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Generalized Co – Symplectic manifold of second class

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Abstract

The generalized co- symplectic manifold of second class was defined by Mishra, 1991 .In the present paper some theorems related to it has been discussed.The theorems related to its Nijenhuis tensor and the normal are also studied.

1. Introduction

Let there exit an odd-dimensional differentiable manifold M_n , of differentiability class C^∞ on which there are defined a tensor field F of type (1,1) , a vector field U and a 1- form u , satisfying for arbitrary vector fields X, Y, Z,…….

$$\text{a) } \overline{\overline{X}} + X = u(X)U, \quad \text{b) } \overline{X} = F(X), \quad \text{c) } \overline{U} = 0, \tag{1.1}$$

$$\text{d) } u\left(\overline{X}\right) = 0 \quad \text{e) } u(U) = 1, \quad \text{f) } n \text{ is odd } = 2m + 1.$$

Then M_n is called an almost contact manifold.

An Almost contact manifold M_n , on which a metric tensor g satisfying

$$\text{a) } 'F(X, Y) = g(\overline{X}\, Y) = -g(X, \overline{Y}), \quad \text{b) } g(X, U) = u(X) \tag{1.2}$$

has been introduced is called an almost contact metric manifold

An almost contact metric manifold satisfying

$$(D_x\, 'F)(Y, Z) = u(Y)(D_x u)\left(\overline{Z}\right) - u(Z)(D_x u)\left(\overline{Y}\right) \tag{1.3}$$

is called a generalized co – symplectic manifold [1]

An almost contact metric manifold on which U satisfies

$$\text{a) } (D_x u)\left(\overline{Y}\right) = -\left(D_{\overline{X}}\, u\right)(Y) = \left(D_Y\, u\right)\left(\overline{X}\right) \iff \tag{1.4}$$

$$\text{b) } (D_x u)(Y) = \left(D_{\overline{X}}\, u\right)\left(\overline{Y}\right) = -\left(D_Y\, u\right)(X), \quad \text{and}$$

$$\text{c) } D_U F = 0$$

is said to be generalized co – symplectic manifold of first class or (nqs) manifold whose equation is

$$(D_x\, 'F)(Y, Z) = u(Y)\left(D_Z\, u\right)\left(\overline{X}\right) + u(Z)\left(D_{\overline{X}}\, u\right)(Y). \tag{1.5}$$

If on an almost contact metric manifold U satisfies

$$\text{a) } (D_x u)\left(\overline{Y}\right) = \left(D_{\overline{X}}\, u\right)(Y) = -\left(D_Y\, u\right)\left(\overline{X}\right) \iff \tag{1.6}$$

$$\text{b) } (D_x u)(Y) = -\left(D_{\overline{X}}\, u\right)\left(\overline{Y}\right) = -\left(D_Y\, u\right)(X), \quad \text{and}$$

$$\text{c) } D_U F = 0$$

Then U is said to be of second class and the generalized co- symplectic manifold of second class has its equation as [1]

$(D_x{}'F)(Y,Z) = u(Y)\left(D_{\overline{X}}\,u\right)(Z) + u(Z)\left(D_Y\,u\right)\left(\overline{X}\right).$ (1.7)

In generalized co- symplectic manifold of second class the Nijenhuis tensor is given by

a) $N(X,Y) = \left(D_{\overline{X}}F\right)(Y) - \left(D_{\overline{Y}}F\right)(X) - \overline{\left(D_XF\right)(Y)} + \overline{\left(D_YF\right)(X)}$ (1.8)

b) $'N(X,Y,Z) = (D_{\overline{X}}{}'F)(Y,Z) - (D_{\overline{Y}}{}'F)(X,Z)$
$+ (D_x{}'F)(Y,\overline{Z}) - (D_x{}'F)(X,\overline{Z})$

where,

$'N(X,Y,Z) == g(N(X,Y),Z).$ (1.9)

An almost contact metric manifold is Normal if $\underset{0}{N}(X,Y) = 0$ [2] where,

$\underset{0}{N}(X,Y) = (D_{\overline{X}}{}'F)(Y,Z) - (D_{\overline{Y}}{}'F)(X,Z) + (D_x{}'F)(Y,\overline{Z})$ (1.10)
$- (D_x{}'F)(X,\overline{Z}) + u(Z)\{(D_X\,u)(Y) - (D_Y\,u)(X)\}.$

Using (1.6) b) in this equation we get in generalized co – symplectic manifold of second class

$\underset{0}{N}(X,Y) = (D_{\overline{X}}{}'F)(Y,Z) - (D_{\overline{Y}}{}'F)(X,Z) + (D_x{}'F)(Y,\overline{Z})$ (1.11)
$- (D_x{}'F)(X,\overline{Z}) + 2u(Z)(D_X\,u)(Y).$

1. Some Theorems on generalized Co – Symplectic manifold of second class

Theorem 2.1 - On a generalized co- symplectic manifold of second class F Was killing iff

$u(Y)(D_{\overline{X}}u)(Z) + u(X)(D_{\overline{Y}}u)(Z) = 0.$ (2.1)

Proof: - We have from (1.7)

$(D_x{}'F)(Y,Z) + (D_y{}'F)(X,Z) = u(Y)\left(D_{\overline{X}}\,u\right)(Z) + u(Z)\left(D_Y\,u\right)\left(\overline{X}\right)$
$+ u(X)(D_{\overline{Y}}u)(Z) + u(Z)(D_X\,u)\left(\overline{Y}\right).$

Using (1.6) a) and (2.1) we get

$(D_x{}'F)(Y,Z) + (D_y{}'F)(X,Z) = 0$

which proves the statement. The converse is also true.

Theorem 2.2 - On a generalized co – symplectic manifold of second class we have

$'N(X,Y,Z) = d\,'F(X,Y,\overline{Z}) + 2u(Z)(D_X\,u)(Y).$ (2.2)

Proof: - From (1.8) b) we have

$'N(X,Y,Z) - d\,'F(X,Y,\overline{Z}) = (D_{\overline{X}}{}'F)(Y,Z) - (D_{\overline{Y}}{}'F)(X,Z) - (D_{\overline{Z}}{}'F)(X,Y).$

By virtue of (1.7), this equation assumes the form

$'N(X,Y,Z) - d\,'F(X,Y,\overline{Z}) = u(Y)(D_{\overline{\overline{X}}}u)(Z) + u(Z)\left(D_Y\,u\right)\left(\overline{\overline{X}}\right)$
$- u(X)(D_{\overline{\overline{Y}}}u)(Z) - u(Z)(D_X\,u)\left(\overline{\overline{Y}}\right)$
$- u(X)(D_{\overline{\overline{Z}}}u)(Y) - u(Y)(D_X\,u)\left(\overline{\overline{Z}}\right).$

Using (1.6) b) we get 2.2.

Theorem 2.3- Let us put

a) $'T(X,Y,Z) = (D_{\overline{X}}{}'F)(Y,Z) + (D_x{}'F)(Y,\overline{Z}) - u(Z)(D_X\,u)(Y)$ (2.3)

b) $u(X)(D_Z\,u)(Y) + u(Z)(D_Y\,u)(X) = 0.$

Then on generalized co symplectic manifold of second class we have

a) $'T(X,Y,Z) - 'T(Y,X,Z) = 'N(X,Y,Z) + 2u(Z)(D_Y u)(X)$ (2.4)

b) $'T(X,Y,Z) + 'T(Z,Y,X) = 0$

Proof: - In the consequence of (2.3) a) we have

$$'T(X,Y,Z) - 'T(Y,X,Z) = (D_{\bar{X}}\,'F)(Y,Z) + (D_x\,'F)(Y,\bar{Z}) - u(Z)(D_X u)(Y)$$
$$-(D_{\bar{Y}}\,'F)(X,Z) - (D_Y\,'F)(X,\bar{Z}) + u(Z)(D_Y u)(X)$$

Using (1.8) b) and (1.6) b) we get (2.4) a).

By virtue of (2.3), (1.7) and (1.6) a, b) we have

$$'T(X,Y,Z) = -2u(Y)(D_X u)(Z). \tag{2.5}$$

Then,

$$'T(X,Y,Z) + 'T(Z,Y,X) = -2u(Y)(D_X u)(Z) - 2u(Y)(D_Z u)(X)$$

Using (1.6) b) we find (2.4) b).

Theorem 2.4 - If we put

a) $'N(X,Y,Z) = 0$ and (2.6)

b) $u(Y)(D_X u)(Z) + u(Z)(D_Y u)(X) = 0$

Then on a generalized co- symplectic manifold of second class we have

$$(D_Z u)(Y) = 0 \tag{2.7}$$

Proof: - In the consequences of (1.6) and (1.7)

$'N(X,Y,Z) = 0$ is equivalent to

$$u(Y)(D_{\bar{\bar{X}}} u)(Z) + u(Z)(D_Y u)(\bar{\bar{X}}) + u(Z)(D_{\bar{\bar{Y}}} u)(X) + u(X)(D_Z u)(\bar{\bar{Y}})$$
$$+ u(Y)(D_{\bar{X}} u)(\bar{Z}) + u(X)(D_{\bar{Z}} u)(\bar{Y}) = 0.$$

Using (1.6) and (2.6)b) in the above equation we get (2.7).

3. Normal Property

Theorem 2.5- The generalized co – symplectic manifold of second class is normal if

$$u(Y)(D_X u)(Z) + u(Z)(D_Y u)(X) = 0. \tag{2.8}$$

Proof: - On generalized co - symplectic manifold of second class we have

$$\underset{0}{N}(X,Y) = (D_{\bar{X}}\,'F)(Y,Z) - (D_{\bar{Y}}\,'F)(X,Z) + (D_x\,'F)(Y,\bar{Z})$$
$$(D_x\,'F)(X,\bar{Z}) + 2u(Z)(D_X u)(Y).$$
$$= u(Y)(D_{\bar{\bar{X}}} u)(Z) + u(Z)(D_Y u)(\bar{\bar{X}}) + u(Z)(D_{\bar{\bar{Y}}} u)(X)$$
$$+ u(X)(D_Z u)(\bar{\bar{Y}}) + u(Y)(D_{\bar{X}} u)(\bar{Z}) + u(X)(D_{\bar{Z}} u)(\bar{Y})$$
$$+ 2u(Z)(D_X u)(Y).$$

Using (1.6) a, b) we get

$$\underset{0}{N}(X,Y) = -2u(Y)(D_X u)(Z) - 2u(Z)(D_Y u)(X)$$
$$+ 2u(X)(D_Y u)(Z) + 2u(Z)(D_X u)(Y).$$

Using (2.8) in this equation we get $\underset{0}{N}(X,Y) = 0$, which is the statement.

Theorem 2.6- If the generalized co – symplectic manifold of second class is normal and satisfying

$$u(Y)(D_X u)(Z) + u(x)(D_Z u)(Y) = 0 \tag{2.9}$$

then we get

$(D_X\ u)(Y) = 0$. (2.10)

Proof :- $\underset{0}{N}(X, Y) = 0$ is equivalent to

$$(D_{\overline{X}}{}' F)(Y, Z) - (D_{\overline{Y}}{}' F)(X, Z) + (D_x{}' F)(Y, \overline{Z})$$

$$- (D_x{}' F)(X, \overline{Z}) + 2\,u(Z)\,(D_X\ u)(Y) = 0$$

In the consequences of (1.6) and (1.7) this equation assumes the form

$$2\,u(Y)\,(D_X\ u)(Z) + 2\,u(x)\,(D_Z\ u)(Y) + 4\,u(Z)\,(D_X\ u)(Y) = 0$$

Using (2.9) we get (2.10).

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