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The decay process of dusty fluid turbulence

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Abstract

In this paper, the decay of dusty fluid turbulence in a rotating system is discussed. The expression for total turbulent energy is obtained analytically.

Introduction

The motion of dusty fluid occurs in the movement of dust-laden air, in problems of fluidization, in the use of dust in a gas cooling system and in the sedimentation problem of tidal rivers. When the motion is referred to axes, which rotate steadily with the bulk of the fluid, the Coriolis force and centrifugal force must be supposed to act on the fluid. The Coriolis force due to rotation plays an important role in a rotating system of turbulent flow while the centrifugal force with the potential is incorporated into the pressure. Deissler (1958, 1960) generalized a theory "Decay of homogeneous turbulence for times before the final period". Saffman (1962) derived an equation that described the motion of a fluid containing small dust particles. Dixit and Upadhyay (1989), Kishore and Dixit (1979) and Kishore and Singh (1984) discussed the effect of Coriolis force on acceleration covariance in ordinary and MHD turbulent flows. Shamomura and Yoshizawa (1986) and Shimomura (1986, 1989) also discussed the statistical analysis of turbulent viscosity, turbulent scalar flux and turbulent shear flows, respectively in a rotating system by two-scale Direct-interaction approach. Kishore and Upadhyay (2000) studied the decay of MHD turbulence in a rotating system. Sarker and Islam (2001) also studied the decay of dusty fluid turbulence before the final period in a rotating system using 2 and 3 point correlation equations. Shukla (2007) discussed the decay equation for MHD dusty turbulence. By analyzing the above theories we have studied the decay of dusty turbulent flow in a rotating system using 3 and 4 point correlation equations.

2. Correlation and Spectral Equations

Equation of motion of dusty fluid turbulence in a rotating system at the points, p, p', and p''' are:

$$\frac{\partial u_i}{\partial t} + \frac{\partial (u_i u_m)}{\partial x_m} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + v\frac{\partial^2 u_i}{\partial x_m \partial x_m} - 2\varepsilon_{nmi}\Omega_n u_i + f(u_i - v_i) \tag{2.1}$$

$$\frac{\partial u_j'}{\partial t}+\frac{\partial\left(u_j' u_m'\right)}{\partial x_m}=-\frac{1}{\rho}\frac{\partial p'}{\partial x_i'}+\nu\frac{\partial^2 u_j'}{\partial x_m'\partial x_m'} \tag{2.2}$$

$$-2\varepsilon_{pmj}\Omega_p u_j'+f\left(u_j'-v_j'\right)$$

$$\frac{\partial u_k''}{\partial t}+\frac{\partial\left(u_k'' u_m''\right)}{\partial x_m''}=-\frac{1}{\rho}\frac{\partial p''}{\partial x_k''}+\nu\frac{\partial^2 u_k''}{\partial x_m''\partial x_m''} \tag{2.3}$$

$$-2\varepsilon_{qnj}\,\Omega_q u_k''+f\left(u_k''-v_k''\right)$$

$$\frac{\partial u_l'''}{\partial t}+\frac{\partial\left(u_l''' u_m'''\right)}{\partial x_m'''}=\frac{1}{\rho}\frac{\partial p'''}{\partial x_l'''}+\nu\frac{\partial^2 u_l'''}{\partial x_m'''\partial x_m'''} \tag{2.4}$$

$$-2\varepsilon_{mlj}\Omega_r u_l'''+f\left(u_l'''-v_l'''\right).$$

Multiplying Equation 2.1. by $u_j'u_k''u_l'''$ Equation 2.2 by $u_j\ u_k''u_l'''$, and Equation 2.4 $u_i u_j' u_k''$ then adding and taking ensemble average writing in terms of the independent variables r,r' and r'' as:

$$\frac{\partial}{\partial t}\left\langle uu_i'u_k''u_l'''\right\rangle-\frac{\partial}{\partial r_m}\left\langle u_iu_mu_j'u_l''\right\rangle-\frac{\partial}{\partial r_m''}\left\langle u_iu_mu_j'u_k''u_l'''\right\rangle$$

$$\frac{\partial}{\partial t}\left\langle uu_j'u_m''u_k''u_l'''\right\rangle+\frac{\partial}{\partial r_m'}\left\langle u_iu_j'u_k''u_m''u_l'''\right\rangle+\frac{\partial}{\partial r_m''}\left\langle u_iu_j'u_k''u_l'''u_m''\right\rangle$$

$$=-\frac{1}{\rho}\left(-\frac{\partial}{\partial t}\left\langle pu_j'u_k''u_l'''\right\rangle-\frac{\partial}{\partial r_m'}\left\langle pu_j'u_k''u_m''u_l'''\right\rangle-\frac{\partial}{\partial r_i''}\left\langle pu_j'u_k''u_l'''\right\rangle+\frac{\partial}{\partial r_j}\left\langle u_lp'u_k''u_l''\right\rangle\right.$$

$$\left.+\frac{\partial}{\partial r_k'}\left\langle u_iu_j'p''p_l'''\right\rangle+\frac{\partial}{\partial r_l''}\left\langle u_iu_j'u_k''p'''\right\rangle\right)$$

$$+2v\left(\frac{\partial^2\left\langle u_iu_j'u_k''u_l'''\right\rangle}{\partial r_m\partial r_m}+\frac{\partial^2\left\langle u_iu_j'u_k''u_l'''\right\rangle}{\partial r_m\partial r_m'}+\frac{\partial^2\left\langle u_iu_j'u_k'u_l''\right\rangle}{\partial r_m\partial r_m''}+\frac{\partial^2\left\langle u_iu_j'u_k''u_l'''\right\rangle}{\partial r_m'\partial r_m}\right.$$

$$\left.+\frac{\partial^2\left\langle u_iu_j'u_k''u_l'''\right\rangle}{\partial r_k'\partial r_m''}+\frac{\partial^2\left\langle u_iu_j'u_k''u_l''\right\rangle}{\partial r_k''\partial r_m''}\right)$$

$$-2\left(\varepsilon_{nmi}\Omega_n\left\langle u_iu_j'u_k''u_j''\right\rangle+\varepsilon_{pmj}\Omega_p\left\langle u_iu_j'u_k''u_j''\right\rangle+\varepsilon_{qmk}\Omega_q\left\langle u_ju_j'u_k''u_j'\right\rangle\varepsilon_{mnl}\Omega_r\left\langle u_iu_j'u_k''u_j'\right\rangle\right)$$

$$+f\left(-\left\langle u_iu_j'u_k''u_j'''\right\rangle-\left\langle u_iu_j'u_k''u_l'''\right\rangle-\left\langle u_iu_j'u_k''u_l'''\right\rangle-\left\langle u_iu_j'u_k''v_l'''\right\rangle+4\left\langle u_iu_j'u_k''u_l'''\right\rangle\right) \tag{2.5}$$

By using

$$\frac{\partial}{\partial x'_m}=\frac{\partial}{\partial r_m},\frac{\partial}{\partial x''_m}=\frac{\partial}{\partial r'_m},\frac{\partial}{\partial r''_m} \text{ and } \frac{\partial}{\partial x_m}=-\frac{\partial}{\partial r_m}-\frac{\partial}{\partial r'_m}-\frac{\partial}{\partial r''_m}$$

In order to convert Equation (2.5) to spectral form, using nine-dimensional Fourier transforms (Sultana *et al.*, 2006) we obtain substituting the preceding relations into Equation (2.5), we get

$$\frac{d}{dt}(\gamma_{jkl})+2v(k^2+k_m k'_m k''_m+k'^2+k'_m k''_m+k''^2)\gamma_{ijkl}$$

$$=[i(k_m+k'_m+k''_m)\gamma_{imjkl}(k,k',k'')-ik_m\gamma_{jmkl}(-k-k'-k'',k',k'')$$

$$-ik'_m\gamma_{kmijl}(-k-kk'-k''-k,k'')-ik''_m\gamma_{lmijl}(-k-k'-k'',k')]$$

$$-i(k_i+k'_i+k''_i)\delta_{ijl}(k,k',k'')+ik_j\delta_{ikl}(-k-k'-k'',k',k'')$$

$$+ik'_k\delta_{ijl}(-k-k'-k'',k,k'')+ik''_i\delta_{ijl}(-k-k'-k'',k,k')]$$

$$-2(\in_{mni}\Omega_n+\in_{pmj}\Omega_p+\in_{qmk}\Omega_q\in_{nml}\Omega_r)\gamma_{ijkl}$$

$$+f[4\gamma_{ijkl}(k,k',k'')-\gamma,\delta_{jkl}(k,k',k'')-\gamma_j\delta_{ikl}(-k-k'-k'',k',k'')$$

$$-\gamma_k\delta_{ijl}(-k-k'-k'',k,k'')-\gamma_l\delta_{ijk}(\gamma_l\delta_{ijk}(-k-k'-k'',k,k')] \tag{2.6}$$

To obtain a relation between the terms on the right side of Equation (2.6), derived from the quadruple correlation terms, pressure terms, rotational terms and the dust particle terms in Equation (2.5), take the divergence of the equation of motion and combine with the continuity equation to give

$$\frac{1}{\rho}\frac{\partial^2 p}{\partial x_m \partial x_m}=-\frac{\partial^2(u_m u_n)}{\partial x_m \partial x_m} \tag{2.7}$$

Multiplying Equation (2.7) by $u'_j u''_k u''_l$, taking ensemble average and writing the resulting equation in terms of the independent variables r and r', gives

$$\frac{1}{\rho}\left(\frac{\partial^2\langle pu'_j u''_k u'''_l\rangle}{\partial r_m \partial r_m}+2\frac{\partial^2\langle pu'_j u''_k u'''_l\rangle}{\partial r_m \partial r'_m}+2\frac{\partial^2\langle pu'_j u''_k u'''_l\rangle}{\partial r_m \partial r''_m}+\frac{\partial^2\langle pu'_j u''_k u'''_l\rangle}{\partial r'_m \partial r'_m}\right.$$

$$\left.+2\frac{\partial^2\langle pu'_j u''_k u'''_l\rangle}{\partial r_m \partial r''_m}+2\frac{\partial^2\langle pu'_j u''_k u'''_l\rangle}{\partial r''_m \partial r''_m}\right)$$

$$=-\left(\frac{\partial^2\langle u_m u_n u'_j u''_k u'''_l\rangle}{\partial r_m \partial r_m}+\frac{\partial^2\langle u_m u_n u'_j u''_k u'''_l\rangle}{\partial r_m \partial r'_m}+\frac{\partial^2\langle u_m u_n u'_j u''_k u'''_l\rangle}{\partial r_m \partial r''_m}\right.$$

$$+\frac{\partial^2\left\langle u_m u_n u_j^{'} u_k^{''} u_l^{'''}\right\rangle}{\partial r'_m \partial r_m} ++ \frac{\partial^2\left\langle u_m u_n u_j^{'} u_k^{''} u_l^{'''}\right\rangle}{\partial r'_m \partial r'_m}$$

$$\left. + \frac{\partial^2\left\langle u_m u_n u_j^{'} u_k^{''} u_l^{'''}\right\rangle}{\partial r'_m \partial r''_m} + \frac{\partial^2\left\langle u_m u_n u_j^{'} u_k^{''} u_l^{'''}\right\rangle}{\partial r''_m \partial r_m} + \frac{\partial^2\left\langle u_m u_n u_j^{'} u_k^{''} u_l^{'''}\right\rangle}{\partial r''_m \partial r'_m} + \frac{\partial^2\left\langle u_m u_n u_j^{'} u_k^{''} u_l^{'''}\right\rangle}{\partial r''_m \partial r''_m}\right) \tag{2.8}$$

The Fourier transform of Equation (2.8) is

$$\frac{1}{\rho}\delta_{jkl} = \frac{(k_m k_n + k_m k_n^{'} + k_m k_n^{''} + k_m k_n + k_m^{'} k_n^{''} + k_m^{''} k_n^{'} + k_m^{''} k_n^{''})\gamma_{mnjkl}}{(k^2 + 2k_m k_m^{'} + 2k_m k_m^{''} + k'^2 + 2k_m^{'} k_m^{''} + k''^2)} \tag{2.9}$$

Equations (2.6) and (2.9) are the spectral equations corresponding to the 4 point correlation equations. The spectral equations corresponding to the three point correlation equations are:

$$\frac{d}{dt}(k_k \beta_{iik}) + 2\nu(k^2 + k_l k_l^{'} + k^2)\, k_k \beta_{iik}$$

$$= ik_k(k_l + k_l^{'})\beta_{ilik}(k,k') - ik_k k_l \beta_{ilik}(-k-k',k')$$

$$-ik_k k_l^{'} \beta_{klii}(-k-k',k) - \frac{1}{\rho}[-ik_k(k_i + k_i^{'})\,\alpha_{ik}(k,k')$$

$$+ik_k k_i \alpha_{ik}(-k-k',k') + ik_k k_k^{'} \alpha_{ii}(-k-k',k)]$$

$$-2k_k[\varepsilon_{mli}\Omega_m + \varepsilon_{nli}\Omega_n + \varepsilon_{qlk}\Omega_q]\beta_i \beta_i^{'} \beta_k^{''} + Rfk_k \tag{2.10}$$

where

$$R\beta_i\beta_i^{'}\beta_k^{''} = 3\left\langle \beta_i\beta_i^{'}\beta_k^{''}\right\rangle - \left\langle \gamma_i\beta_i^{'}(k)\beta_k^{''}(k')\right\rangle - \left\langle -\gamma_i\beta_i^{'}(-k-k')\right\rangle$$

$$-\left\langle \gamma_k\beta_i^{'}(-k-k')\beta_i^{''}(k)\right\rangle, \quad \text{(say)}$$

R is an arbitrary constant and

$$-\frac{1}{\rho}\alpha_{ik} = \frac{k_l k_m + k_l^{'} k_m + k_l^{'} k_m^{'}}{k^2 2k_l k_l^{'} + k'^2}\,\beta_{lmik} \tag{2.11}$$

Here, the spectral tensors are defined by

$$\left\langle u_i u_j^{'}(r) u_k^{''}(r')\right\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \beta_{ijk}(k,k')\exp[i(k.r + k'.r')]\,dk\,dk' \tag{2.12}$$

$$\left\langle u_i u_l u_j'(r) u_k''(r')\right\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \beta_{iljk}(k,k')\exp[i(k.r+k'.r')]\,dk\,dk' \tag{2.13}$$

$$\left\langle p u_j'(r) u_k''(r')\right\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \alpha_{jk}(k,k')\exp[i(k.r+k'.r')]\,dk\,dk' \tag{2.14}$$

A relation between β_{iljk} and γ_{ijkl} can be obtained by letting r'' = 0 in Equation 2.12 and comparing the result with Equation 2.13,

$$\beta_{iljk}(k,k') = \int_{-\infty}^{\infty} \gamma_{ijkl}(k,k',k'')\,dk'' \tag{2.15}$$

The spectral equation corresponding to the 2 point correlation equation in presence of dusty fluid in a rotating system is

$$\frac{d}{dt}\phi_{i,i} + (2\nu k^2 - Qf + 2\varepsilon_{mki}\Omega_m + 2\varepsilon_{nki}\Omega_n)\,\phi_{i,i}$$

$$= ik_k\,\phi_{iki}(k) - ik_k\phi_{ikj}(-k) \tag{2.16}$$

where $\phi_{i,i}$ and ϕ_{ikl} are defined by

$$\left\langle u_i u_j'(r)\right\rangle = \int_{-\infty}^{\infty} \phi_{ij}(k)\exp(ikr)\,dk \tag{2.17}$$

and

$$\left\langle u_i u_k u_j'(r)\right\rangle = \int_{-\infty}^{\infty} \phi_{ij}(k)\exp(ikr)\,dk \tag{2.18}$$

The relation between ϕ_{ikj} and ϕ_{ijk} obtained by letting r' = 0 in Equation 2.13 and comparing the result with Equation 2.18 is

$$\phi_{ikj}(k) = \int_{-\infty}^{\infty} \beta_{ijk}(k,k')\,dk' \tag{2.19}$$

Solution

Equation 2.9 shows that if the terms corresponding to the quintuple correlations are neglected, then the pressure force terms also must be neglected. Thus neglecting first and second terms on the right side of Equation 2.6, the equation can be integrated between t_l and t to give

$$\gamma_{ijkl} = (\gamma_{ijkl})_1 \exp[\{-2\nu(k^2 + k_m k_m' + k_m k_m'' + k'^2 + k_m' k_m'' + k''^2)$$

$$+2(\varepsilon_{nmi}\Omega_n + \varepsilon_{pmj}\Omega_r + \varepsilon_{qnk}\Omega_q + \varepsilon_{nml}\Omega_i) - sf\}(t-t_1)] \tag{3.1}$$

where

$$S\gamma_{ijkl} = 4\gamma_{ijkl}(k,k',k'') - \gamma_i\delta_{jkl}(k,k',k'') - \gamma_j\delta_{ikl}(-k-k'-k'',k',k'')$$

$$-\gamma_k\delta_{ijl}(-k-k'-k'',k,k'') - \gamma_l\delta_{ijk}(-k-k'-k'',k,k') \tag{3.2}$$

is an arbitrary constant and (□$_{ijkl}$) is the value of □$_{ijkl}$ at $t = t_1$. The quantity (□$_{ijkl}$) can be considered also as the value of □$_{ijkl}$ at small values of k, k' and k'', at least for times when the quintuple correlations are neglected.

Equations (2.15) and (3.1) can be converted to scalar form by contracting the indices i and j, as well as k and l. Substitution of Equations 2.11, 2.15 and 3.1 into the three point scalar Equation 2.10 results in

$$k_k\beta_{iik} = (k_k\beta_{iik})_0 \exp[\{-2\nu(k^2 + k_l k_l' + k'^2) + 2(\varepsilon_{mli}\Omega_m + \varepsilon_{nli}\Omega_n + \varepsilon_{qlk}\Omega_q) - Rf\}(t-t_0)]$$

$$+\frac{\pi^{3/2}[a]_1}{\nu}\left\{\omega^{-1}\exp\left[-\omega^2\left(\frac{3}{4}k^2 + \frac{1}{2}k_l k_l' + \frac{3}{4}k'^2\right)\right.\right.$$

$$+\{2(\varepsilon_{nmi}\Omega_n + \varepsilon_{pmi}\Omega_p + \varepsilon_{qmk}\Omega_q + \varepsilon_{nml}\Omega_r) - Sf\}(t-t_t)]$$

$$+2\left(\frac{1}{4}k^2 + \frac{1}{2}k_l k_l' + \frac{1}{4}k'^2\right)^{1/2}\exp[-\omega^2 + k_l k_l' + k'^2)$$

$$+\{2(\varepsilon_{nmi}\Omega_n + \varepsilon_{pmi}\Omega_p + \varepsilon_{qmk}\Omega_r) - Sf\}(t-t_1)]$$

$$\times\int_0^m \exp(x^2)dx\} + \frac{\pi^{3/2}[b]_1}{\nu}\{\left(\frac{3}{4}k^2 + k_l k_l' + k'^2\right)$$

$$+\{2(\varepsilon_{mni}\Omega_n + \varepsilon_{pmi}\Omega_p + \varepsilon_{qlk}\Omega_q + \varepsilon_{nnl}\Omega_r) - Sf\}(t-t_1)]$$

$$+k\exp[-\omega^2(k^2 + k_l k_l' + k'^2) + \{2(\varepsilon_{nmi}\Omega_n + \varepsilon_{pmi}\Omega_q + \varepsilon_{qmk}\Omega_q + \varepsilon_{nml}\Omega_r)$$

$$-Sf\}(t-t_1)]\int_0^{\frac{1}{2}\omega k}\exp(x^2)dx\}$$

$$+\frac{\pi^{\frac{1}{2}}[c]_1}{\nu}[-\omega^{-1}\exp[-\omega^2(k^2 + k_l k_l' + \frac{3}{4}k'^2)$$

$$+\{2(\varepsilon_{mni}\Omega_n + \varepsilon_{pmi}\Omega_p + \varepsilon_{qnk}\Omega_q + \varepsilon_{nnl}\Omega_r) - Sf\}(t-t_1)]$$

$$+k'\exp[-\omega^2(k^2 + k_l k_l' + k'^2) + \{2(\varepsilon_{nmi}\Omega_n + \varepsilon_{pmi}\Omega_p + \varepsilon_{nml}\Omega_r)$$

$$-Sf\}(t-t_1)]\int_0^{\frac{1}{2}\omega k}\exp(x^2)dx\} \tag{3.3}$$

where

$$\omega=[2\nu(t-t_1)]^{1/2}$$

In order to simplify the calculations, we shall assume that $[a]_1 = 0$: that is, we assume that a function sufficiently general to represent the initial conditions can be obtained by considering only the terms involving $[b]_1$ and $[c]_1$.

The substitution of Eq. (2.19) and (3.3) in Equation (2.16) and setting k^2 results in

$$\frac{dE}{dt}+\left(2\nu k^2+2\varepsilon_{mki}\Omega_m+2\varepsilon_{nki}\Omega_n-Qf\right)E=W \tag{3.4}$$

where

$$\begin{aligned}
W &= k^2\int_{-\infty}^{\infty}2\pi i[k_i\beta_{iik}(k,k')-k_k\beta_{iik}(-k,-k')]_0\exp[\{-2\nu(k^2+k_1k_1+k^2)\\
&+2\left(\varepsilon_{mli}\Omega_m+\varepsilon_{nli}\Omega_n+\varepsilon_{qlk}\Omega_q\right)-Rf\}(t-t_0)dk'\\
&+k^2-\int_{-\infty}^{\infty}\frac{2\pi^2 i}{\nu}[b(k,k')-b(-k,-k')]_1\{-2^{-1}\exp[-w^2\left(\frac{3}{4}k^2+k_1k_1'+k^2\right)\\
&+2\left(\varepsilon_{nmi}\Omega_m+\varepsilon_{pmi}\Omega_p+\varepsilon_{qnk}\Omega_q+\varepsilon_{rml}\Omega_r\right)-Sf\}(t-t_1)]\\
&+k\ \exp[-w^2\left(k^2+k_1k_1'+k^2\right)+\{2\left(\varepsilon_{nmi}\Omega_n+\varepsilon_{pmi}\Omega_p+\varepsilon_{qnk}\Omega_q+\varepsilon_{mnl}\Omega_r\right)-Sf\}\Big)\\
&\int_0^{\frac{1}{2}wk}\exp(x^2)dx\}dk'\\
&+k^2-\int_{-\infty}^{\infty}\frac{2\pi^2 i}{\nu}[c(k,k')-c(-k,-k')]_1\{-w^2\left(k^2+k_1k_1'+\frac{3}{4}k^2\right)\\
&+2\left(\varepsilon_{nmi}\Omega_n+\varepsilon_{pmi}\Omega_p+\varepsilon_{nml}\Omega_r\right)-Sf\}(t-t_1)]\\
&+k'\ \exp[-w^2\left(k^2+k_1k_1'+k^2\right)+\{2\left(\varepsilon_{nmi}\Omega_n+\varepsilon_{pmi}\Omega_p+\varepsilon_{qnk}\Omega_q+\varepsilon_{nml}\Omega_r\right)-Sf\}\Big)\\
&\int_0^{\frac{1}{2}wk'}\exp(x^2)dx\}dk'
\end{aligned} \tag{3.5}$$

The quantity E is the energy spectrum function, which represents contributions from various wave numbers or eddy sizes to the total energy. W is the energy transfer function, which is responsible for the transfer of energy between wave numbers.

In order to find the solution completely and following Deissler (1960), we assume that

$$(2\pi)^2 i[k_k\beta_{iik}(k,k')-k_k\beta_{iik}(-k,-k')]_0=-\beta_0(k^4k'^6-k^6k'^4) \quad (3.6)$$

For the bracketed quantities in Equation (3.5), we let

$$\frac{4\pi^{7/2}}{\nu}i[b(k,k')-b(-k,-k')]_1=\frac{4\pi^{7/2}}{\nu}i[c(k,k')-c(-k,-k')]_1=-2r_1(k^6k^8-k^8k^6) \quad (3.7)$$

where the bracketed quantities are set equal in order to make the integrands in Eq. (3.5) antisymmetric with respect to k and k'.
By substituting Eq. (3.6) and (3.7) in Eq. (3.5) remembering

$dk'=2\pi k'^2 d(\cos\theta)dk'$ $k_i k_i'=kk'\cos\theta$, (θ is the angle between vectors k k and k') and carrying out the integration with respect to θ, we get

$$W=\int_0^\infty\Bigg[\frac{\beta_0\left(k^4k'^6k'^4\right)kk'}{2\nu(t-t_0)}\{\exp[\{-2\nu(k^2+kk'+k'^2)$$

$$+2\left(\varepsilon_{mli}\Omega_m+\varepsilon_{nlj}\Omega_n+\varepsilon_{qlk}\Omega_q\right)-Rf\}\ (t-t_0)]$$

$$-\exp[\{-2\nu(k^2-kk'+k'^2)+2\left(\varepsilon_{nli}\Omega_m+\varepsilon_{nlj}\Omega_n+\varepsilon_{qlk}\Omega_q\right)$$

$$-RF\}\ (t-t_0)]-\gamma_1\frac{\left(k^6k'^8-k^8k'^6\right)kk'}{v(t-t_1)}$$

$$\times\ \omega^{-1}\exp[-\omega^2\left(\frac{3}{4}k^2+kk'+k'^2\right)+2\left(\varepsilon_{mni}\Omega_m+\varepsilon_{pmj}\Omega_p+\varepsilon_{qmk}\Omega_q+\varepsilon_{nml}\Omega_r\right)-Sf\}(t-t_1)]$$

$$-\ \omega^{-1}\exp[-\omega^2\left(\frac{3}{4}k^2+kk'+k'^2\right)+2\left(\varepsilon_{nmi}\Omega_m+\varepsilon_{pmj}\Omega_p+\varepsilon_{qmk}\Omega_q+\varepsilon_{nml}\Omega_r\right)-Sf\}(t-t_1)]$$

$$+\ \omega^{-1}\exp[-\omega^2\left(k^2+kk'+\frac{3}{4}k'^2\right)+2\left(\varepsilon_{nmi}\Omega_m+\varepsilon_{pmj}\Omega_p+\varepsilon_{qmk}\Omega_q+\varepsilon_{nml}\Omega_r\right)-Sf\}(t-t_1)]$$

$$-(\omega^{-1}\exp[-\omega^2\left(k^2+kk'+\frac{3}{4}k'^2\right)+2\left(\varepsilon_{nmi}\Omega_m+\varepsilon_{pmj}\Omega_p+\varepsilon_{qmk}\Omega_q+\varepsilon_{nml}\Omega_r\right)-Sf\}(t-t_1)]$$

$$+\{k\exp[-\omega^2\left(k^2+kk'+k'^2\right)+\{2\left(\varepsilon_{nmi}\Omega_n+\varepsilon_{pmj}\Omega_p+\varepsilon_{qmk}\Omega_q+\varepsilon_{nml}\Omega_r\right)-Sf\}(t-t_1)]$$

$$+k\exp[-\omega^2\left(k^2+kk'+k'^2\right)+\{2\left(\varepsilon_{nmi}\Omega_n+\varepsilon_{pmj}\Omega_p+\varepsilon_{qmk}\Omega_q+\varepsilon_{nml}\Omega_r\right)-Sf\}(t-t_1)]$$

$$\times \int_0^{\frac{1}{2}} \exp(x^2)dx$$

$$+\{k'\exp[-\omega^2\left(k^2+kk'+k'^2\right)+\{2\left(\varepsilon_{nmi}\Omega_n+\varepsilon_{pmj}\Omega_p+\varepsilon_{qmk}\Omega_q+\varepsilon_{nml}\Omega_r\right)-Sf\}(t-t_1)]$$

$$-k'\exp[-\omega^2\left(k^2+kk'+k'^2\right)+\{2\left(\varepsilon_{nmi}\Omega_n+\varepsilon_{pmj}\Omega_p+\varepsilon_{qmk}\Omega_q+\varepsilon_{nml}\Omega_r\right)-Sf\}(t-t_1)]$$

$$\times \int_0^{\frac{1}{2}\omega k'} \exp(x^2)dk' \qquad (3.8)$$

where

$$\omega=[2\nu(t-t_1)$$

The integrand in this equation represents the contribution to the energy transfer at a wave number k, from the wave number k'. The integral is the total contribution to W and k, from all wave numbers. Carrying out the indicated integration with respect to k' in Eqn. (3.8), results in

$$W = W_\beta + W_\gamma \qquad (3.9)$$

where

$$W_\beta=\frac{\left(\frac{\pi}{2}\right)^{\frac{1}{2}}\beta_0}{256\varpi^{\frac{3}{2}}(t-t_0)^{\frac{5}{2}}}\exp\left[\left(-\frac{3}{2}\varepsilon^2\right)\left(105\varepsilon^6+45\varepsilon^8-19e^{10}-3\varepsilon^{12}\right)\right.$$

$$+\{2\left(\varepsilon_{nli}\Omega_m+\varepsilon_{nlj}\Omega_n+\varepsilon_{qlk}\Omega_q\right)-Rf\}\ (t\text{-}t_0)] \qquad (3.10)$$

$$W_\gamma=-\frac{\gamma_1}{\nu^{10}\left(t-t_1\right)^{10}}\left[\frac{\pi^{\frac{1}{2}}}{16}\exp\left[(-\eta^2)\left(\frac{3}{128}\eta^{16}+\frac{3}{8}\eta^{14}+\frac{21}{64}\eta^{12}-\frac{105}{16}\eta^{16}-\frac{945}{128}\eta^8\right)\right.\right.$$

$$+\{2\left(\varepsilon_{nmi}\Omega_m+\varepsilon_{pmj}\Omega_p+\varepsilon_{qmk}\Omega_q+\varepsilon_{rml}\Omega_r\right)-Sf\}\ (t\text{-}t_0)]$$

$$+\frac{2\pi^{\frac{1}{2}}}{\sqrt{3}}\exp\left[\left(-\frac{4}{3}\eta^2\right)\ \left(\frac{160}{19683}\eta^{16}+\frac{40}{729}\eta^{14}-\frac{14}{27}\eta^{12}-\frac{455}{162}\eta^{10}-\frac{35}{18}\eta^8\right)\right.$$

$$+\{2\left(\varepsilon_{nmi}\Omega_n+\varepsilon_{pmj}\Omega_p+\varepsilon_{qmk}\Omega_q+\varepsilon_{rml}\Omega_r\right)-Sf\}\ (t\text{-}t_1)]$$

$$-\frac{\left(\frac{\pi}{2}\right)^{\frac{1}{2}}}{16}\exp\left[\left(-\frac{3}{2}\eta^2\right)\ \int_0^{\frac{\eta}{\sqrt{2}}}\exp(y^2)dy\left(\frac{3}{64}\eta^{17}+\frac{3}{4}\eta^{15}+\frac{21}{32}\eta^{13}-\frac{105}{8}\eta^{11}-\frac{945}{32}\eta^9\right)\right.$$

$$+\{2\left(\varepsilon_{nmi}\Omega_n+\varepsilon_{pmj}\Omega_p+\varepsilon_{qmk}\Omega_q+\varepsilon_{rml}\Omega_r\right)-Sf\}\ (t\text{-}t_1)]$$

$$+\frac{\pi}{2}\exp\left[\left\{\left(\frac{3}{2}\eta^2\right)(5.386\eta^8+9.118\eta^{10}+1.1017\eta^{12}+0.179\eta^{14}\right.\right.$$

$$-0.03106\eta^{16}-0.004942\eta^{18}-3.615\times10^{-4}\eta^{20}\times1.890\times10^{-5}\eta^{22}-7.561\times10^{-7}\eta^{24}$$

$$-2.447\times10^{-8}\eta^{26}-6.64\times10^{-10}\eta^{28}-1.55\times10^{-11}\eta^{30}\ .....)$$

$$+\{2\left(\varepsilon_{nmi}\Omega_n+\varepsilon_{pmj}\Omega_p+\varepsilon_{qmk}\Omega_q+\varepsilon_{rml}\Omega_r\right) \tag{3.11}$$

$$\eta=\nu^{\frac{1}{2}}(t-t_1)^{\frac{1}{2}}k \text{ and } \varepsilon=\nu^{\frac{1}{2}}\left(t-t_0\right)^{\frac{1}{2}}k$$

The quantity W□ is the contribution to the energy transfer arising from consideration of the 3 point correlation equation: W□ arises from consideration of the 4 point equation. Integration of Eq. (3.9) over all wave numbers shows

$$\int_{-\infty}^{\infty}W dk=0$$

That indicating the expression for W satisfies the conditions of continuity and homogeneity.
In order to obtain the energy spectrum function E, we integrate Equation (3.4) with respect to time. This integration results in

$$E=E_j+E_\beta+E_\gamma \tag{3.12}$$

$$E_j=\frac{J_0\varepsilon^4}{3\pi\nu^2(t-t_0)^2}\exp[-2\varepsilon^2+\{2(\varepsilon_{mki}\Omega_m+\varepsilon_{nkj}\Omega_n)-Qf\}(t-t_0)] \tag{3.13}$$

$$E_\beta=\frac{(2\pi)^{\frac{1}{2}}}{256\nu^{\frac{15}{2}}(t-t_0)^{\frac{13}{2}}}\exp\left(-\frac{3}{2}\varepsilon^2\right)\left(-15\varepsilon^6-12\varepsilon^8+\frac{7}{8}\varepsilon^{10}+\frac{16}{3}\varepsilon^{12}\right)$$

$$-\frac{32}{3\sqrt{2}}e^{13}\exp\left(-\frac{\varepsilon^2}{2}\right)\int_0^{\frac{\pi}{\sqrt{2}}}\exp(y^2)dy) \tag{3.14}$$

$$+\{2(\varepsilon_{mli}\Omega_m+\varepsilon_{qlk}\Omega_q)-Rf\}(t-t_0)]$$

$$E_\gamma=-\frac{\gamma_1}{v^{10}(t-t_1)^9}\left\{\frac{\pi^{\frac{1}{2}}}{32}\exp[(-\eta^2)\left(\frac{189}{64}\eta^8+\frac{1029}{256}\eta^{10}+\frac{287}{256}\eta^{12}\right.\right.$$

$$\left.+\frac{95}{512}\eta^{14}+\frac{71}{512}\eta^{16}+\frac{71}{512}\eta^{18}\exp(-\eta^2)\ [Ei(\eta^2)-0.5772]\right)$$

$$+\{2(\varepsilon_{nmi}\Omega_n+\varepsilon_{pmj}\Omega_q+\varepsilon_{rnl}\Omega_r)-Sf\}(t-t_1)]$$

$$+\left(\frac{\pi}{3}\right)^{\frac{1}{2}}\exp[\left(\frac{4}{3}\eta^2\right)\left(\frac{7}{9}\eta^8+\frac{497}{324}\eta^{10}+\frac{1001}{1458}\eta^{12}+\frac{761}{4374}\eta^{14}\right.$$

$$+\frac{1693}{19683}\eta^{16}-\frac{3926}{59049}\eta^{18}\exp\left(-\frac{2}{3}\eta^{2}\right)\left[-Ei\left(\frac{2}{3}\eta^{2}\right)-0.5772\right]\Bigg)$$

$$+\{2(\varepsilon_{nmi}\Omega_{n}+\varepsilon_{pmj}\Omega_{p}+\varepsilon_{qnk}\Omega_{q}+\varepsilon_{rnl}\Omega_{r})-Sf\}(t-t_{1})]$$

$$+\frac{\pi^{\frac{1}{2}}}{2}\exp[\left(-\frac{3}{2}\eta^{2}\right)(0.2307\eta^{10}+0.3632\eta^{12}+0.1502\eta^{14}+0.04463\eta^{16}$$

$$-0.01326\eta^{18}\exp\left(-\frac{1}{2}\eta^{2}\right)\left[Ei\left(\frac{1}{2}\eta^{2}\right)-0.5772\right]$$

$$+2.459\times10^{-3}\eta^{18}+2.935\times10^{-4}\eta^{20}+2.846\times10^{-5}\eta^{22}$$

$$+2.52\times10^{-6}\eta^{24}+1.69\times10^{-7}\eta^{26}+1.25\times10^{-8}\eta^{28}$$

$$+5.80\times10^{-10}\eta^{30}+4.00\times10^{-11}\eta^{32}+.........)$$

$$+\{2(\varepsilon_{nmi}\Omega_{n}+\varepsilon_{pmj}\Omega_{p}+\varepsilon_{qnk}\Omega_{q}+\varepsilon_{rnl}\Omega_{r})-Sf\}(t-t_{1})]$$

$$+\frac{1}{2}\pi^{\frac{1}{2}}\exp[\left(-\frac{3}{2}\eta^{2}\right)(1.077\eta^{8}+2.414\eta^{10}+1.408\eta^{12}+0.4416\eta^{14}+0.1898\eta^{16}$$

$$-0.9899\eta^{18}\exp\left(-\frac{1}{2}\eta^{2}\right)-0.5772]+6.575\times10^{-4}\eta^{18}$$

$$+3.27\times10^{-5}\eta^{20}+1.270\times10^{-6}\eta^{22}+4.03\times10^{-8}\eta^{24}$$

$$+1.08\times10^{-9}\eta^{26}+2.50\times10^{-11}\eta^{28}+5.09\times10^{-13}\eta^{30}+...)+$$

$$+\{2(\varepsilon_{nmi}\Omega_{n}+\varepsilon_{pmj}\Omega_{p}+\varepsilon_{qnk}\Omega_{q}+\varepsilon_{rnl}\Omega_{r})-Sf\}(t-t_{1})] \quad (3.15)$$

The quantity E_1 is the energy spectrum function for the final period, where E□ and E□ are the contributions to the energy spectrum arising from consideration of the three and four point correlation equations, respectively.

Equation (3.12) can be integrated over all wave numbers to give the total turbulent energy

$$\frac{1}{2}\langle u_{i}u_{i}\rangle=\int_{-\infty}^{\infty}E\,dk \quad (3.16)$$

$$\frac{\langle u_{i}u_{i}\rangle}{2}=\frac{j^{\frac{14}{9}}\nu^{\frac{5}{9}}}{\beta^{\frac{5}{9}}}\left[\frac{1}{32(2\pi)^{\frac{1}{3}}}\;T^{-\frac{5}{2}}\exp[\{Qf\text{-}2(\varepsilon_{mkl}\Omega_{m}-\varepsilon_{nkj}\Omega_{n})\}(t-t_{0})]\right.$$

$$+0.2296T^{-7}\;\exp[\{Rf\text{-}2(\varepsilon_{mli}\Omega_{m}+\varepsilon_{nlj}\Omega_{n}+\varepsilon_{qlk}\Omega_{q})\}(t-t_{0})]$$

$$+6.18\frac{\gamma_{j}\nu^{\frac{5}{9}}\nu_{0}^{\frac{5}{9}}}{\beta_{0}^{\frac{14}{5}}}\left(\frac{t\text{-}t_{1}}{t\text{-}t_{0}}\right)^{-\frac{19}{2}}T^{-\frac{5}{2}}\exp[\{Sf\text{-}2(\varepsilon_{nmi}\Omega_{m}+\varepsilon_{pmj}$$

$$+\varepsilon_{qmk}\Omega_{q}+\varepsilon_{rml}\Omega_{r})\}\,(t\text{-}t_{1})] \quad (3.17)$$

where

$$\langle u^2 \rangle = C_1 T^{-\frac{5}{2}} \exp[\{Qf - 2(\varepsilon_{mkl}\Omega_{nkj}\Omega_n)\}(t - t_0)]$$

$$+C_2 T^{-7} \exp[\{Rf - 2(\varepsilon_{mli}\Omega_m + \varepsilon_{nlj}\Omega_n + \varepsilon_{qlk}\Omega_q)](t - t_0)]$$

$$+C_2 T^{-\frac{19}{2}} \left(\frac{t - t_1}{t - t_0}\right)^{-\frac{19}{2}} \exp[\{Sf - 2(\varepsilon_{nmi}\Omega_n + \Omega_n + \varepsilon_{pmj}\Omega_p$$

$$+\varepsilon_{qmk}\Omega_q + \varepsilon_{nml}\Omega_r)\}(t - t_1)] \quad (3.18)$$

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