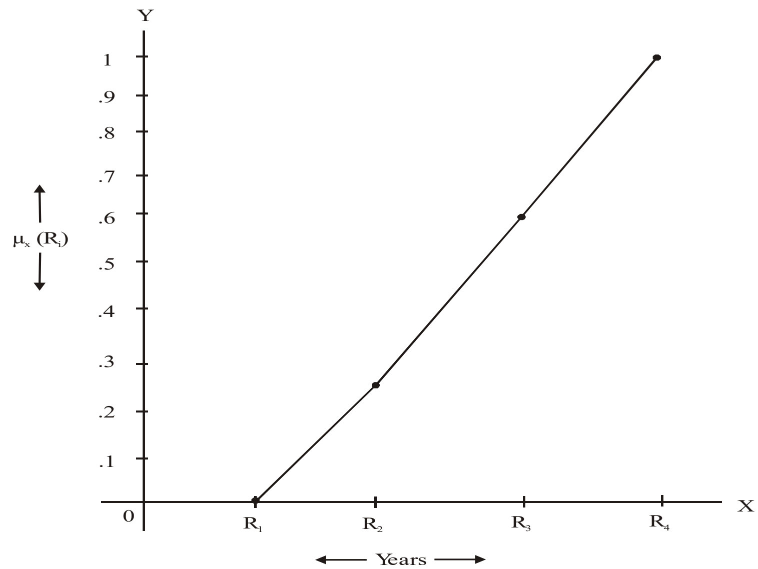
Corresponding fuzzy relational matrix is given by:

$$\begin{bmatrix} 0 & 0 & .61 & 0 & .36 & .84 & 0 & 0 \\ .36 & .70 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & .84 & 0 & 1 & 1 \\ 1 & 1 & 1 & .76 & 1 & 0 & 1 & 1 \end{bmatrix}$$

Corresponding row sum matrix is given by

$R_1 = 1.81$
$R_2 = 3.06$
$R_3 = 4.84$
$R_4 = 6.76$
Min of $[R_1, R_2, R_3, R_4] = 1.81$
Max of $[R_1, R_2, R_3, R_4] = 6.76$

Fuzzy membership functions are given by $\mu_x (R_1) = 0$, $\mu_x (R_2) = .25$, $\mu_x (R_3) = 0.61$, $\mu_x (R_4) = 1$



Graph-3

Hence R_4 is getting the highest membership grade 1. Next highest grade is attained by R_3 and that is .61

For □ = .7, we have

$$\begin{bmatrix} 35 & 34 & 7.5 & 2.5 & 1.5 & 2 & 5 & 2.5 \\ 37.5 & 40 & 6.0 & 3.5 & 1.0 & 2.5 & 6 & 2.5 \\ 38.5 & 42.5 & 7.0 & 2.5 & 1.75 & 1.5 & 7.5 & 3.5 \\ 40 & 41.5 & 9.0 & 3 & 2.25 & 1.5 & 7 & 4 \end{bmatrix}$$

μ_j	Mij	37.700	39.500	7.400	2.900	1.620	1.870	6.370	3.120
M_j	Mj	1.5	2.5	.88	.38	.38	.38	.87	.62
□*M_j	λMj	1.050	1.750	.616	.266	.266	.266	.609	.434
μ_j-□□*M_j	Mj-λMj	36.650	37.750	6.784	2.634	1.354	1.604	5.761	2.686
μ_j+□□*M_j	Mj+λMj	38.750	41.250	8.016	3.166	2.886	2.136	6.979	3.554

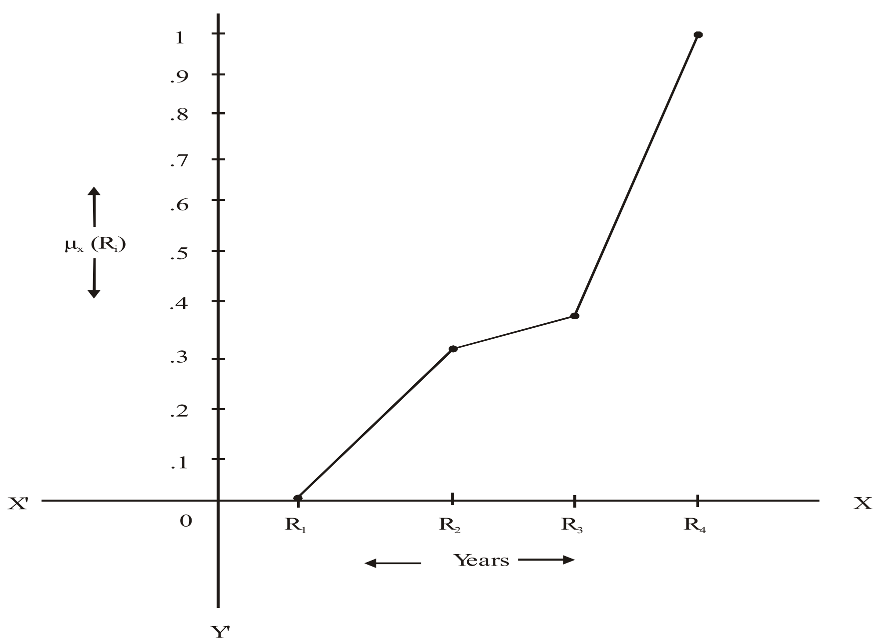
Corresponding FRM is given by

$$\begin{bmatrix} 0 & 0 & .57 & 0 & .09 & .74 & 0 & 0 \\ .40 & .64 & 0 & 1 & 0 & 1 & 0 & 0 \\ .88 & 1 & .17 & 0 & .25 & 0 & 1 & 0 \\ 1 & 1 & 1 & .68 & .57 & 0 & 1 & 1 \end{bmatrix}$$

Corresponding row sum matrix is given by

$R_1 = 1.40$
$R_2 = 3.04$
$R_3 = 3.30$
$R_4 = 6.25$
Min of $[R_1, R_2, R_3, R_4] = 1.40$
Max of $[R_1, R_2, R_3, R_4] = 6.25$

Fuzzy membership functions are given by $\mu_x(R_1) = 0$, $\mu_x(R_2) = .34$, $\mu_x(R_3) = 0.39$, $\mu_x(R_4) = 1$. Corresponding graph is given by:



Graph-4

For □=0.9 we have

$$\begin{bmatrix} 35 & 34 & 7.5 & 2.5 & 1.5 & 2 & 5 & 2.5 \\ 37.5 & 40 & 6.0 & 3.5 & 1.0 & 2.5 & 6 & 2.5 \\ 38.5 & 42.5 & 7.0 & 2.5 & 1.75 & 1.5 & 7.5 & 3.5 \\ 40 & 41.5 & 9.0 & 3 & 2.25 & 1.5 & 7 & 4 \end{bmatrix}$$

μ_j	37.700	39.500	7.400	2.900	1.620	1.870	6.370	3.120
M_j	1.5	2.5	.88	.38	.38	.38	.87	.62
□*M_j	1.350	2.250	.792	.342	.342	.342	.783	.558
μ_j-□□*M_j	36.350	37.250	6.608	2.558	1.278	1.528	5.587	2.562
μ_j+□□*M_j	39.050	41.750	8.192	3.242	1.962	2.212	7.153	3.678

Corresponding FRM is given by:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & .32 & .69 & 0 & 0 \\ .42 & .61 & 0 & 1 & 0 & 1 & .26 & 0 \\ .79 & 1 & .87 & 0 & .69 & 0 & 1 & .84 \\ 1 & .93 & 1 & .93 & 1 & 0 & .90 & 1 \end{bmatrix}$$

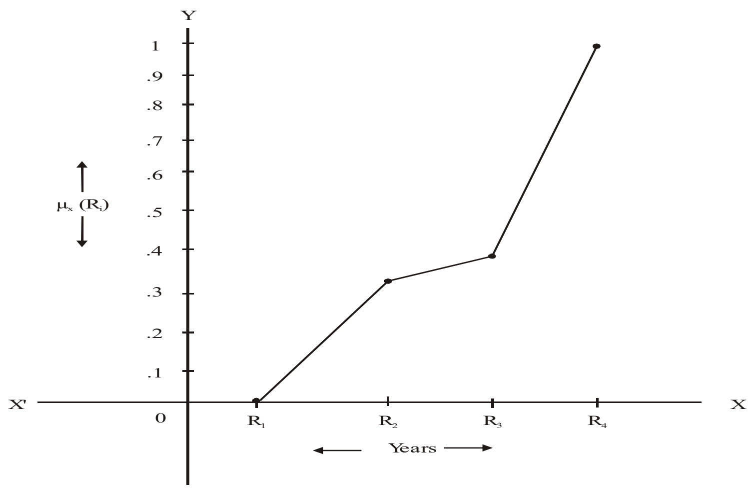
Corresponding row sum matrix is given by

$$\begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix} = \begin{bmatrix} 1.01 \\ 3.29 \\ 5.19 \\ 6.76 \end{bmatrix}$$

Min of $[R_1, R_2, R_3, R_4] = 1.01$

Max of $[R_1, R_2, R_3, R_4] = 6.76$

Membership grades are $\mu_x(R_1) = 0$, $\mu_x(R_2) = .39$, $\mu_x(R_3) = 0.72$, $\mu_x(R_4) = 1$. Corresponding graph is given by:



Graph-5

The combined fuzzy matrix for all the values of $\lambda \in [0,1]$ is given below

	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8
Y_1	0	0	2.86	0	.77	4.27	0	0
Y_2	2.23	3.78	0	5	0	5	.26	0
Y_3	4.67	5	1.87	0	3.78	0	5	3.84
Y_4	5	4.93	5	4.30	4.57	0	4.90	5

Corresponding row sum of the above matrix is given by

$R_1 = 7.90$

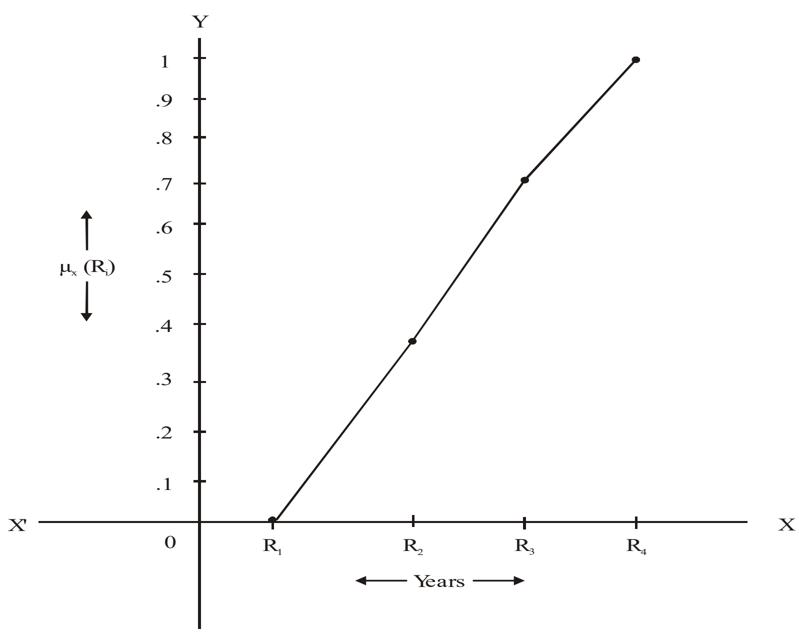
$R_2 = 16.27$

$R_3 = 24.16$

$R_4 = 35.75$

Using membership function the corresponding values of μ_x (Ri) is 0, .30, .59, 1 respectively.

Graphical illustration is given below-



Graph-6

Conclusion

Here we introduce a fuzzy relational matrix model to the problem of employees and employers in BHEL and this finds best form of relationship between the employees and the employers. The row data is obtained by the experts' opinion from the BHEL as well as feeling of the employees. Using the data from BHEL, we carry out the analysis of the data via F.R.M. model and found the best form of relationship between employees and employers. This has been done in two ways.

In the first stage the data was converted into a relational map and the relational matrix was obtained from the relational map and using this the average relational matrix is obtained.

In the second stage the average relational matrix is converted into fuzzy relational matrix using different parametersλ, $\lambda \in [0,1]$, the best form of relationship is obtained. This is also explicitly described by the graphs. Here we have considered eight attributes of the employees and four effects of employers, viz the net worths of the years from 2005 to 2008. The production levels of each year are treated as rows and the various attributes of employee are treated as columns in the fuzzy matrix.

The highest membership grade gives the best form of relationship between employer and employees, which maximizes the production level of BHEL. Using different parameter λ = .1, .3, .5, .7 and .9 with the row sum of the fuzzy matrix, the fourth row sum R_4 i.e. year 2008 gives maximum net worth and employer gets maximum satisfaction. From the combined fuzzy matrix, we observe that the row sum R_4 gets the highest membership grade that is the value 1 which is in the year 2008, giving the maximum net worth, with maximum satisfaction of employers and the row sum R_3 gets the next highest membership grade .72 that is in the year 2007. Now from the analysis of combined fuzzy matrix the row sum R_1 gets the lowest membership grade which is zero, that is, in the year 2005, the employee's satisfaction was poor with minimum net worth.

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