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Propagation of explosion waves in a dusty gas

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Abstract

The propagation of the variable energy explosion wave in a dusty gas is studied by using the method of similarity solution. The variations in the flow variables behind the shock are calculated numerically and the results obtained here are compared with those in absence of solid particles.

Introduction

The study of the problem of propagation of explosion waves in a mixture of gas and solid particles is of great interest to the research workers due to its applications in various branches of science and technology Pai *et al.* (1980), Higashino and Suzuki (1980), Narasimhulu *et al.* (1985) have investigated the flow field behind a propagating blast wave in a mixture of gas and solid particles by using the gas mixture modal of Pai (1977) and studied the behaviour of such waves in presence of solid particles. Rogers (1958), Freeman (1968), Dirctor and Dabora (1977) have studied the propagation of variable energy explosion waves in which the energy input varies proportional to some power of time. In present study, our aim is to consider two phase flow of a mixture of gas and small solid particles and study the propagation of variable energy explosion wave. In order to get some essential features of shock propagation, small solid particles are considered as a pseudo-fluid and it is assumed that the equilibrium flow condition is maintained in the flow field and that the viscous stress and heat conduction of the mixture are negligible (Pai *et al.* 1980).

2. Basic Equations

The one dimensional unsteady flow field in a mixture of gas and small solid particles is a function of two independent variables, the space coordinate r and the time t. In absence of viscous stress and heat conduction the equations governing the flow in spherical symmetry are given by

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \rho \frac{\partial u}{\partial r} + \frac{2 \rho u}{r} = 0 \qquad (2.1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} = 0 \qquad (2.2)$$

$$\frac{\partial e}{\partial t} + u\frac{\partial e}{\partial r} - \frac{p}{\rho 2}\left(\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r}\right) = 0 \quad (2.3)$$

where u is the velocity of the mixture along the direction of r, p the pressure of the mixture, ρ the density of the mixture and e is the internal energy of the mixture per unit mass.
The equation of state of the mixture under equilibrium condition is

$$p = \frac{\left(1 - k_{\rho}\right)\rho R'T}{1 - Z} \quad (2.4)$$

where kp is the mass concentration of the solid particles in the mixture taken as constant in the whole flow field, Z the volume fraction of the solid particles, R′ the gas constant and T the temperature.
The relation between kp and Z is given by Conforto (2001)

$$k_{\rho} = \frac{Z\rho_{sp}}{\rho} \quad (2.5)$$

where ρ sp is the species density of the solid particles.
The volumetric fraction of the dust in the mixture at a considered state (ρ,T) is given by

$$\frac{Z}{\rho} = \frac{Z_0}{\rho_0} = \text{Constt} \quad (2.6)$$

where Z0 and ρ0 are the initial values of Z and ρ.
Z0 is given by the relation.

$$Zo = \frac{k_p}{(1 - k_p)G + k_p} \quad (2.7)$$

where $G = \frac{\rho_{sp}}{\rho_g}$ is the ratio of the density of the solid particles to the species density of the gas.
The internal energy of the mixture is related to the internal energies of the two species and may be written as

$$e = \frac{(1 - Z)}{(\Gamma - 1)}\frac{p}{\rho} \quad (2.8)$$

with $\Gamma = \frac{\gamma + \delta\beta}{(1 + \delta\beta)}$, $\delta = \frac{k_p}{1 - k_p}$, $\beta = \frac{c_{sp}}{c_v}$, $\gamma = \frac{c_p}{c_v}$

where csp is the specific heat of the solid particles, cp the specific heat of gas at constant pressured and cv the specific heat of the gas at constant volume.
The equilibrium speed of sound in a mixture of gas and solid particles may be written as

$$c = \left[\frac{\Gamma p}{\rho(1 - Z)}\right]^{1/2} \quad 2.9$$

3. Boundary Conditions-

Consider a shock wave be propagating into a mixture of perfect gas and small solid particles of constant density at rest. The shock conditions across the shock are (Steiner and Hirschler (2002), Ojha and Srivastava (2007))

$$\rho_1(U - u_1) = \rho_0 U \tag{3.1}$$

$$p_1 + \rho_1(U - u_1)^2 = p_0 + \rho_0 U^2 \tag{3.2}$$

$$\frac{1}{2}(U - u_1)^2 + \frac{(\Gamma - Z_1)p_1}{(\Gamma - 1)\rho_1} = \frac{1}{2}U^2 + \frac{(\Gamma - Z_0)p_0}{(\Gamma - 1)\rho_0} \tag{3.3}$$

where the subscript 0 and 1 refer to the value of the quantities in front of and behind the shock and U denotes the velocity of the shock in gas mixture at rest.

From (3.1) – (3.3) we may write

$$\frac{\rho_1}{\rho_0} = \frac{\Gamma + 1}{(\Gamma - 1 + 2Z_0) + \frac{2\Gamma}{\gamma M^2}} \tag{3.4}$$

$$\frac{u_1}{U} = \frac{2\left(1 - Z_0 - \frac{\Gamma}{\gamma M^2}\right)}{\Gamma + 1} \tag{3.5}$$

$$\frac{p_1}{\rho_0 U^2} = \frac{1}{\gamma M^2} + \frac{2\left(1 - Z_0 - \frac{\Gamma}{\gamma M^2}\right)}{\Gamma + 1} \tag{3.6}$$

where $M^2 = \frac{U^2}{\gamma p_0 / \rho_0}$

In absence of solid particles (3.4)- (3.6) reduce to shock conditions in ordinary perfect gas. When velocity of the shock wave is initially large the above conditions are given by (Steiner and Hirschler 2002)

$$\frac{\rho_1}{\rho_0} = \frac{\Gamma + 1}{\Gamma - 1 + 2Z_0} \tag{3.7}$$

$$\frac{u_1}{U} = \frac{2(1 - Z_0)}{\Gamma + 1} \tag{3.8}$$

$$\frac{p_1}{\rho_0 U^2} = \frac{2(1 - Z_0)}{\Gamma + 1} \tag{3.9}$$

4. Similarity Solutions

Let the solution of the problem exist in similarity form as

$$u = U f(x) \tag{4.1}$$

$$\rho = \rho_0 g(x) \tag{4.2}$$

$$p = \rho_0 U^2 h(x) \tag{4.3}$$

where x is the similarity variable given by $x = \frac{r}{R}$, R denotes the shock radius which is a function of time only. The shock velocity U is given by

$$U = \frac{dR}{dt} \quad (4.4)$$

The total energy of the flow may be written as

$$E = 4\pi \int_o^R \left(\frac{1}{2}\rho U^2 + \rho e\right) r^2 dr \quad (4.5)$$

using similarity transformations in (4.5) we get

$$E = 4\pi \rho_0 R^3 U^2 \int_{x_0}^1 \left[\frac{1}{2} g f^2 + \left(\frac{1 - Z_0 g}{\Gamma - 1}\right) h\right] x^2 dx \quad (4.6)$$

where x_0 is the co-ordinate of the expanding surface. Suppose that the total energy is allowed to vary with time such that

$$E = E_0 t^q \;, \quad q \geq 0 \quad (4.7)$$

where E_0 and q are constants.

From (4.4) and (4.6) it follows that the motion of the shock front is given by the equation.

$$R^{3/2} \frac{dR}{dt} = \left(\frac{E_0}{4\pi \rho_0 J}\right)^{1/2} t^{q/2} \quad (4.8)$$

where

$$J = \int_{x_0}^1 \left[\frac{1}{2} g f^2 + \frac{h(1 - Z_0 g)}{\Gamma - 1}\right] x^2 dx \quad (4.9)$$

Equation (4.8) on integration yields

$$R = \left(\frac{5}{q+2}\right)^{2/5} \left(\frac{E_0}{4\pi \rho_0 J}\right)^{1/5} t^{(q+2)/5} \quad (4.10)$$

It is clear from equation (4.10) that the value of q=3 corresponds to the uniform expansion of a surface. The assumption $E=E_0 t^q$ includes the blast waves when q=0. Therefore, the solution of physical significance appears for the values of q which are between 0 and 3.

After using the similarity transformation equations, the equations (2.1) - (2.3) take the forms-

$$(x - f)\frac{dg}{dx} = g\left(\frac{df}{dx} + \frac{2f}{x}\right) \quad (4.11)$$

$$(x - f)\frac{df}{dx} = \frac{1}{g}\frac{dh}{dx} + \left(\frac{q-3}{q+2}\right) f \quad (4.12)$$

$$(x - f)\frac{dh}{dx} = \frac{\Gamma h}{1 - Z_0 g}\left(\frac{df}{dx} + \frac{2f}{x}\right) + 2h\left(\frac{q-3}{q+2}\right) \quad (4.13)$$

At the shock front the boundary conditions of the problem is given by

$$g(1) = \frac{\Gamma + 1}{(\Gamma - 1 + 2Z_0) + \dfrac{2\Gamma}{\gamma M^2}} \quad (4.14)$$

$$f(1) = \frac{2\left(1 - Z_0 - \dfrac{\Gamma}{\gamma M^2}\right)}{\Gamma + 1}$$

(4.15)

$$h(1) = \frac{1}{\gamma M^2} + \frac{2(1-Z_0) - \frac{2\Gamma}{\gamma M^2}}{\Gamma + 1} \tag{4.16}$$

In case of strong shocks these boundary conditions take the forms

$$g(1) = \frac{\Gamma + 1}{\Gamma - 1 + 2Z_0} \tag{4.17}$$

$$f(1) = \frac{2(1-Z_0)}{\Gamma + 1} \tag{4.18}$$

$$h(1) = \frac{2(1-Z_0)}{\Gamma + 1} \tag{4.19}$$

By solving equations (4.11), (4.12) and (4.13) for $\frac{df}{dx}$, $\frac{dg}{dx}$ and $\frac{dh}{dx}$ we have:

$$\frac{df}{dx} = \frac{x-f}{(x-f)^2 - \frac{\Gamma h}{g(1-Z_0 g)}} \left[\left(\frac{q-3}{q+2} \right) f + \frac{2\Gamma f h}{x(x-f)g(1-Z_0 g)} + \frac{2h}{(x-f)g} \left(\frac{q-3}{q+2} \right) \right] \tag{4.20}$$

$$\frac{dg}{dx} = \frac{g}{(x-f)} \left[\left(\frac{df}{dx} + \frac{2f}{x} \right) \right] \tag{4.21}$$

$$\frac{dh}{dx} = g \left[(x-f)\frac{df}{dx} - \left(\frac{q-3}{q+2} \right) f \right] \tag{4.22}$$

Table 1

k_p	δ	Γ	Z_o		
			G=1	G=10	G=100
0	0	1.4000	0	0	0
0.1	0.1111	1.3600	0.1	0.010989	0.0011099
0.3	0.4286	1.2799	0.3	0.04109	0.004267

Variation of δ, Γ and Z_o with k_p for β=1.

5. Results and Discussion

Table 1 gives the variation of δ, Γ and Z_o with G=1, 10, 100 for the values of k_p =0, 0.1, 0.3 with β=1. Equations (4.20), (4.21) and (4.22) are integrated numerically with boundary conditions (4.14), (4.15) and (4.16) for weak shocks and boundary conditions (4.17), (4.18) and (4.19) for strong shocks respectively for the values of Z_o given in table 1 and for M^2 =10, γ= 1.4 and q = 2.5 and results are plotted in figures 1(a), 1(b), 1(c) and 2(a), 2(b), 2(c) for weak and strong shocks respectively.Figures 1(a) and 2(a) give the variation of velocity behind the shock wave with boundary conditions for weak and strong shocks respectively. These figures show that the velocity increases behind the shock in presence and absence of solid particles. For low compressibility i.e. for G=1 the velocity increases faster than those in case of high compressibility. For the high values of the compressibility the variation in velocity is negligible in comparison to the velocity in absence of solid particles.Figures 1(b) and 2(b) give the variation of density with initial conditions for weak and strong shocks respectively behind the shock in presence and absence of solid particles These figures show that decrease in density becomes faster behind the shock as compressibility increases. In case of low compressibility the decrease in density is so slow that it is not possible to differentiate with the decrease in density in absence of solid particles. Fig 1(c) and 2(c) give variation of pressure behind the shock for weak and strong shocks respectively in absence and presence of solid particles. These figures show that pressure first increases and then decreases behind the shock in both the case i.e. in absence and presence of solid particles as we move towards the centre of explosion. At low compressibility the increase in pressure is faster in comparison to high compressibility. Thus, the variations in flow variables behind the shock are similar in case of weak and strong shocks both and the presence of solid particles affects significantly the behaviour of the flow behind the shock wave.

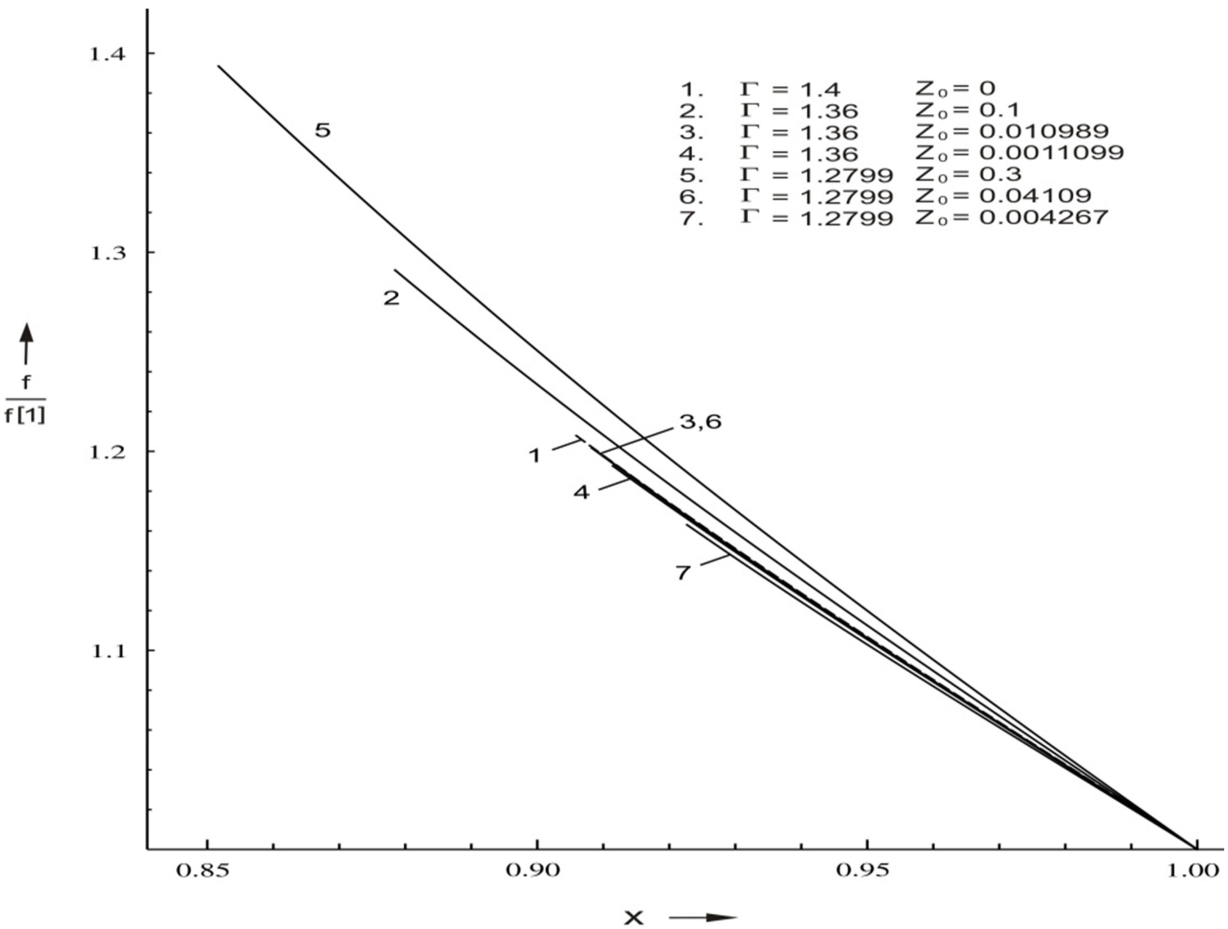

Figure 1(a) : variation of f/f[1] w.r.t. x for weak shock.

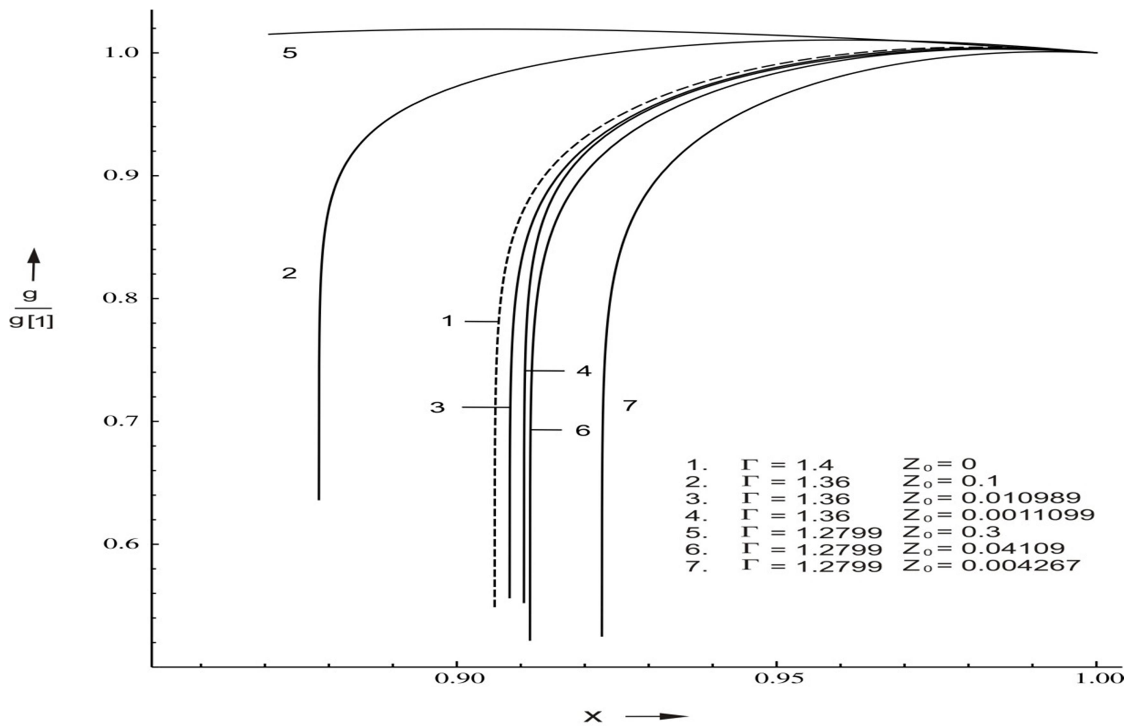

Figure 1(b) : variation of g/g[1] w.r.t. x for weak shock.

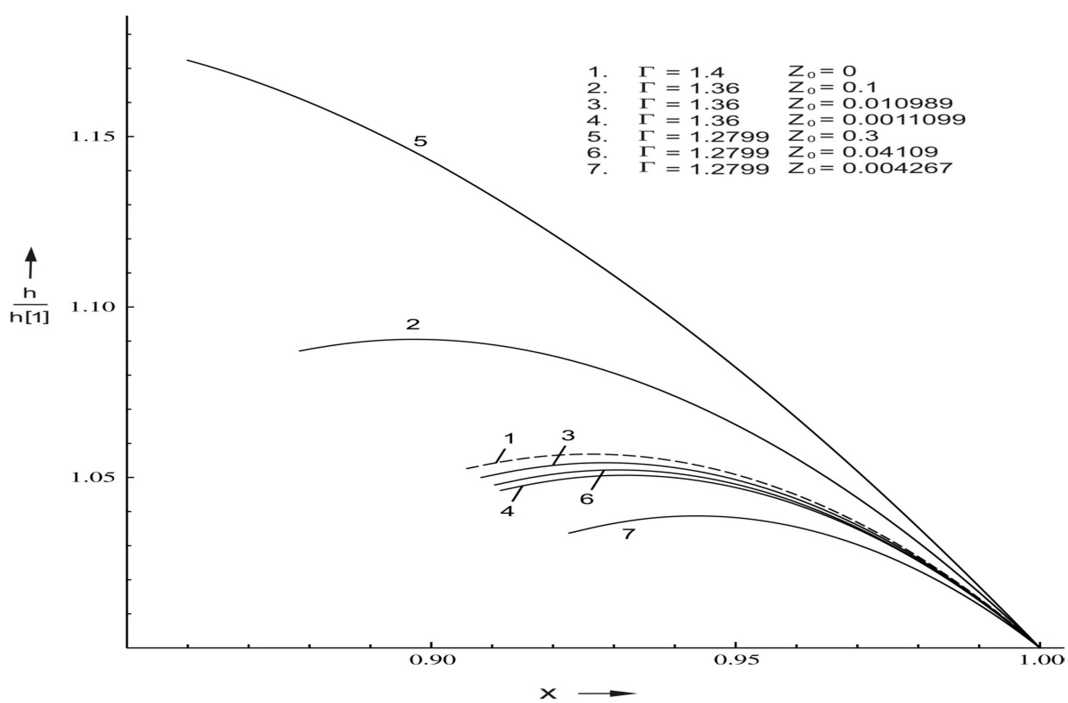

Figure 1(c) : variation of h/h[1] w.r.t. x for weak shock.

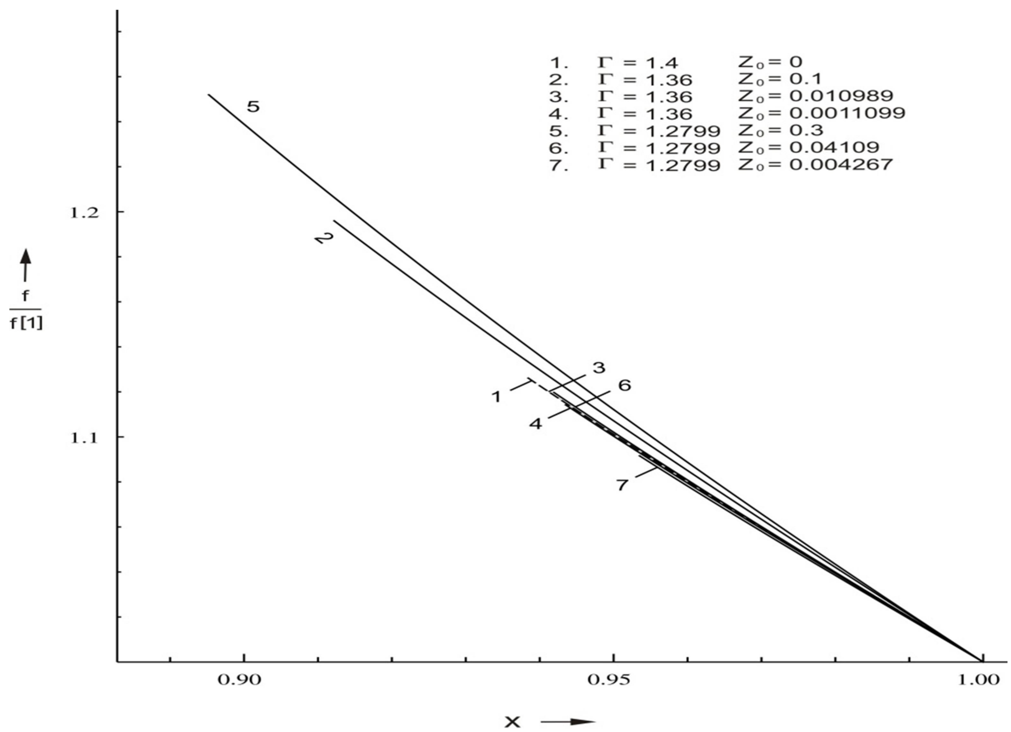

Figure 2(a) : variation of f/f[1] w.r.t. x for strong shock.

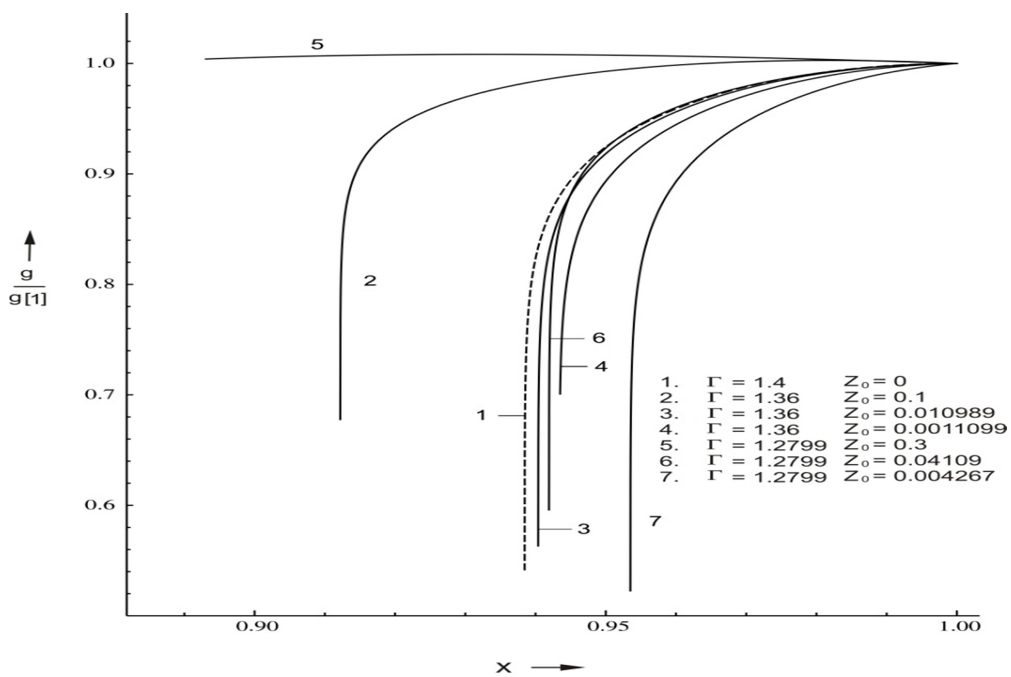

Figure 2(b) : variation of g/g[1] w.r.t. x for strong shock.

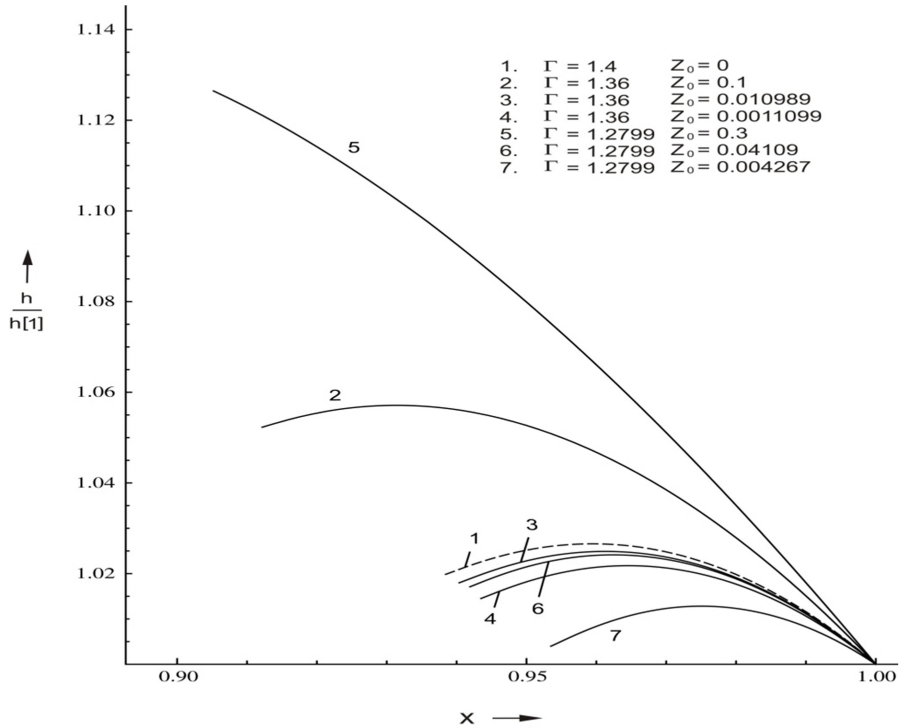

Figure 2(c) : variation of h/h[1] w.r.t. x for strong shock.

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