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Quarter symmetric metric connection in a locally conformal kahler manifold

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Abstract

Quarter symmetric metric connection was defined and studied by Golab (1975). In 1980, R.S. Mishra and S.N. Pandey (1979) studied this connection in Kahlerian and Sasakian manifold. Yano and Imai (1979) studied the curvature properties of quarter symmetric metric connection. In the present paper we have taken a quarter symmetric metric connection in a Locally Conformal Kahler (L.C.K.) manifold and obtained some of the properties of its curvature tensor.

Keywords- Quarter symmetric, matric connection. locally conformalKahle manifold

1. Introduction

Let M be a real 2n-diemnsional Hermitian manifold with metric g, a complex structure F and fundamental form 'F given by 'F (X, Y) = g (FX, Y). Then M is a locally conformal Kahler manifold (an l.c.k. manifold) if at any point of M there exists a neighbourhood in which a conformal metric.

$$g' = e^{-2\rho} g \text{ is a Kahler one i.e. } \quad \nabla' = (e^{-2\rho} F) = 0 \qquad d\rho = \quad \alpha \tag{1.1}$$

where ∇' is the covarian derivative with respect to g' [1].

We also have [1]

$$(\nabla_X\ 'F)\ (Y, Z) = -\ \beta\ (Y)\ g\ (X, Z) + \beta\ (Z)\ g(X, Y)\ -\ \alpha\ (y)\ 'F\ (x, Z) + \alpha(Z)\ 'F\ (X, Y) \tag{1.2a}$$

$$(\nabla_X \alpha)\ Y - (\nabla_Y \alpha)\ X = 0 \tag{1.2b}$$

for any vector field X, Y and Z tangent to M, where ∇ denotes the covariant differentiate with respect to g and the 1-form α and β are defined by

$$\alpha(X) = g\ (X, \alpha) \text{ and } \qquad \beta(X) = -\ \alpha(FX) \tag{1.3}$$

In an l.c.k. manifold, K. Matsumato [1] defined a symmetric tensor field P (X, Y) as

$$P(X, Y) = -(\nabla_X \alpha) Y - \alpha(X)\alpha(Y) + \|\alpha\|^2 \frac{g(X, Y)}{2} \tag{1.4}$$

where $\|\alpha\|$ denotes the length of the lee form α with respect to g.

Kashiwada (1982) obtained the following formulas on a l.c.k. manifold.

$$(\nabla_X \beta)(Y) = -\beta(X)\alpha(Y) + \beta(Y)\alpha(X) - \|\alpha\|^2 F(X, Y) + {}'F(\nabla_X \alpha, Y) \tag{1.5a}$$

$$\nabla_r \beta^r = 0 \tag{1.5b}$$

2. Quarter symmetric metric connection in a l.c.k. manifold

Golab (1980) defined a quarter symmetric metric connection in a Riemannian manifold as a linear connection ∇ whose torsion tensor S is given by

$$S(X, Y) = p(Y)\, t(X) - p(X)\, t(Y) \tag{2.1}$$

where p is a 1- from and t is a tensor of type (1,1) In 1980, Mishra and Pandey (1991) introduced this connection by replacing t(X) by F (X).
In the present paper we have replaced the 1-form p by the lee form α Golabo, 1975 and taken the torsion tensor as

$$S(X, Y) = \alpha(Y) FX - \alpha(X) FY \tag{2.2}$$

Then the quarter symmetric metric connection $\overset{\circ}{\nabla}$ with torsion tensor (2.2) is given by

$$\overset{\circ}{\nabla}_X Y = \nabla_X Y - \alpha(X) FY \tag{2.3}$$

Proposition (2.1) In a l.c.k. manifold with quarter symmetric metric connection $\overset{\circ}{\nabla}$ we have the following relations:

$$\overset{\circ}{\nabla}_X F \quad (Y) = (\nabla_X F)(Y) \tag{2.4a}$$

$$\left(\overset{\circ}{\nabla}_X \alpha\right)(Y) - \left(\overset{\circ}{\nabla}_Y \alpha\right)(X) = \beta(X)\alpha(Y) - \beta(Y)\alpha(X) \tag{2.4b}$$

$$\left(\overset{\circ}{\nabla}_X \beta\right)(Y) + \left(\overset{\circ}{\nabla}_Y \beta\right)(X) = {}'F(\overset{\circ}{\nabla}_X \alpha, Y) + {}'F(\overset{\circ}{\nabla}_Y \alpha, X) \tag{2.4c}$$

$$\left(\overset{\circ}{\nabla}_r \beta^r = \right) \|\alpha\|^2 \tag{2.4d}$$

Proof: Putting FY for Y in (2.3) and using (2.3) again we get (2.4a), (2.4b) is obtained directly from (1.2b). Using (2.3) we get

$$\left(\overset{\circ}{\nabla}_X \beta\right)(Y) = \nabla_X \left(\beta \ (Y)\right) + \alpha(X)\alpha(Y) \tag{2.5}$$

With the help of (1.5a), (2.3) gives (2.4c) Now (2.5) gives

$$\overset{\circ}{\nabla}_X \beta = \nabla_X \beta + \alpha(X)\alpha$$

Contracting and using (1.5b) we set (2.4d)

3. Curvature tensor of a quarter symmetric metric connection

Let $\overset{\circ}{R}$ and R be the curvature tensors of the connection $\overset{\circ}{\nabla}$ and ∇ respectively. Then from (1.2a), (1.2b) and (2.3), we get

$$\overset{\circ}{R}(X, Y, Z) = R(X, Y, Z) + \alpha(X)(\nabla_y F)(Z) - \alpha(Y)(\nabla_X F)(Z) \qquad (3.1a)$$

$$\overset{\circ}{R}(X, Y, Z) = R(X, Y, Z) + \alpha(X)[-\beta(Z)Y + g(Y, Z)\beta - \alpha(Z)FY + {}'F(Y, Z)\alpha] - \alpha(Y)[-\beta(Z)X + g(X, Z)\beta - \alpha(Z)FX + {}'F(X, Z)\alpha] \qquad (3.1b)$$

Now, denoting the Ricci tensor of $\overset{\circ}{\nabla}$ and ∇ as $\overset{\circ}{R}$ic and Ric respectively and contracting (3.1b) with respect to X, we get

$$\text{Ric}(Y, Z) = \text{Ric}(Y, Z) + (n-3)\alpha(Y)\beta(Z) + \beta(Y)\alpha(Z) + {}'F(Y, Z)\|\alpha\|^2 \qquad (3.2)$$

Which gives

$$\overset{\circ}{R}(Y) = R(Y) + (n-3)\alpha(Y)\beta + \beta(Y)\alpha + \|\alpha\|^2 FY \qquad (3.3)$$

where $\overset{\circ}{R}$ic (Y, Z) = g ($\overset{\circ}{R}$ (Y), Z) and Ric (Y, Z) = g (R (Y), Z). Contracting (3.3), we get

$$\overset{\circ}{r} = r \qquad (3.4)$$

where $\overset{\circ}{r}$ and r are the scalar curvatures with respect to $\overset{\circ}{\nabla}$ and ∇ respectively.

T. Kashiwada [5] proved the following theorem.

Theorem (3.1), In a l.c.k. manifold $M^n(\phi, g, \alpha)$ $(n \neq 2)$ the relation $R(\phi X, \phi Y, \phi Z, \phi W) = R(X, Y, Z, W)$ holds good if and only if the Ricci tensor is hybrid. Here prove the following Lemma.

3.1 Lemma The Ricci tensor with respect to the quarter symmetric connection $\overset{\circ}{\nabla}$ in a l.c.k. manifold $M^n(F, g, \alpha)$ is hybrid iff Ric is hybrid and

$$\beta(Y)\alpha(Z) + \alpha(Y)\beta(Z) = 0 \qquad (3.5a)$$

$$\alpha(Y)\alpha(Z) = \beta(Y)\beta(Z) = 0 \qquad (3.5b)$$

Proof Equating (3.2) gives

$$\overset{\circ}{R}\text{ic}(Y, Z) - \text{Ric}(\bar{Y}, \bar{Z}) = [\text{Ric}(Y, Z) - \text{Ric}(\bar{Y}, \bar{Z})] - (n-2)[\beta(Y)\alpha(Z) + \alpha(Y)\beta(Z)]$$

which shows that $\overset{\circ}{R}$ ic hybrid iff Ric is hybrid, and (3.5a) is satisfied. Putting FY for Y in (3.5) a and using $\beta(X) = -\alpha(FX)$, we get (3.5b)

3.2 Theorem In a l.c.k. manifold $M^n(F, g, \alpha)$ $(n \neq 2$ the relation

$$\overset{\circ}{R}(X, Y, Z, W) = R(FX, FY, FZ, FW) \qquad (3.6)$$

Holds well if and only if (3.5) is satisfied.

Proof from (3.1)b), we get

$$\overset{\circ}{R}(X, Y, Z, W) - \overset{\circ}{R}(FX, FY, FZ, FW) = R(X, Y, Z, W) - R(FX, FY, FZ, FW) - g(Y, W)[\alpha(X)\beta(Z) + \beta(X)\alpha(Z)] + g(Y, Z)[\alpha(X)\beta(W) + \beta(X)\alpha(W)]$$

$+ g(X, W) [\alpha (Y) \beta (Z) + \beta (Y) \alpha (Z)]$
$- g(X, Z) [\alpha (Y) \beta (W) + \alpha (W) \beta (Y)]$
$- 'F(Y, W) [\alpha (X) \alpha (Z) - \beta (X) \beta (Z)]$
$+ 'F(Y, Z) [\alpha (X) \alpha (W) - \beta (X) \beta (W)]$
$+ 'F(X, W) [\alpha (Y) \alpha (Z) - \beta (Y) \beta (Z)]$
$- 'F(X, Z) [\alpha (Y) \alpha (W) - \beta (Y) \beta (W)]$

With the help of theorem (3.1) and Lemma (3.1) we proves

3.3 Proposition In a l.c.k. manifold M^n (F, g, α), the curvature tensor of the quarter symmetric metric connection $\overset{\circ}{\nabla}$ satisfies the following properties.

$$\overset{\circ}{R}(X, Y, z, \alpha) + \overset{\circ}{R}(Y, Z, X, \alpha) + \overset{\circ}{R}(Z, X, Y, \alpha) \tag{3.7a}$$

$$= 2 [\alpha (X) 'F (Y, Z) + \alpha (Y) 'F (Z, X) + \alpha (Z) 'F (X, Y)]||\alpha||^2$$

$$\overset{\circ}{R}(X, Y, Z, \beta) + \overset{\circ}{R}(Y, Z, X, \beta) + \overset{\circ}{R}(Z, X, Y, \beta) = 0 \tag{3.7b}$$

$$C^3_1 \overset{\circ}{R} = 0 \tag{3.7c}$$

Proof With the help of (3.1b) and the Bianchi identity, we get (3.7a) and (3.7b). Contracting (3.1b) with respect to Z we get (3.7c).

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