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On pairwise open-ness in bitopological spaces

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Abstract

The chief aim of this paper is to enunciate the profound concepts of pairwise open-ness in a bitopological space. Several researchers in the field of bitopological spaces have been utilizing the name 'pairwise open set' but so for no one could have glanced at this point appropriately so as to the continuity, compactness, connectness as well as separation can be comprehended in the most suitable form in a bitopological space. Having been made the most appropriacy and suitability of open ness, three types of pairwise open sets for a bitopological space have been sought out in way that these can be utilized in the whole discussion of bitopolization.

Key words- S_1-pairwise open sets, S_2-pairwise open sets and S_3-pairwise open sets, adjoint topology, bijoint topology and dijoint topology.

1. Introduction (Pairwise open-ness in bitopological space)

The pedestal of topology on which its whole discussion is based depends in the manipulation of defining way of open sets that through which mode the subsets of a space X are being called open. After having been owed the selected subsets of X as open sets, the standpoint of the topological discussion appears to be clear for further description. In bitopological discussion also, the types of open-ness of subsets of X should be clarified by considering the most oppropriate way of graplric. Actually, a mathematician is nothing but a good zero level designer who elucidates sometimes a very subtle fact of theology and a topologist is a man who does not know the difference between a doughnut and a coffee cup. If we go back in the field of researches of bitopological spaces, then we will find that a number of researchers have tried to define a pairwise open set in a bitopological space but, however, the orignator of this topic (J.C.Kelly) has said nothing about this. In deed, by the originator, the bitopological description, whatsoever have been discussed is based the way of cross containment definition as well as propositions and theorems. The views are follows:

Definition (1.01) (Pairwise Housdorff Spaces)

A Spaces (X, T_1, T_2) is said to be pairwise Hausdorff if, for two distinct point x and y, there are a T_1-neighbourhood U of x and a T_2 – neighbourhood V fo y such that $U \cap V = \phi$.

Definition (1.02) (Pairwise regular Spaces):
In a space (X, T_1, T_2), T_1 is said to be regular with respect to T_2 if, for each point x in X and each T_1-closed set F such that $x \notin F$, there are T_1- open set G and T_2- open set H such that $x \in G$, $F \subseteq H$ and $G \cap H = \phi$. The space (X, T_1, T_2) is said to be pairwise regular if T_1 is reguler with respect to T_2 and vice-versa.

Definition (1.03) (Pairwise normal Spaces)
A space (X, T_1, T_2) is said to be pairwise normal if, given a T_1-closed set A and a T_2-closed set B with $A \cap B = \phi$, there exists a T_2- open set U and a T_1-open set V such that $A \subseteq U$, $B \subseteq V$ and $U \cap V = \phi$. Equivalently, (X, T_1, T_2) is pairwise normal if, given a T_2-closed set C and a T_1-closed set D such that $C \subseteq D$, there are a T_1-open set G and a T_2-closed set F such that $C \subseteq G \subseteq F \subseteq D$.

Theorem. [Urysohn's Lemma]: If (X, T_1, T_2) is pairwise normal, them given a T_2- closed set F and a T_1-closed set H with $F \cap H = \phi$, there exists a real valued function f on X such that
$f(x) = o\ (x \in F)$, $f(x) = 1 (x \in H)$ and $o \leq f(x) \leq 1\ (x \in H)$,
f(x) is T_1-upper semi-continuous and T_2-lower semi-continuous.

Having observed the definitions of pairwise regular spaces and pairwise normal spaces, it is quite obvious that manipulation of existing open sets have been adjusted by considering the cross containment tact. Perhaps, the originator of this topic may have thought before giving the ideas of these definitions that T_1-open sets and T_2-open should not appear jointly in a bitopological space. No doubt, this type of bitopolization has eminence importance for conjoining two sets to obtain the various analogous results of general topology. But after having heard the name of the topic 'a bitopological space' one may consider the open sets of the space by renaming it as 'pairwise open sets'. Some years ago and recently too, a number of reseachers have tried to define the pairwise compactness in a bitopological space having made some slight differences in the following fashions:

Definition (1.04) [Peter Fletcher, Hughes B. Hoyle, C.W.Patty (FHP) – (2)]
A cover U of a bitopological space (X, T_1, T_2) is said to be a T_1T_2-open cover of X if $U \subset T_1 \cup T_2$. If , in addition, U contains at least one non-empty set of T_1 and at least one non-empty set of T_2, it is called pairwise open covering of X. If every pairwise open covering of X has a finite sub-cover, then the space is called pairwise compact.

Definition (1.05) [Y.W.Kim – (3)]
Let (X, T_1, T_2) be a bitopological space and let V be a non-empty member of T_2. Then $T_1(V) = \{O, X, \{U \cup V: U \in T_1\}\}$ is a topology on X called adjoint topology of X with respect to V. The space (X, T_1, T_2) is said to be (1,2) compact iff $(X, T_1(V))$ is compact for every non-empty member V of T_2. If , in addition, X is (2,1) – compact, we say that (X, T_1, T_2) is p-compact.

Definition(1.06) [J. Swart]
A cover U of X is said to be T_1T_2 – open cover if $U \subset T_1 \cup T_2$. The space (X, T_1, T_2) is said to be pairwise compact if every T_1T_2-open cover of X has a finite sub cover.

Definition (1.07) [Birson)]
The bitopological space (X, T_1, T_2) is said to be T_1-compact with respect to T_2 if each T_1-open covering of X can be reducible to a finite T_2-open sub-covering of X. The space is said to be B-compact if it is T_1-compact with respect to T_2 and T_2- compact with respect to T_1.

Out of these definitions of pairwise compactness several others also have tried to give the concept of pairwise compactness in different ways, but the initially originated theory on bitolopogical spaces by

Kelly, J.C. (1) says nothing about pairwise compactness, that it should be defined by considering pairwise open sets. Recently(2010), a paper has been published by Int. J. Contemp. Maths(6) with the names of the authers O.Ravi, S.Pious Missier and T.Salai Parkunan in which an attempt has been made to define the pairwise open sets and pairwise closed sets in the followng way:

Definition (1.08.): A subset S of a bitopologicl space (X T_1, T_2) is called $T_{1,2}$–open if S=A∪B, where A $\in T_1$ and B $\in T_2$. The complement of $T_{1,2}$–open set is said to be $T_{1,2}$–closed

The authors of this paper have defined also the various terminologies of general topologies such as closure and interior of a set by applying exactly the same concept on $T_{1,2}$–open set and $T_{1,2}$–closed set as described in general topology. We are not vigorously opposing this aspect of open-ness of a set, but this idea may need to be reformation for the most appropriative and suitable presentation. In this discussion, we are going to express the various ideas of pairwise open-ness of a set for a bitopological space and that sort of expression will be used also in the whole discussion.

Definition (1.09.) [S_1 – Pairwise open set]

A subset G of X will be said to be $T_{12(A)}$ – open if it can be expressed as $G = A \cup Bi$, Where $A \in T_1$ is fixed, $B_i \in T_2$, $A \neq \phi$, $A \neq X$, $B_i \neq X$.

A subset H of X will be said to be $T_{21(B)}$ – open if it can be expressed as $H = B \cup A_i$, Where $B \in T_2$ is fixed, $A_i \in T_1$, $B \neq \phi$, $B \neq X$, $B_i \neq X$.

In fact, $T_{12}(A) = \{\phi, X, A \cup B_i : A \in T_1, B_i \in T_2\}$ and

$T_{21}(B) = \{\phi, X, B \cup A_i : B \in T_2, A_i \in T_1\}$

are topologies on X relative to A and B respectively. We are verifying that $T_{12}(A)$ is a topology on X.

(i) $\phi \in T_{12}(A), X \in T_{12}(A)$

(ii) Let $F = \{A \cup B_i : i \in I\}$ be an arbitrary indexed family of members of $T_{12}(A)$. Then $\bigcup_{i \in I} (A \cup B_i)\ A \cup (\bigcup_{i \in I} B_i)$ and since $\bigcup_{i \in I} B_i \in T_2$, therefore $A \cup (\bigcup_{i \in I} B_i) \in T_2, (A)$.

(iii) Let $A \cup B_1$ and $A \cup B_2$ be any two members of $T_{12}(A)$. Then $(A \cup B_1) \cap (A \cup B_2) = A \cup (B_i \cap B_2)$ and since $B_i \cap B_2 \in T_2$, therefore $(A \cup B_1) \cap (A \cup B_2) \in T_{12}(A)$.

The complement of the corresponding open sets will be called analogously closed sets.

Now, In this it is quite evident that we are getting two types of family of topologies like $A = \{ T_{12}(A) : A \in T_1 \}$ and $B = \{ T_{21}(B) : B \in T_2 \}$ in which $A \cap B$ may not be empty. The geometrical aspect of pairwise open spheres suggests to a assume that the open sets of T_1 and T_2 should not be disjoint at all for all the members of T_1 and T_2 whereas some of the members of T_1 and T_2 may be alike. So far, our knowingness is concern we assert that no any researcher in the field of bitopological spaces has clarified the distinction between pairwise open sets and a pair of open sets of T_1 and T_2 . In our work we are going to enunciate the difference between them explicitly. As accordance with the originator (Kelly, 1963) of the field of bitopological spaces, the cross- containment definitions of pairwise regular spaces and pairwise normal spaces indicates to consider the pairwise open sets in such a way that it should be open in both the topologies. The FHP (Peter Fletcher, Hughes B. Hoyle and C.W Patty) description on 'The Comparison of Topologies' convinces that the two topologies T_1 and T_2 may be equal under certain conditions. Therefore the concept of 'pairwise open set' might be considered as a set which is open in both the topologies. Again, if $A \in T_1$ and $B_i \in T_2$, then $A \cup B_i$ should be assumed as pair of open sets of T_1 and T_2 provided that $A \notin T_2$, and $B_i \notin T_2$. If $A \cup B_i$ belongs to both the topologies T_1 and T_2 , then $A \cup B_i$ should be considered as a 'pairwise open set'. However, if $A \cup B_i$ belongs to both the topologies $T_{12}(A)$ and $T_{21}(B)$, then it should be considerd as bipairwise open set.

If $A \cup B_i$ does not belongs to $T_1 \cap T_2$, then it should be simply called a pair of open sets of T_1 and T_2

Definitions (1.10) [S_2 - pariwise open sets]

A subset G of X will be said to a pairwise open set of kind S_2 if G is expressed as $G = A_i \cup B_i$, where $A_i \in T_1$, $B_i \in T_2$, $A_i \neq \phi$, $B_i \neq \phi$, $A_i \neq X$, $B_i \neq X$, provided that $A_i \cup B_i \in T_1 \cap T_2$, otherwise it will be called only a pair of open sets of T_1 and T_2.

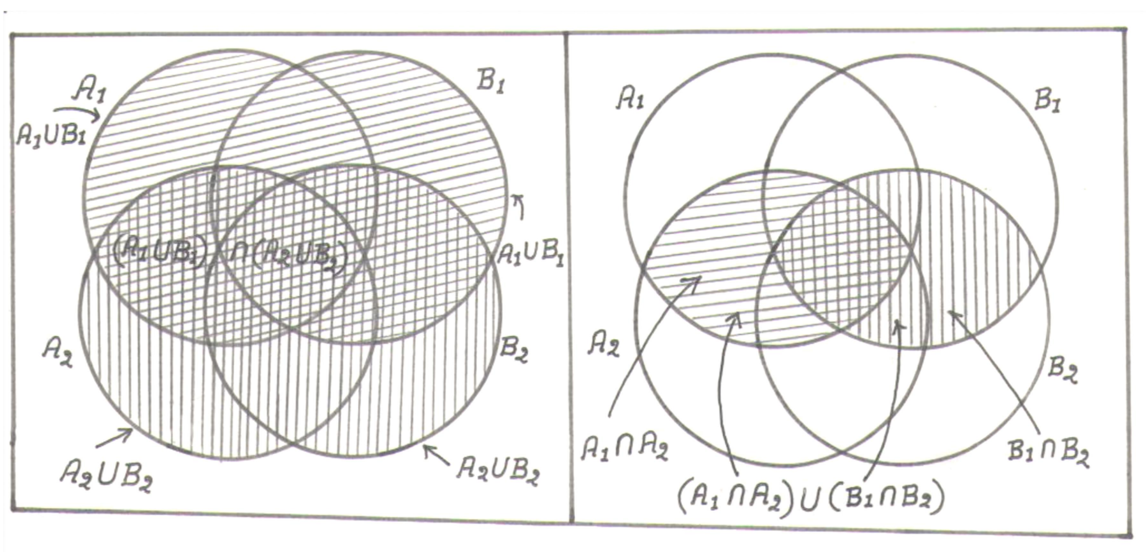
In fact, the collection $T_{AB} = \{\phi, X, A_i \cup B_i: A_i \in T_1, B_i \in T_2\}$ of subsets of X forms a topology on X. For,

(i) $\phi \in T_{\overline{AB}}$, $X \in T_{\overline{AB}}$,

(ii) If $\{A_i \cup B_i: A_i \in T_1, B_i \in T_2\}$ be an arbitrary family of members of T_{AB}, then $\bigcup_{i \in I} (A_i \cup B_i) = (\bigcup_{i \in I} A_i) \cup (\bigcup_{i \in I} B_i)$ and since $(\bigcup_{i \in I} A_i) \in T_1$, $(\bigcup_{i \in I} B_i) \in T_2$,

Therefore $\bigcup_{i \in I} (A_i \cup B_i) \in T_{AB}$.

(iii) Let $A_1 \cup B_1$ and $A_2 \cup B_2$ be any two members of T_{AB}. Then $(A_1 \cup B_1) \cap (A_2 \cup B_2) = (A_1 \cup A_2) \cup$



$(B_1 \cup B_2)$ and since $A_1 \cup A_2 \in T_1$, $B_1 \cup B_2 \in T_2$, therefore $(A_1 \cup B_1) \cap (A_2 \cup B_2) \in T_{\bar{AB}}$.

Thus $T_{\bar{AB}} = \{\phi, X\ A_i \cup B_i: A_i \in T_1, B_i \in T_2\}$ is topology on X called bijoint topology of X. Evidently, the subsets of X of the form $A_i \cup B_i$ are open sets with respect to $T_{\bar{AB}}$. In bitopological discussion, the open-ness of $A_i \cup B_i$ with respect to $T_{\bar{AB}}$ has no meaning of pairwise open-ness at all. Now , it is markable fact that the subsets of X of the form $A_i \cup B_i$ is not necessary to be open in X with respect to either T_1 or T_2 or both. If $A_i \cup B_i$ is open in X with respect to T_1, then it will be said to TA_1B - open; if it is open in X with respect to T_2, then it will be said to be TAB_2 - open; and if it is open inX with respect to both the topologies T_1 and T_2 , then it will be said to be pairwise open. If $A_i \cup B_i$ is neither open with respect to T_1 nor with respect to T_2 , then it will be said to be simply a pair of open sets.

The complement of the corresponding open sets will be called analogously closed sets

Definition (1.11) [S_3 -pairwise open sets]:

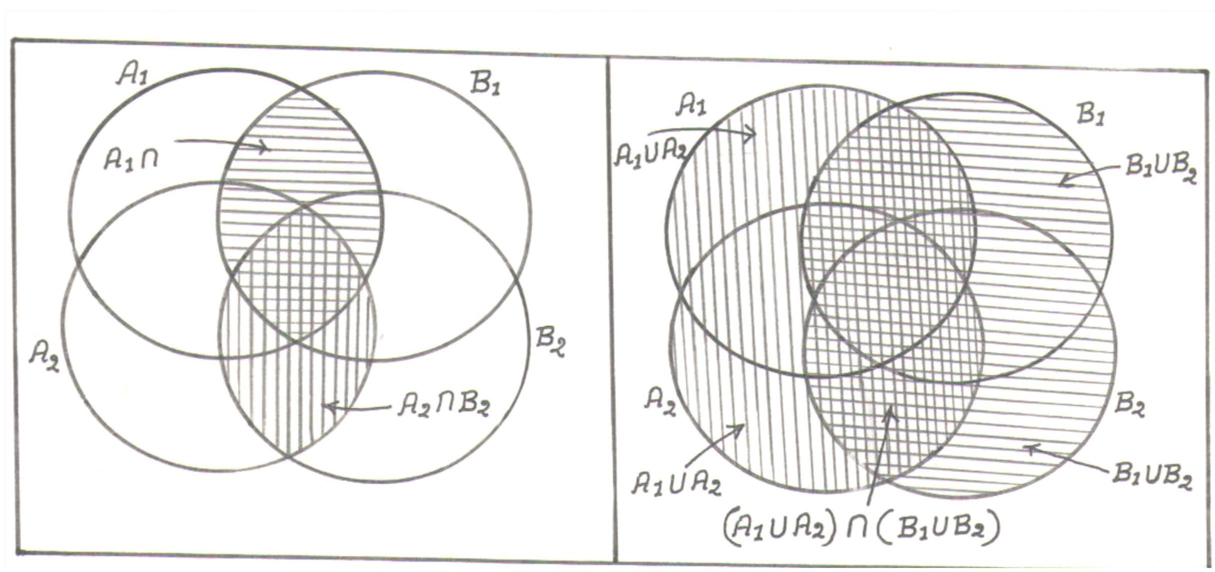
A subset G of X will be said to be pairwise open set of kind S_3 if G be expressed as $G = A_i \cap B_i$, $A_i \neq \phi$, $B_i \neq \phi$, $A_i \neq X$, $B_i \neq X$, provided that $A_i \cap B_i \in T_1 \cap T_2$, otherwise it will be said to be only a pair of open set produced by T_1 and T_2 .

In deed, here in this case also, the family $T_{\overline{AB}} = \{\phi\ A_i \cap B_i : A_i \in T_1, B_i \in T_2 \}$forms a topology on X. For,

(i) $\phi \in T_{\overline{AB}}$, $X \in T_{\overline{AB}}$.

(ii) If $\{A_i \cap B_i : A_i \in T_1, B_i \in T_2 \}$ be an arbitrary family of $T_{\overline{AB}}$, then

$\bigcup_{i \in I} (A_i \cup B_i) = (\bigcup_{i \in I} A_i) \cap (\bigcup_{i \in I} B_i)$ and since $\bigcup_{i \in I} A_i \in T_1$, $\bigcup_{i \in I} B_i \in T_2$, therefore $(\bigcup_{i \in I} A_i) \cap (\bigcup_{i \in I} B_i) \in T_{\overline{AB}}$.



(iii) Let $A_1 \cap B_1$ and $A_2 \cap B_2$ be nay two members of $T_{\overline{AB}}$. Then obviously, we have $(A_1 \cap B_1) \cap (A_2 \cap B_2) = (A_1 \cap A_2) \cap (B_1 \cap B_2)$ and since $A_1 \cap A_2 \in T_1$, $B_1 \cap B_2 \in T_2$, therefore $(A_1 \cap A_2) \cap (B_1 \cap B_2) \in T_{\overline{AB}}$.

Thus $T_{\overline{AB}}$, $\{\phi\ A_i \cap B_i : A_i \in T_1, B_i \in T_2 \}$ is a topology on X called dijoint topology of X. Obviously the subsets of X of the from $A_i \cap B_i$ are open sets with respect to the topology $T_{\overline{AB}}$.

Following the description illustrated in S_2 pariwise open-ness, we have at this point too the analogous elucidation can be given. The notable point is that the set of the form $A_i \cap B_i$ may or may not be open in X with respect to either T_1 or T_2 both. If $A_i \cap B_i$ is open in X with to T_1, then it will be said to be $T_{\overline{A1B}}$ -open ; if it is open in X with respect to T_2, then it will be said to be $T_{\overline{AB2}}$ -open; and if it is open in X with respect to T_1 and T_2 both then it will be said to be S_3 pairwise open. If $A_i \cap B_i$ is neither open in X with respect to T_1 nor with respect to T_2, then it should be recalled only by saying a pair of open sets guided by T_1 and T_2.

The complement of the corresponding open sets will be called analogously closed sets

References

1. (1971). Total dis-connectedness in bitopological spaces and product of bito-polical spaces. *Proc.Kon.Ned. Akad*.v. Wetensch, Amsterdam, A74135-145
2. Lane, E (1967). Bitopological spaces and quasi-unifoem spaces. Proc. Lon. Math soc, 17 2:241-256 MR 34: 5054
3. Singal, A.S. and A.S.Arya (1971). 10O n pariwise almost regular spaces. *Glasnik Math*, 6: 335-343.
4. Balachandran, K; Sundsram, P; Maki, H. (1991). On generalized conti- nuous maps in bitopological spaces: Fac. Sci Kochi Univ. (math), 12, 5-13
5. Fletcher, P., H.B.Hoyle and C.W.Patty (1969). Comparison of topologies By: *Duke Math. J*.36, 325-331.
6. Kelly, J.C. (1963). Bitopological space. *Proc. Lon. Math. Soc*. (3):1372-89
7. Lellis Thivagar, M; Ravi, O. (2004). On stronger forms of (1, 2)* -quotient mappings in bitopological spaces.*Internat. J.Math. Game theory and Algebra.* 14(6):481-492.
8. Mukherjee, M.N. (1985). On pairwise S-closed bitopological spaces. *Journal of Math. Sci*, 8, 729-745
9. Patty, C.W. (1966). Bitopological Space.The umiversity of north Carolona at Chapel Hill.
10 Ravi, O., S.Pious (2010).On Bitopological (1, 2).Generalized Homeomorphisos Missier and T.Salani Parkunan: Int. *Journal of Contemp. Math.sciences*. 5 (11): 543-557

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